Novel metamodel-based optimization approach for game theoretical analyses in mobility ecosystems and its demonstration via a case study about toll competition

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SHORT SUMMARY

First, a concept of novel metamodel-based optimization, in which a transport economics inspired model acts as a metamodel over an underlying set-up of directly interfaced transport models, is discussed. It is hypothesized that this approach allows to solve transportation problems, at a feasible computational cost, with due consideration to relevant interactions between different stakeholders/(sub)systems. Then, a case study concerning toll competition between a city and its neighboring rural municipalities is developed as a proof of concept. The metamodel for this case study involves the two players optimizing their objectives based on a schematic network and simplified cost and demand functions, whereas the underlying set-up is a Static Traffic Assignment over the physical network with physical origin-destination demand. Preliminary results suggest that this approach offers the lower computational effort required for the analysis of scenarios involving significant interactions between the stakeholders/(sub)systems and when such interactions are not important, it can be a less accurate but faster alternative to contemporary approaches.

Keywords: Game theoretical interactions in mobility, Pricing and capacity optimization, Transport economics and policy, Metamodel based optimization, Static Traffic Assignment

1. INTRODUCTION

Road pricing has been an interesting topic for transport engineers, transport economists, policy makers and other stakeholders. The idea of toll optimization truly reveals the inherently permeable boundaries of the transportation **sub**systems as well as the entire transportation system itself. For a comprehensive analysis of the impacts of a particular tolling scheme, one should account not only for the interactions between the transportation **sub**systems/stakeholders like **travelers**: having distributed decisions concerning when, where, via which mode and route to travel, **mobility service providers**: public or private with decisions concerning their pricing and service level, and **local network operators**: decisions about traffic restrictions, infrastructure investments, the toll itself etc. but also the interactions with other systems e.g., network operators from neighboring governments, fiscal system, housing market, urban design/ land-use etc. Such comprehensiveness is vital for analyzing both short- and long-term impacts of a proposed change, like a tolling scheme, in any domain of the mobility ecosystem.
Thus, for a particular tolling scheme, the challenge for modelers is to 1) identify relevant (sub)systems/stakeholders and their interactions. 2) develop models that take into account these interactions in a more elaborate way than fixed inputs or only unidirectional influences. 3) develop mechanisms for computation of consistent impacts on all interrelated (sub)systems/stakeholders. Due to the required complexity and computational power, such solutions are non-existent to the best knowledge of the authors. Nevertheless, there have been attempts to approach a solution based on three major approaches:

1. **All-encompassing digital twins of all interacting subsystems:** These are highly detailed and disaggregate models in which every relevant player is modelled at a micro

level, e.g., each person is modelled individually, each vehicle is propagated through the network discretely etc. The most renowned example of this approach is the Multi-Agent Transport Simulation (MATSim) framework (Horni et al., 2016). Such frameworks have definite advantages as they lead to a natural way of reaching consistency and can provide information at a very disaggregate level to the stakeholders allowing analysis from equity perspective as well in addition to the efficiency perspective. However, a substantial effort is required for developing, extending, calibrating, maintaining and even interpreting these models. Further, the computational effort required for game-theoretical analyses involving multiple stakeholders with different objectives (which is typical of transport and its interconnected systems) can be prohibitively expensive. For reference, reaching user equilibrium for just travelers alone, with a fixed demand and only the choice of adapting their routes, took 2 hours and 13 minutes for a model with 899 zones, 8136 nodes and 20670 links (Fourie, 2010). The required computation power would increase exponentially for each additional choice/optimization variable and for each additional objective-optimizing stakeholder/(sub)system who enters the competition/game (Paul Ortmann, 2022). A notable example is MATSim4UrbanSim (Nicolai, 2013). It is an integration of MATSim and UrbanSim (a simulation system for supporting planning and analysis of urban development). Even though, the objective was not to attain equilibrium between the two systems, but rather to study the dynamic disequilibrium path, the case studies could only be performed on 1-10% random samples in order to keep the computational effort feasible. In scenarios with more stakeholders and faster dynamics, where analyzing equilibrium point is much more important, the computational load can only be higher.

2. **Tailor-made simplified conceptual models:** On the other end of the spectrum lie the simplified conceptual models which are often used by transport economists (De Borger and Proost, 2021) (Proost and Westin, 2014) (De Borger et al., 2007). These are comparatively easer to develop, calibrate, maintain, and interpret and often even analytical results can be obtained. It is also quite easy to tailor them and focus only on the relevant stakeholders/(sub)systems and their interactions for a particular problem. Even complex optimization problems involving objective maximization by different levels of competing governments, in both Stackelberg and Cournot competitions, can be analytically solved using this approach. However, they involve extensive simplifications of the underlying subsystems, e.g., by using schematic networks, by using linear relations which are usually only locally valid etc. and therefore, extrapolations far beyond reference points are not trust-worthy. Additionally, they only provide highly aggregate and schematic results which is, usually, not sufficient for aiding actual decisions.
3. **Tailor-made directly interfaced traditional mono-disciplinary models:** Traditionally, a toll optimization problem is solved by using a bi-level optimization framework in which Static Traffic Assignment (STA) becomes the inner loop and then toll is altered in the outer loop (Ekström et al., 2014). Nowadays, to account for interactions with other (sub)systems and to endogenize the hitherto exogenous inputs, dedicated interfaces between relevant mono-disciplinary transportation models are being built within the inner loop. The connection between activity-based demand model and STA in SPM-VLA: Strategisch Personen Model Vlaanderen and the connection between UrbanSim and PTV Visum (Fourie, 2010) can be considered as examples of this approach. This approach provides flexibility of interfacing only the relevant models for a specific problem and allows to capture less-trivial relationships (e.g., non-linear, non-convex) while producing more disaggregate outputs suitable for aiding actual decisions. However, despite their faster execution speed, solving complex optimization problems involving several stakeholders can still be extremely sluggish because: a) As the number (or level of detail) of models increase, attaining consistency via circular (bi-directional) dedicated interfaces

for a particular iteration becomes computationally quite expensive. b) Due to the possibility of only marginal steps in the optimization variables, the risk of getting stuck in local stationary points is quite high.

It can be appreciated that none of these three approaches offers the combination of scalability, detail, and flexibility required by contemporary transportations problems.
On a separate but related note, it has been shown that a simpler special purpose meta-model can guide an underlying micro traffic simulator to an optimal toll much more efficiently than direct bi-level optimization or even optimization with general purpose meta models (Osorio and Atasoy, 2021). Inspired from the proven feasibility of this concept and the complementary characteristics of the 2nd and 3rd approaches mentioned above, we aim to use a transport economics inspired conceptual model as a metamodel to find an optimal toll for an underlying set-up of directly interfaced traditional transport models. In this way, the underlying set-up only has the computational load related to achieving consistency between the directly interfaced models, whereas the computational load for optimization lies completely at the metamodel level. At every iteration, the underlying set-up is used to (re-)calibrate the metamodel which includes a simpler and more aggregate version of all the relevant stakeholders/(sub)systems. Toll optimization is performed for the meta-model and the optimal tolls are transmitted to the underlying set-up. At the new tolls, the underlying set-up is evaluated again, and the meta-model is recalibrated at the new point. This sequence is repeated until a certain level of convergence is achieved in the optimal toll values. **The objective** of this conference paper is to present the development and results of a proof of this concept. For this initial exercise, the underlying set-up is an STA with elastic demand for a fictional city (inspired from the Flemish city of Aarschot) and the meta-model focusses on finding game-theoretical equilibrium for the scenario in which the city municipality charges a cordon toll and the neighboring rural municipalities charge a per km toll. At this stage, the objective is not to find the most accurate optimal tolls but rather to determine 1) the accuracy and 2) the speed of execution of this framework. This will allow to have a perspective on 1) how much can such a metamodel-based optimization be trusted and 2) till what extent will it be computationally feasible to add other models e.g., demand model, mobility-as-a-service model, land-use etc. in the underlying set-up. This can verify our hypothesis that this approach allows to have a comprehensive analysis of a particular tolling scheme.

2. METHODOLOGY

The fictional problem considered for the proof of concept is as follows: the city municipality is looking to impose a non-discriminatory cordon toll for all private motor vehicles with the intention of curbing the use of city infrastructure by transit traffic. With the intention to avoid the rerouting of the transit traffic to local infrastructure of neighboring villages, the rural municipalities come together to charge a per km toll for all private motor vehicles.
The framework has three main parts: 1) The underlying STA 2) the metamodel 3) the interface between underlying set-up and the metamodel. **Figure 1** shows a block diagram representing a basic instance of this framework.

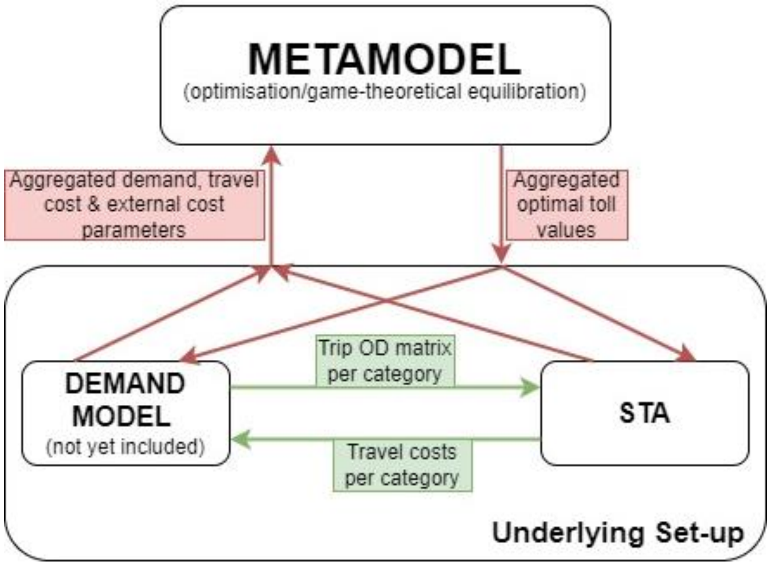

Figure 1

Underlying Set-up

The underlying set-up for this exercise consists of only the STA for a fictional city. The network is extracted from OpenStreetMap using OSMnx (Boeing, 2017) with Aarschot as center and a buffer of 20 kms. The network is shown in **Figure 2**. It has 56585 nodes and 186802 links. The OD matrix and the zoning have been derived from demand data given by the Flemish department of Mobiliteit en Openbare Werken (MOW). The zoning is shown in **Figure 3**. There are 215 zones, and the OD matrix is 215 by 215. For the traffic assignment, both deterministic and stochastic assignment methods are used. The assignment is performed in PTV Visum and in an in-house assignment tool called dyntapy (Ortmann and Tampère, 2022).

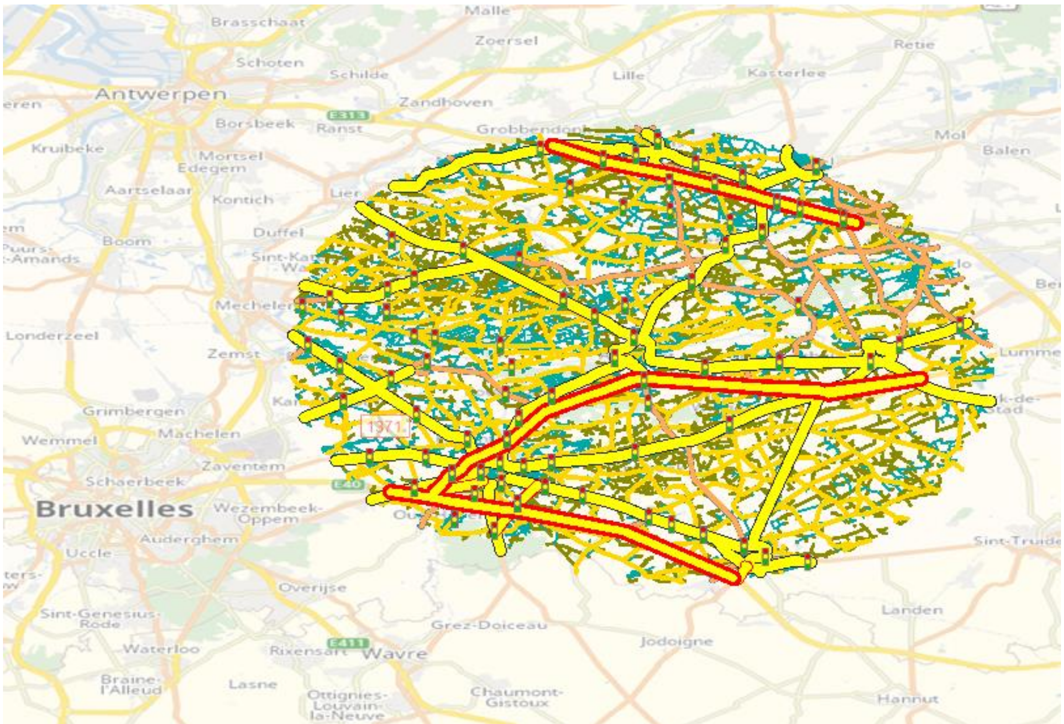

Figure 2

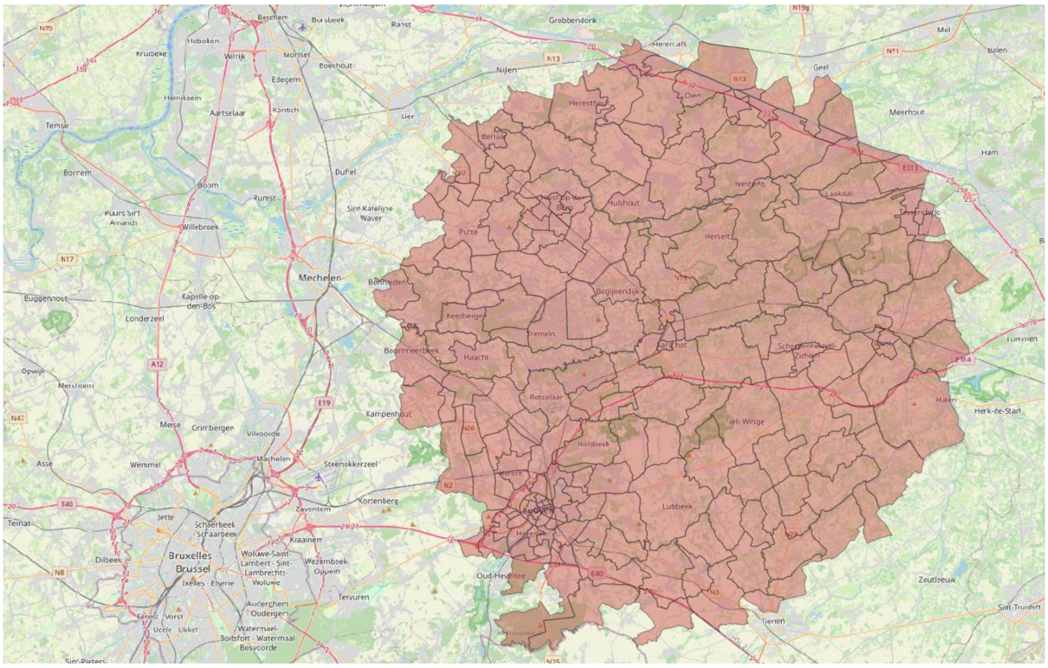

Figure 3

Metamodel

The metamodel is inspired from transport economics models (De Borger and Proost, 2021). The schematic network is shown in **Figure 4** with node and link labels shown in blue and green respectively. The schematic OD pairs are listed with explanation in Table 1. The links 21 and 22 represent the links through the city and are mentioned as ‘r’ in Table 1, the links 31 and 32 represent the ring of the city and are mentioned as ‘R’ in Table 1 and the links 6, 41, 42, 43 and 7 are the links passing through neighboring villages and are mentioned as ‘M’ in **Table 1**.

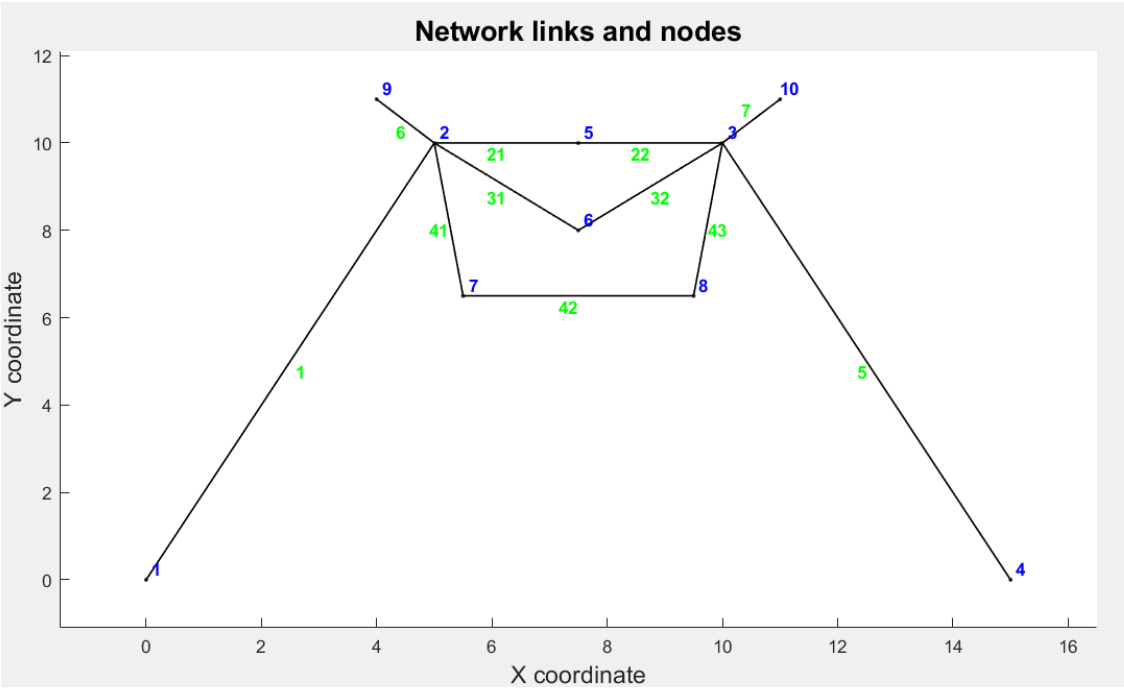

Figure 4

S.No.	OD pair (from_to)	Available paths	Explanation (from_to)
1	1_4	M,R,r	external_external
2	1_7	M	external_closeneighborhood
3	1_8	M,R,r	external_oppneighborhood
4	9_4	M,R,r	oppneighborhood_external
5	10_4	M	closeneighborhood_external
6	1_5	r	external_city
7	5_4	r	city_external
8	9_10	M,R,r	neighborhood_neighborhood (opp.)
9	9_7	M	neighborhood_neighborhood (close)
10	8_10	M	neighborhood_neighborhood (close)
11	9_5	r	neighborhood_city
12	5_10	r	city_neighborhood

Table 1

The city municipality and the rural municipalities are modeled as two players in a game theoretical competition with each of them having partially conflicting objective functions. Each player decides its toll variable considering its own optimization problem and taking the other's toll as fixed. This cycle is repeated until they are in a game-theoretical (Nash) equilibrium. The equilibrium tolls are then transmitted to the underlying set-up.

Calibration Interface

At the beginning of each new metamodel optimization routine, the metamodel is (re-)calibrated by using the underlying set-up at equilibrium/optimal tolls from previous metamodel optimization routine. This is done by the interface (represented by red lines in **Figure 1**). Main tasks of the interface are to calibrate the schematic OD matrix (with elasticities) of the meta-model using the detailed physical OD matrix (with assumed elasticities), calibrate generalized cost parameters of the schematic links using the actual costs of the physical links and calibrate external cost parameters that go into the objective functions of the two players using the physical traffic flow values.

3. RESULTS AND DISCUSSION

Firstly, results are obtained for the case when the two payers i.e., the two municipalities act as a single player controlling toll variables of both (i.e., toll_radial, toll_ring and toll_rural) and optimize the sum of individual objective functions. Presently, the interface between the metamodel and the underlying set-up is in development. The standalone results of the underlying set-up of STA with LUCE equilibrium assignment are shown in **Figure 5** and **Figure 6**, and the standalone results of the metamodel optimization with initial parameters are shown in **Figure 7**. It is to be noted that a comparison of **Figure 5** (or Figure 6) and **Figure 7** is not meaningful as the former represents untolled base scenario, whereas the latter represents situation after optimal tolls have been implemented. Moreover, the metamodel parameters have not yet been calibrated using the underlying set-up (via the interface). Nevertheless, it can be appreciated that in the optimal case, a higher percentage of traffic routes via the ring as compared to the radial road (see **Figure 7**). This is not the case in base scenario (see **Figure 6**) which justifies why a city municipality will contemplate tolling in this situation.

Once the interface is fully developed, the next objective is to compare the results of the metamodel-based optimization with those obtained when the same problem is solved via a direct bi-

level single player optimization (loop inside a loop) and to quantify the accuracy and execution speed of the new framework. Preliminary results hint that the new framework will give less accurate results but at a much lower computational effort. For reference, the STA and the metamodel optimization took 136 seconds and 2 seconds respectively on a computer with Intel(R) Core(TM) i7-9750H CPU @ 2.60GHz and 32 GB RAM. This lower computation effort will allow to add additional (sub)systems/stakeholders which will, ultimately, allow to solve more realistic problems without assumptions of fixed inputs.
Then, the next objective is to extend the framework to the cases 1) when the two players are in Stackelberg competition and 2) when the two players are in a Nash-Cournot competition.

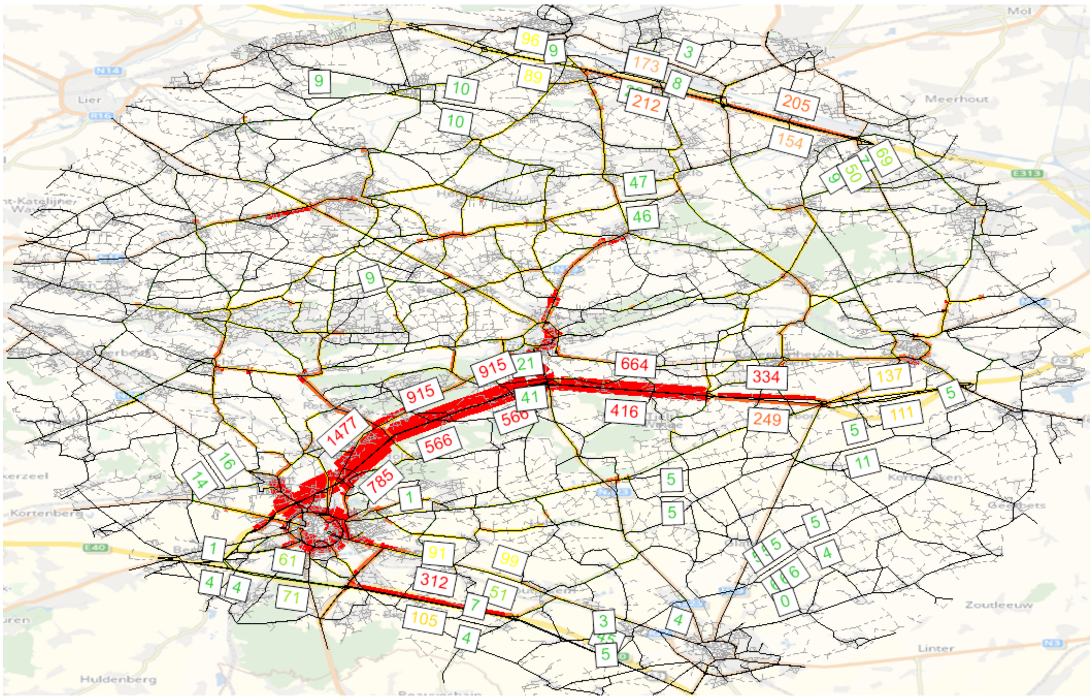

Figure 5: STA traffic flows in the overall network (base situation)

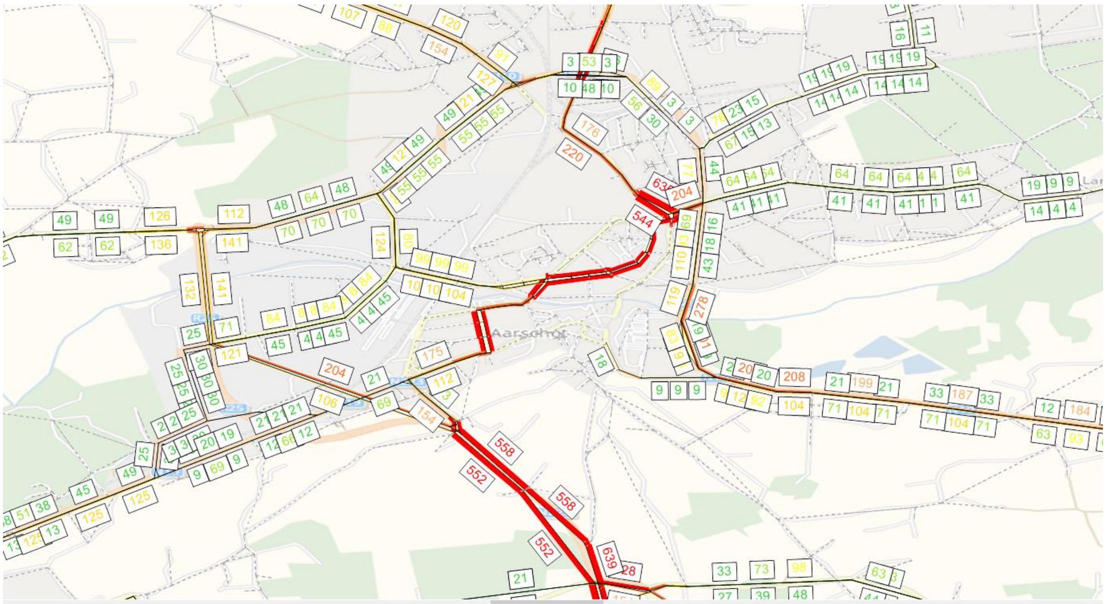

Figure 6: STA traffic flows (zoomed in on Aarschot)

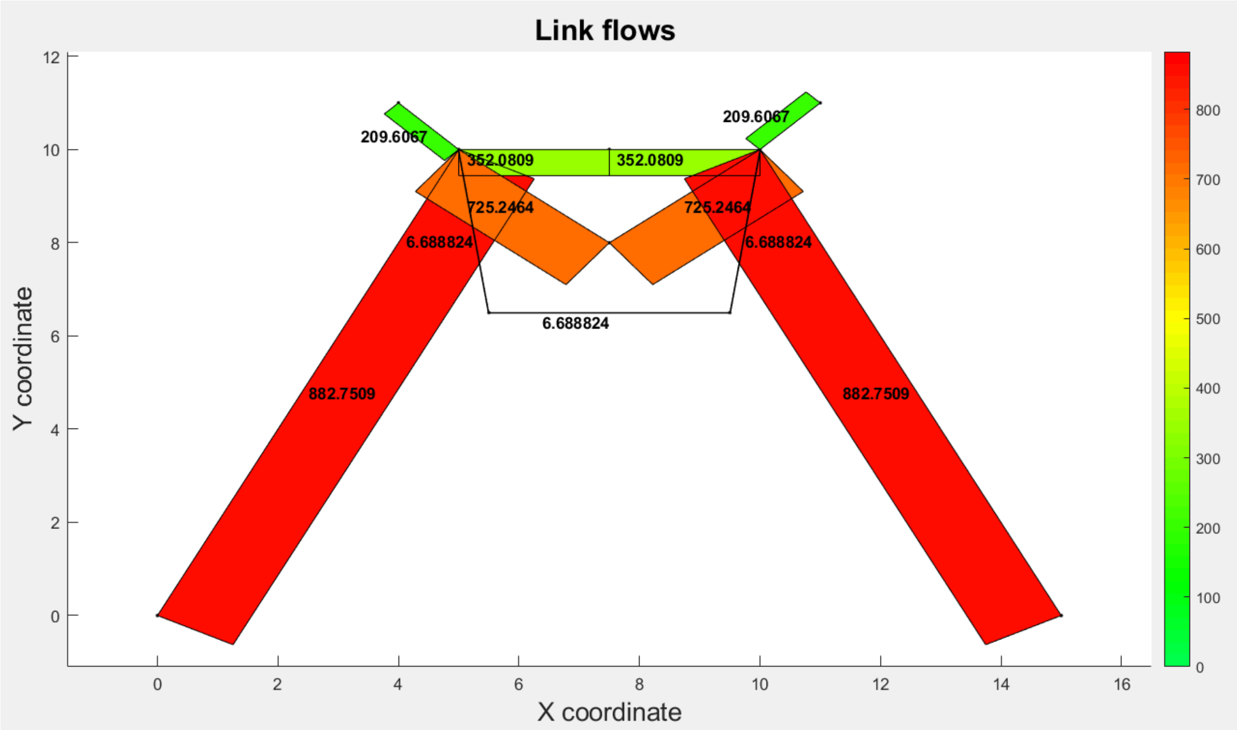

Figure 7: Metamodel traffic flows with optimal tolls

4. CONCLUSIONS

In this paper, a new metamodel-based optimization approach is discussed in which a transport economics inspired metamodel is used for optimizing the underlying directly interfaced traditional transport models. It is hypothesized that this approach allows to solve transportation problems, at a feasible computational cost, with due consideration to relevant interactions between different stakeholders/(sub)systems. A case study about a toll competition between a fictional city municipality and its neighboring rural municipalities, with only an STA as the underlying set-up, is developed and presented to serve as a proof of concept of this approach. Preliminary results suggests that the problem can be solved at a much lower computational cost but with less accuracy in terms of optimality of tolls. Thus, it serves as a motivation for developing the approach further by interfacing additional models in the underlying set-up.
The real application of this approach is in aiding decision-making related to scenarios where there are significant interactions between different stakeholders/(sub)systems. At the same time, in case of scenarios where such interactions are not relevant, this approach serves as a less accurate but faster alternative to contemporary transportation engineering approaches.

ACKNOWLEDGMENT

The first author of this paper is an FWO PhD Strategic Basic Research fellow, and this research is funded as part of the FWO Project titled “Novel metamodel-driven model interaction framework for multi-actor bi-level problems in mobility” (Project Number: 1S44922N).

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