Investigating the Relationship between Efficiency and Criticality: Some Experimental Findings

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SHORT SUMMARY

Resilience is a complex term, bearing multiple definitions and resilience-related metrics. Two of the most common metrics are efficiency and criticality. Efficiency examines resilience from a topological aspect and considers the shortest paths between the nodes, whereas criticality considers transportation-related variables such as travel demand. In the present research, the interrelation between efficiency and criticality is examined through a series of simulations on the Athens testbed. For the quantification of efficiency and criticality, an iterative approach is used where, per iteration, one link of the network was removed. The outputs of the experiments are then statistically analyzed. Findings reveal that the relation between efficiency and criticality hinders some complex dynamics that are approached by incorporating traffic flow in the investigation of their relation. As a result, a polynomial relationship between the ratio of criticality and efficiency, and traffic flow of each link per functional class of the links is developed.

Keywords: Criticality, Efficiency, Resilience.

1. INTRODUCTION

Various studies concerning resilience can be found in the literature. All studies aim to examine resilience under different perspectives, such as the event considered in each of them, as well as the extent of the network that is examined (Wan et al., 2018), (Zhou, Wang, and Yang, 2019). A variety of resilience-related metrics that can be found in the literature, namely adaptability, preparedness, recoverability, vulnerability, and criticality. These metrics can be specified in different scales, ranging from a link level (components-based resilience analysis) to network level (network-based resilience analysis) (Wan et al., 2018), (Zhou, Wang, and Yang, 2019), (Sun, Bocchini, and Davison, 2020), (Leobons, Gouvêa Campos, and de Mello Bandeira, 2019). They may also overlap (e.g., robustness-vulnerability) or act complementary to each other (e.g., redundancy vs criticality). Nevertheless, the research on the degree to which these measures are correlated or can be further associated in a functional relationship to depict a much more complete understanding of the resilience of a transport network is still at an early stage, especially in cases of link level failures, where both the structural properties of the network as well as the route choice and feasibility can be affected.

A few attempts have been made for the correlation of criticality and efficiency to be further examined. The first attempt was made in (Nagurney and Qiang, 2008), where the concept of criticality is introduced, by incorporating traffic flow and the generalized cost in a purely topological metric. In this research, as far as the correlation of criticality and efficiency is concerned, it is only mentioned that said metrics are equal if, and only if, the demand of the network is so low that the results of the Path Assignment algorithm are the same as a shortest path approach. Another research which aimed to examine the relation between topological and flow/demand-related resilience measures is (Almotahari and Yazici, 2020). In this research, authors examined the relation between various metrics, however certain functions which permeate the relation between the metrics examined in this paper are not provided. In both researches mentioned, there is no further categorization of the links of the network and, therefore, of the relation between criticality and vulnerability according to the functional class of the links.

Present research aims to provide a methodology for evaluating the resilience of a complex urban transportation network under the emergence of link level failures (based on their functional class) by jointly examining the efficiency, vulnerability, and criticality, and modeling their interrelationships using macroscopic simulation and further statistical modeling. These interrelationships are developed and evaluated in the Athens city-center urban transportation network using macroscopic simulation and statistical modeling. Even though efficiency and criticality examine different aspects of the networks in terms of their resilience, the proposed framework aims to bridge the gap between them and provide further insight in the possible relation between them. The remainder of the paper is structured as follows; In Section 2, the methodological approach followed is presented. In Section 3 the results of the experiments are discussed and the existence or absence of any correlation among vulnerability and criticality is investigated. Finally, in Section 4, the main findings of this research are presented and discussed, as well as some future research initiatives are established.

2. METHODOLOGY

Problem formulation

The focus of the present research is to understand the importance of each link of the road network in its resilience, based on both its topological features, its participation in the distribution of traffic flow in the urban transportation network, and its functional class. To achieve this, a four-level approach is followed in the present research, as illustrated below (Figure 1).

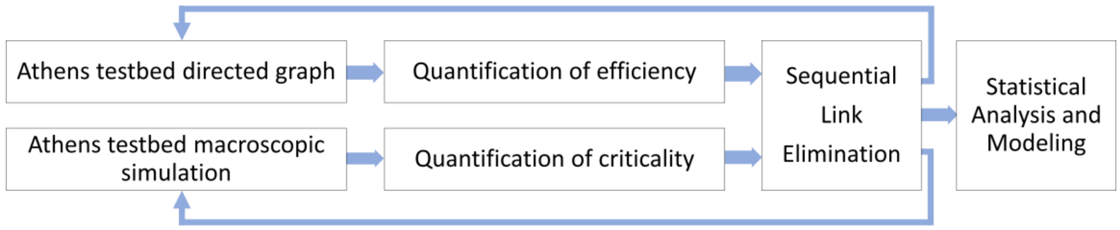

Figure 1: Methodological approach.

In the present research, efficiency and criticality are further examined. For the investigation of these metrics the transportation network needs to be represented as a graph (Brandes, 2001), (Latora and Marchiori, 2001), (Nagurney and Qiang, 2008), (Mattsson and Jenelius, 2015), (Vogiatzis and Pardalos, 2015), (Guze, 2019). Therefore, it is important to define a directed graph $G(N, L)$ with N nodes and L links. Efficiency is defined in the relevant literature as the average across all node pairs of the reciprocals of the node pair distances (Brandes, 2001), (Latora and Marchiori, 2001). Efficiency can be quantified as described in Equation 1.

$$E = \frac{1}{N(N-1)} \sum_{i \neq j \in N} \frac{1}{d_{i,j}}, \quad (1)$$

where, N is the number of nodes and $d_{i,j}$ is the shortest distance between nodes i and j. Equation 1 associates the extraction of an element of graph G, either a node or an edge (despite the fact that this approach is mostly node-oriented), with the performance of the network in terms of its shortest paths. It is still a topological metric, but the extraction of an element of G can lead in variations of E.

Concerning the quantification of criticality (Nagurney & Qiang, 2008), a User Equilibrium Assignment is calculated for the transportation network and the Unified Network Performance Measure as described in the following equation:

$$\varepsilon = \varepsilon(G,d) = \frac{\sum_{w \in W} \frac{d_w}{\lambda_w}}{n_w}, \quad (2)$$

where d_w is the demand for the w pair of the Origin-Destination matrix, λ_w is the generalized cost (mostly the travel time) for the w pair of the Origin-Destination matrix, and n_w is the total number of Origin-Destination pairs.

Then, each of the transportation network's links are sequentially removed and then the User Equilibrium Assignment and the Unified Network Performance Measure are recalculated for the new state. The Network Component Importance which further examines the importance of each of the links that have been removed from the network is calculated by making use of Equation 3.

$$l(g) = \frac{\Delta\varepsilon}{\varepsilon} = \frac{\varepsilon(G,d) - \varepsilon(G-g,d)}{\varepsilon(G,d)} \quad (3)$$

In equation 3, $\varepsilon(G-g,d)$ is the value of the Unified Network Performance Measure after the removal of a link. Criticality is a metric which is associated only with the links of the network, and it is considered as a demand-based and demand-oriented metric which reflects the impact that the removal of a link has in the network in terms of its performance and its capability to serve demand.

Athens testbed

The use case for the experiments is the inner-ring urban transportation network of Athens, Greece. The network consists of 1293 nodes/ intersections and 2572 edges/ links. Said network is used both as a directed graph and as a macroscopic transportation network. The directed graph of the network is designed and used in a graph-oriented programming package and the macroscopic transportation network is used in Aimsun Next traffic simulator. Figure 2 below illustrates the inner-ring urban transportation network of the city of Athens used in the present research.

Figure 2: Athens testbed.

3. RESULTS AND DISCUSSION

In the present section, the results occurred from the experiments in the Transportation Network Simulation Platform are presented and analyzed. The resilience metrics statistical analysis and modeling include the preliminary statistical analysis of the metrics and the statistical analysis made for fitting a function that best describes the relationship between criticality and efficiency.

Preliminary statistical analysis for efficiency and criticality

Concerning efficiency, the results are presented in Figure 3. Figure 3 illustrates the efficiency metric for the removal of each of the links of graph G. Value E_i is directly associated with the link removed from the graph per iteration. It is worth noting that the values of efficiency per iteration E_i are close to the value of E_0 (value of the efficiency metric without the removal of any structural element of graph G); thus, efficiency is low scattered (Table 1) The fact that efficiency is low scattered can be justified from the fact that in the framework of the present research only the removal of sole links have been investigated and not the removal of other structural elements like nodes or paths (set of links).

Figure 3: Efficiency after the removal of each link from graph G.

In terms of centrality, the results are presented in Figure 4. Figure 4 illustrates the Network Component Importance index $l(g)$ of each link of the network.

Figure 4: Network Component Importance index for the testbed.

Table 1 presents the preliminary statistical analysis of $l(g)$. It is of interest to note that, in some cases, vulnerability takes negative values. This is known as the Braess paradox (Braess, 1968), (Braess, Nagurney, and Wakolbinger, 2005) and it proves that the removal of links from the graph/network may improve its operational performance.

Table 1: Preliminary statistical analysis for efficiency and criticality

Statistical metrics	Efficiency	Network Component Importance Index
Mean	0.06790	0.01600
Standard Error	0.00000	0.00050
Median	0.06791	0.00839
Standard Deviation	0.00003	0.02336
Sample Variance	0.00000	0.00055
Kurtosis	41.05449	8.49439
Skewness	-4.53810	2.72759
Minimum	0.06747	-0.03987
Maximum	0.06792	0.21563
Confidence Level (95.0%)	0.00000	0.00097

Correlation among efficiency and criticality

Having the latter in mind, the correlation among these metrics is to be examined in the present chapter and more light is to be shed concerning the existence or no of any correlation among efficiency and criticality. It is worth noting here that efficiency and criticality are two different aspects of the same coin. Criticality, and to be more specific the Network Component Importance index, is based on the efficiency index and incorporates in its definition some transportation-related characteristics (traffic demand). Therefore, there lies the existence of any possible correlation among these metrics.

Concerning the present research paper, an attempt is made for the efficiency and Network Component Importance indices to be correlated with the flow of a certain link of the urban transportation network, as well as with the functional class of the link. First, let us define the Link Importance indicator (LI). LI is defined as the ratio of the Network Component Importance Index and the efficiency index (Equation 4).

$$LI = \frac{I(g)}{E} \qquad (4)$$

In the present research, efficiency is a low scattered metric and the Network Component Importance index is both more scattered and with higher values. Therefore, as the magnitude of LI increases, the importance of the removed link/element of the graph is higher and, thus, the network is less resilient.

Another term of importance is flow. Flow is deemed as important because it provides information about the traffic characteristics of the link, and it is highly correlated with the results of the Path Assignment algorithms because it differentiates the path choice per iteration of the Path Assignment algorithm (acts as a weight in the links of the network). In Figure 5, LI is illustrated with the flow of each link.

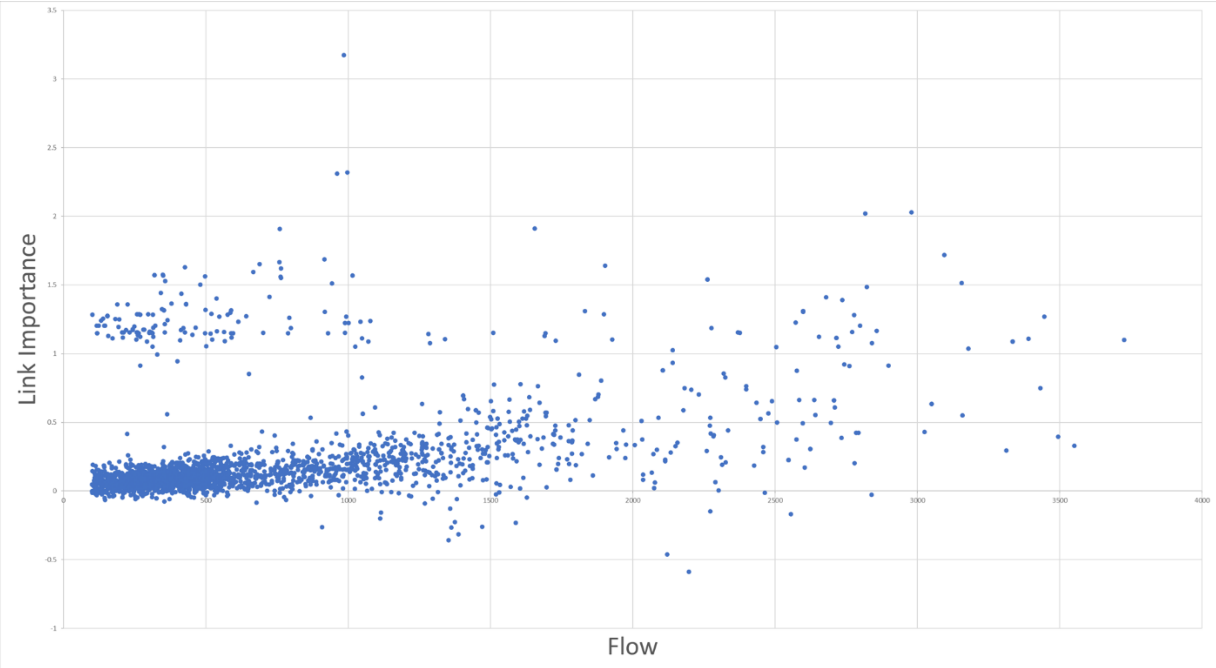

Figure 5: LI versus link traffic volume (veh/h) of each link.

In Figure 6, LI and flow per functional class of the network is illustrated. It is worth mentioning that in Athens testbed three discrete functional classes are defined for the links of the network. The links of the transport network are therefore categorized as trunk, primary, and residential.

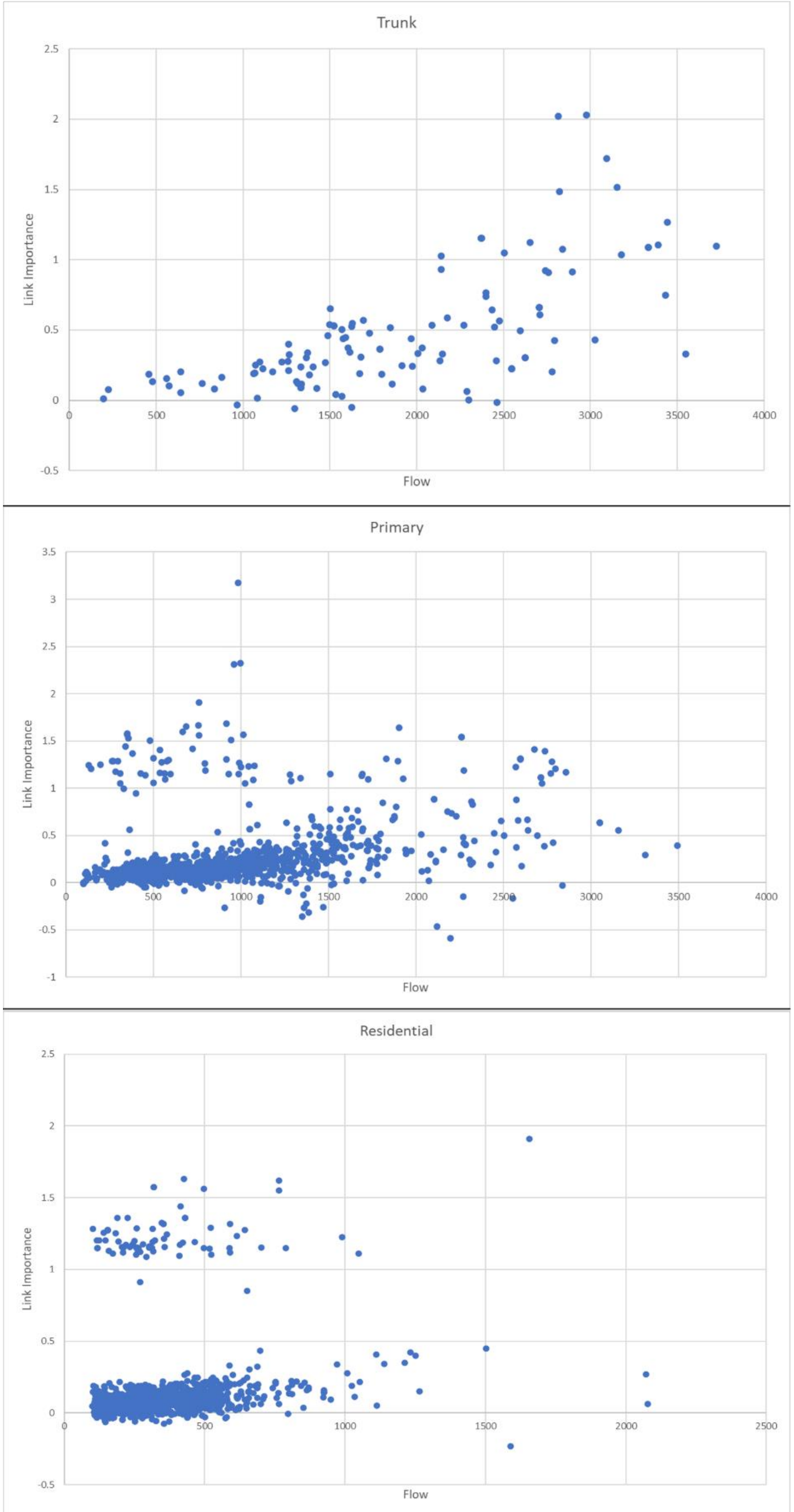

Figure 6: LI versus link traffic volume (veh/h) of each link per its functional class.

For links that are categorized as trunk, no further clustering is needed for the examination of the relation between LI and flow. However, this is not the case with primary and residential links. For each of these functional classes, two (2) clusters appear. Said clusters are determined based on the values of LI and flow. Considering LI, the clustering occurs for LI<1 and for LI>1. LI = 1 is of importance because when LI equals to one (1) it means that the Path Assignment algorithm gives the same result as a simple shortest path algorithm, thus the demand of the network is very low. However, this is not happening in urban transportation networks. Concerning flow, the clustering occurs for flow<1000 and flow>1000. Flow = 1000 is important because for values of flow greater than 1000 congestion starts to appear in the links of the network.
The functions that best express the relation of LI versus flow, per the functional class of the links of the network, are illustrated in Figure 7.

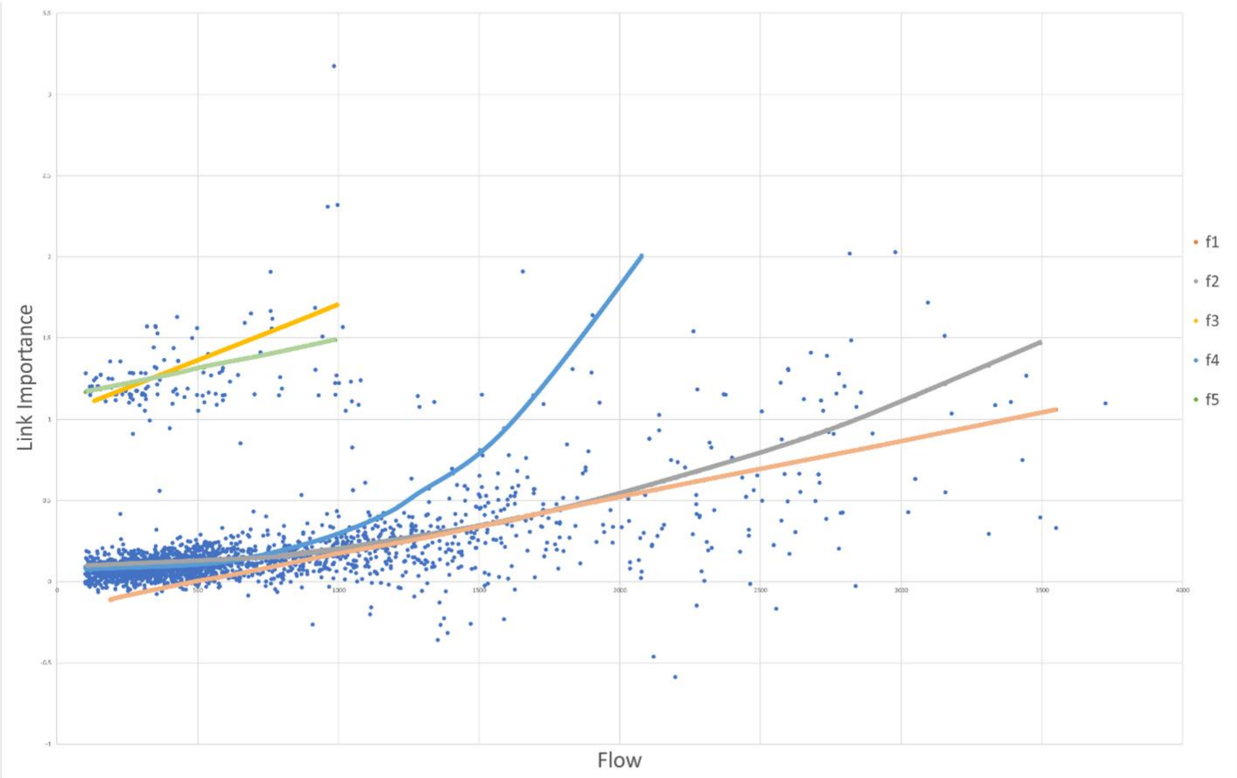

Figure 7: Functions that permeate LI versus link traffic volume (veh/h) of each link per its functional class.

Five (5) functions are provided for the functional classes of the links of Athens inner-ring urban transportation network. Said functions are presented in Equations 5–9. A statistical analysis of these functions is presented in Table 2.

$$LI = 0.4538 + 0.0003 * (flow - 1809.7889) \quad (5)$$

$$LI = 0.1886 + 0.1126 * \left(\frac{flow}{1000}\right)^2 - 0.8434 \quad (6)$$

$$LI = 1.4058 + 0.0007 * (flow - 559.4820) \quad (7)$$

$$LI = 0.93 + 0.2148 * \left(\frac{flow}{1000}\right)^2 - 0.04524 \quad (8)$$

$$LI = 1.2315 + 0.0004 * (flow - 277.3324) \quad (9)$$

Table 2: Goodness of fit of Equations 5-9

Statistical metrics	Equation 5	Equation 6	Equation 7	Equation 8	Equation 9
Residual df	93	782	40	819	56
Scale parameter	0.0827	0.0261	0.1130	0.0065	0.01437
Deviance	7.6895	20.4195	4.5188	5.3048	0.8049
Pearson	7.6895	20.4195	4.5188	5.3048	0.8049
(1/df) Deviance	0.08268	0.0261	0.1130	0.0065	0.0144
(1/df) Pearson	0.0827	0.0261	0.1130	0.0065	0.0144
AIC	0.3660	-0.8049	0.7037	-2.1979	-1.3706
Log likelihood	-15.3832	317.5354	-12.7774	906.3552	41.7472
BIC	-415.8210	-5191.1480	-144.9880	-5491.6110	-226.5799

As mentioned above, each of the equations corresponds to one of the clusters defined through Figures 6 and 7. Equation 5 corresponds to trunk links with LI<1. No restrictions concerning flow exist for links that are characterized as trunks. Equations 11 and 12 refer to links that are characterized as primary. Equation 6 refers to LI<1 with no flow limitations, and Equation 7 refers to LI>1 and flow<1000. The last two (2) equations (Equations 8 and 9) refer to residential links. To be more specific, Equation 8 corresponds to LI<1 and Equation 9 to LI>1 and flow<1000.

4. CONCLUSIONS

This paper aimed to investigate the relationship between efficiency, vulnerability, and criticality for a real-life urban transportation network. For this purpose, a series of experiments were planned and executed in the Athens testbed that consists of both the macroscopic transportation network and the directed graph of Athens city center urban transportation network. The experiments where then executed, and the concept was that per each iteration one link was removed from the network and then the metrics were calculated.

The results collected are then statistically analyzed. The analysis shows that efficiency is both lower scattered and that its values are significantly lower compared to criticality. It is important that the Braess Paradox was present during the preliminary analysis. Then, in order to identify the complex dynamics that permeate said metrics, flow has been incorporated in the process of investigating the relation between efficiency and criticality. The joint analysis of criticality and efficiency point to a polynomial relationship between the ratio of the Network Component Importance index and efficiency index (Link Importance indicator), and traffic flow of each link of the network. Said analysis also highlighted the possibility to further examine the relationship between said metrics by considering the functional class of the links of the transportation network under examination.

Future research could focus on the application of the methodological approach both in other structural elements of the network, as well as in other urban transportation networks. Concerning the application of the methodological approach in other structural elements of the network, central nodes and frequently used paths shall be removed from the network. This can lead to a more realistic approach in terms of a disturbance on an urban transportation network. The application of the present methodological framework on other networks will lead to an examination of the predictive values of the functions proposed in this research. In addition, effort should be put into the investigation of other possible variable which may lead to more significant results. Finally, it would be interesting to examine other resilience-related metrics and further develop the proposed functions for a Generic Resilience Index to be able to be proposed.

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