# A MILP Framework to Solve System Optimum with Link MFD Functions

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## SHORT SUMMARY

This work aims to calculate the optimal routing strategy minimizing total travel time in a transportation network. We solve a mixed integer linear program (MILP) for the system-optimum dynamic traffic assignment with a link-based Macroscopic Fundamental Diagram (SO-MFD-DTA) and with multiple origins and destinations. Previous studies resort to cell transmission model and triangular fundamental diagram to describe traffic dynamics. Here, each link is associated with a single cell characterized by a more advanced cost-function: the link-MFD better describing the speed variation related to the network loading and traffic control impacts. Link MFDs are approximated by piecewise linear functions and adapted to the MILP framework by employing the convex combination method with special ordered sets of type 2 variables. Vehicle holding (VH) that originates from the linearization of traffic model is also addressed. Finally, on a synthetic network, we present the sensitivity analysis on the parameters of the model.

## 1. INTRODUCTION

Optimal routing strategies are a promising tool to improve transportation network management and alleviate congestion. Users usually make their routing decisions based on their self-interest trying to minimize their individual costs. This converges at a network level to the so-called user-equilibrium (Sheffi 1984), which is different from system optimum (Sheffi 1984), when a collective objective, such as total system travel time (TSTT), is minimized. How the efficiency of a system degrades due to the selfish behavior of its agents, also known as the price of anarchy (Roughgarden 2005), amounts to about 10% to 30% (Roughgarden and Tardos 2002) when considering TSTT.

A large body of literature solves the system optimum problem with static cost functions. It is certainly suitable for planning purposes but not accurate enough for real-time applications where congestion dynamics significantly influence travel times. Since real-time applications usually disqualify static approaches for system-wide optimality and traffic assignment, a dynamic optimization framework is then needed to determine the system optimum solution for a dynamic traffic assignment (DTA) problem.

Numerous models proposed in the literature for the DTA problem can be categorized into two main approaches: simulation-based, and analytical. Simulation-based approaches, also known as black-box procedures, can be implemented for realistic size networks. Despite their scalability, such approaches are very time-consuming and may fail to guarantee optimality. On the contrary, analytical approaches guarantee optimality by design. However, capturing the realistic properties of network roads, implications of traffic behaviors, and implementation to realistic size networks constitute the main challenges. Discrete-time analytical models have been widely used in the literature and are usually formulated as mathematical programming problems, such as linear programming (LP) and mixed integer linear programming (MILP) problems (e.g., Ziliaskopoulos

(2000), Aziz and Ukkusuri (2012), Long et al. (2018)). However, to the best of our knowledge, they are all constructed on the assumption that triangular fundamental diagrams (FD) (Newell 1993) are sufficient to represent traffic dynamics. This is certainly true at a given location when looking at equilibrium states, but at the link level, especially in an urban area, many factors such as the impacts of control actions, bottlenecks effect, link characteristics, or driving behaviors influence the shape of the fundamental curve. Many studies have now shown that more advanced functional shapes, also called Macroscopic Fundamental Diagrams (MFD) (Godfrey 1969; Daganzo 2007), better describe traffic behavior on urban corridors. The shape of the link-level MFDs, an example of which is depicted in Figure 1, however, is more complex than the classical FDs. Hence, triangular or trapezoidal approximations can no longer be assumed as good approximations. To characterize the link-level MFDs more precisely and compatible with the framework of LP, we approximate the link-level MFDs with piecewise linear (PWL) functions.

The traffic flow model is no different when replacing triangular FD by link MFD. In particular, exit flow at boundaries is equal to the minimum of the demand and the supply like in the cell transmission model. The minimum operator makes the optimization problem non-convex. To apply an LP framework, flow propagation formulation needs further transformations. We adopt the LP transformation originally proposed by Ziliaskopoulos (2000). This linearization, however, results in the vehicle holding (VH) problem for multiple OD networks as a common issue in many discrete-time SO-DTA models (e.g., Ziliaskopoulos (2000), Nie (2011), Aziz and Ukkusuri (2012)). VH refers to a situation where drivers are reluctant to move forward from upstream to downstream links even if there are vacant spaces in the downstream links. It comes from exit flows being upper-bounded but not tightly equal to the lesser value between demand and supply. To further address the VH problem we adapt the formulation proposed by Long et al. (2018) to non-triangular fundamental diagrams implemented in the framework proposed here. The remainder of this paper is organized as follows. Section 2 presents the methodological framework. Section 3 provides a case study, numerical results, and discussions.

## 2. METHODOLOGY

In this section, we present the methodological framework and mathematical formulation for modeling a link-based SO-MFD-DTA as a MILP. Table 1 describes all the notations used in the formulation.

**Table 1: Notations**

| Notation | Description |
|---|---|
| T | Set of discrete time intervals |
| $\Delta t$ | Time interval duration |
| $C_R$ | Set of origin links |
| $C_S$ | Set of destination links |
| $C$ | Set of links except for the destination links |
| $Q_i^t$ | The maximum flow that can get into or out of link $i$ at time interval $t$ |
| $l_i$ | Length of link $i$ |
| $d^{O,D,t}$ | Demand from origin O to destination D at interval $t$ |
| $x_i^{O,D,t}$ | Number of vehicles in link $i$ during time $t$ oriented from origin O headed to destination D |
| $y_{i,j}^{O,D,t}$ | Number of vehicles moving from link $i$ to link $j$ during time $t$ from origin O to destination D |
| $k_i^{O,D,t}$ | OD Segregated density of link $i$ at time $t$ |
| $k_i^t$ | Aggregated density of link $i$ at time $t$ |

| | |
|---|---|
| $\Gamma(i)$ | Set of successor links of link $i$ |
| $\Gamma(i)^{-1}$ | Set of predecessor links of link $i$ |
| $\Gamma(i)^{O,D}$ | Set of successor links of link $i$ on the paths from origin O to destination D |
| $\Gamma(i)^{-1,O,D}$ | Set of predecessor links of link $i$ on the paths from origin O to destination D |
| $D(k_i^t)$ | Aggregated demand in link $i$ during time t as a function of density |
| $D(k_i^{O,D,t})$ | Segregated demand in link $i$ from origin O to destination D during time t as a function of density |
| $S(k_i^t)$ | Aggregated supply at cell $i$ during time $t$ as a function of density |
| $SF_i^t$ | Sending flow of link $i$ during time $t$ |
| $RF_i^t$ | Receiving flow of link $i$ during time $t$ |

### *Piecewise Linear Link Macroscopic Fundamental Diagram*

In this study, we adopt the convex combination formulation to handle PWL link-MFD in the MILP. This method adds binary variables and new inequalities to the model (Keha et al. 2004). Beale and Tomlin (1969) replaced the binary variables by algorithmically enforcing the branch and bound algorithm to branch only on specific sets of variables which they called special ordered sets of type 2 (SOS2). SOS2 is defined as a set of variables in which at most two variables can be positive, and if two positive they must be consecutive. In this section, we explain how the link MFDs can be approximated by a concave PWL function, see Figure1.

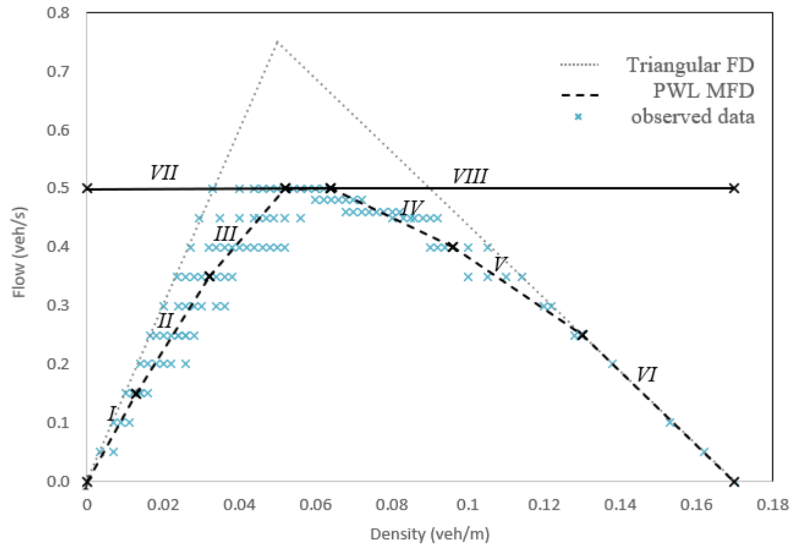


**Figure 1. Link-level MFD, triangular FD approximation, and its PWL MFD counterpart**

Let us suppose that we wish to approximate the link-MFD with four-piece-wise linear curves for both the free-flow and the congested part of the diagram. This results in using 5 breakpoints for each of the two sides of the fundamental diagram. Hence, the diagram includes three free-flow ($I, II,$ *and* $III$) and three congested (IV, V, and VI) branches, together with two horizontal branches *(VII, VIII)* that cut the MFD exactly at the maximum flow. The branches within the free-flow range, together with the horizontal branch VIII, represent the demand function. The branches within the congested range, together with the horizontal branch VII, define the supply function.

We introduce $x_i^{O,D,t}$ as the number of vehicles in link $i$ that are coming from origin O and are headed to destination D during time interval $t$. Accordingly, $k_i^{O,D,t} = \frac{x_i^{O,D,t}}{l_i}$ is the OD segregated density at link $i$ during time interval $t$.

Assume that the PWL demand function is given by its breakpoints: $\left(k_{i,b}^{O,D,t}, F_{i,b}^{O,D,t}\right)$ for $b \in \{1, \dots, B\}$, where B is the number of breakpoints and thus a model parameter. For each link $i$ and each OD pair at time $t$ we can write:

$$k_i^{O,D,t} = \sum_{b=1}^{B} k_{i,b}^{O,D,t} \times \lambda_{i,b}^{O,D,t} \tag{1a}$$

$$D\left(k_i^{O,D,t}\right) = \sum_{b=1}^{B} F_{i,b}^{O,D,t} \times \lambda_{i,b}^{O,D,t} \tag{1b}$$

$$\text{where } \sum_{b=1}^{B} \lambda_{i,b}^{O,D,t} = 1 \tag{2a}$$

$$\lambda_{i,b}^{O,D,t} \geq 0 \ \ \forall b \in \{1, \dots, B\} \tag{2b}$$

$$\lambda_{i,b}^{O,D,t} \ \ b \in \{1, \dots, B\} \ \text{ is SOS2} \tag{3}$$

To enforce (3), one can introduce binary variables $z_{i,b}^{O,D,t}$, $b \in \{0, \dots, B-1\}$ and the constraints:

$$\lambda_{i,0}^{O,D,t} \leq z_{i,0}^{O,D,t} \tag{4a}$$

$$\lambda_{i,b}^{O,D,t} \leq z_{i,b-1}^{O,D,t} + z_{i,b}^{O,D,t} \ \ \forall b \in \{1, \dots, B-1\} \tag{4b}$$

$$\lambda_{i,B}^{O,D,t} \leq z_{i,B-1}^{O,D,t} \tag{4c}$$

$$\sum_{b=0}^{B-1} z_{i,b}^{O,D,t} = 1 \tag{4d}$$

The MIP model given by (1), (2), and (4) with $z_{i,b}^{O,D,t} \in \{0,1\} \ \forall b \in \{1, \dots B-1\}$ is the convex combination formulation for a PWL function (Keha et al. 2004).

Note that, the demand function in (1) expresses the OD segregated demand at link $i$ during time $t$. In order to calculate the total demand at link $i$ during time $t$, one can write:

$$D\left(k_i^t\right) = D\left(\sum_{O,D} k_i^{O,D,t}\right) \tag{5}$$

### *Link transmission model (LTM) based dynamic network constraints*

In this section, we define the constraints of the LTM-based SO-MFD-DTA problem in the framework of MILP. Each link in the network is considered as a cell without internal segmentation. The time horizon is discretized into small time intervals.

At each time step, the sending flow from a link is bounded both by the demand in the link and the maximal outflow capacity at the downstream of the link. Likewise, the receiving flow at each link is restricted both by the supply at the link and the maximal inflow capacity at the upstream of the link. The PWL approximation approach enables us to incorporate the outflow and inflow capacities in the demand and supply functions. The sending and receiving flows of link $i$ during time interval $t$ can be mathematically expressed as follows:

$$SF_i^t = D(k_i^t) \tag{6}$$
$$RF_i^t = S(k_i^t) \tag{7}$$

For an ordinary link $i$ with only one successor link $j$, the number of vehicles moving from link $i$ to link $j$ during time $t$, can be expressed as:

$$y_{i,j}^t = \min\{SF_i^t, RF_j^t\} \times \Delta t = \min\{D(k_i^t),\ S(k_j^t)\} \times \Delta t \tag{8}$$

Note that in a multiple OD network, for any OD specified flow at each link $i$, the sending flow from link $i$ should only be received from its successor links which are on the paths between that specific OD pair. $\Gamma^{O,D}(i)$ and $\Gamma^{-1,O,D}(\mathrm{i})$ respectively denote the successors and predecessors of link $i$ on the paths between origin O and destination D.

The feasible region of equation (8) contains a non-convex set. This nonconvexity was originally relaxed by Ziliaskopoulos (2000) who proposed a method to transform the nonlinearity in the minimum constraint into a set of linear inequalities. Accordingly, for every link $i \in C$, equation (8) can be relaxed into the following system of linear LTM-based flow constraints:

$$\sum_{O,D} \sum_{j \in \Gamma^{O,D}(\mathrm{i})} y_{i,j}^{O,D,t} \leq D(k_i^t) \times \Delta t \qquad O \in C_R, D \in C_S \tag{9}$$

$$\sum_{O,D} \sum_{\mathrm{k} \in \Gamma^{-1,O,D}(\mathrm{j})} y_{\mathrm{k},j}^{O,D,t} \leq S(k_j^t) \times \Delta t \qquad O \in C_R, D \in C_S,\ \forall j \in \Gamma(i) \tag{10}$$

However, the inequality constraints (9)-(10) are not an exact transformation of the nonlinear constraint in equation (8). It is possible that the optimality in the SO-DTA model be obtained in the strict inequality region of constraints in (9)-(10). This solution property corresponds to a situation known as the vehicle holding phenomenon in which vehicles are likely to be held in the link without moving forward, even if there is enough capacity in the successor link (Zheng and Chiu 2010). VH problem represents an unrealistic traffic flow phenomenon and should be properly addressed, see next section.

The OD segregated number of vehicles going from link $i$ to all its successor links during time $t$ should also be constrained by the disaggregated demand at cell $i$. Hence, we have:

$$\sum_{j \in \Gamma^{O,D}(i)} y_{i,j}^{O,D,t} \leq D(k_i^{O,D,t}) \qquad \forall t \in T, O \in C_R, D \in C_S \tag{11}$$

Using equations (9)-(11) and convex combination formulation of $D\left(k_i^{O,D,t}\right),\ D\left(k_i^t\right),\ and\ S\left(k_i^t\right)$ we obtain MILP formulation of the LTM-based flow propagation constraints.

The model also requires traffic flow to satisfy flow conservation and initial constraints. Link mass conservation can be formulated as follows:

$$x_i^{O,D,t} - x_i^{O,D,t-1} + \sum_{j \in \Gamma^{\mathrm{O,D}}(i)} y_{i,j}^{O,D,t-1} = d^{O,D,t-1} \qquad \forall i \in C_R, O \in C_R, D \in C_S, \forall t \in T \tag{12}$$

$$x_i^{O,D,t} - x_i^{O,D,t-1} - \sum_{k \in \Gamma^{-1,\mathrm{O,D}}(i)} y_{k,i}^{O,D,t-1} + \sum_{j \in \Gamma^{\mathrm{O,D}}(i)} y_{i,j}^{O,D,t-1} = 0 \qquad \forall i \in C \backslash C_R, O \in C_R, D \in C_S, t \in T \quad (13)$$

And for the initial constraints we write:

$$x_i^{O,D,t} \geq 0,\ y_{i,j}^{O,D,t} \geq 0 \qquad \forall i \in C \cup C_S, \forall O \in C_R, D \in C_S, t \in T \quad (14)$$
$$x_i^{O,D,0} = 0,\ y_{i,j}^{O,D,0} = 0 \qquad \forall i \in C, \forall j \in C, \forall O \in C_R, D \in C_S, t \in T \quad (15)$$

### *None-Vehicle-Holding (NVH) constraints*

By tightening at least one of the less than or equal to inequalities in (9)-(10) the VH phenomenon can be eliminated (Long et al. 2018). Accordingly, the NVH conditions can be formulated as the following mixed-integer constraints:

$$-\left[\sum_{a=1}^{m_i} \theta_i^a(t)\right] M \leq \sum_{O,D} \sum_{j \in \Gamma^{O,D}(i)} y_{i,j}^{O,D,t} - D(k_i^t) \times \Delta t \qquad \forall i \in C, \forall t \in T \quad (16)$$

$$-\left[\sum_{a=1}^{m_i} \sigma_a^g - \sum_{a=1}^{m_i} (2\sigma_a^g - 1)\, \theta_i^a(t)\right] M \leq \sum_{O,D} \sum_{\mathrm{k} \in \Gamma^{-1,\mathrm{O,D}}(j_g)} y_{\mathrm{k},j_g}^{O,D,t} - S(k_{j_g}^t) \times \Delta t \qquad \forall i \in C, t \in T, \mathrm{g} \in G_i \quad (17)$$

$$\sum_{a=1}^{m_i} 2^a \theta_i^a(t) \leq 2|\Gamma(i)| \qquad \forall i \in C, \forall t \in T \quad (18)$$

$$\theta_i^a(t) \in \{0,1\}, a = 1, \ \dots, m_i \qquad \forall i \in C, \forall t \in T \quad (19)$$

where M is a very large positive value, $G_i = \{1, 2, \dots, |\Gamma(i)|\}$ is an index set for link $i$'s successor links, $j_g$ is the $g$-th link in $\Gamma(i)$, $m_i = \mathrm{argmin}_m\{2^{m+1} \geq 2 + 2 \times |\Gamma(i)|\}$, and $\sigma_a^g$ is 0 or 1, such that $\sum_{a=1}^{m_i} 2^{a-1} \times \sigma_a^g = g$.

Note that, the above formulation results in one constraint on sending flow of link $i$ represented by (16) and $|\Gamma(i)|$ constraints on receiving flow of each of link $i$'s successor links represented by (17). Introducing maximum flow directly in constraints (6)-(7) allows us to reduce exactly by half the number of constraints with respect to the original formulation proposed by Long et al. (2018). Furthermore, the number of binary variables $\theta_i^a(t)$ is decreased by removing the ones responsible for activating the maximal sending and receiving capacities constraints.

### *LTM-based SO-MFD-DTA models in terms of Total System Travel Time (TSTT)*

The objective of a TSTT-SO-DTA model is to minimize the TSTT experienced by all the vehicles in a network. TSTT equals the sum of the number of vehicles on all links within the simulation duration. Therefore, the problem can be formulated as the following LP problem:

$$\min_{x_i^{O,D,t}} TSTT = \sum_{\forall O\in C_R, D\in C_S} \sum_{i\in C} \sum_{t\in T} \Delta t \times x_i^{O,D,t}$$

Subject to: constraints $(9) - (19)$

## 3. RESULTS AND DISCUSSION

### *Simulation setting*

We implement the MILP formulation presented above on a small network similar to the Nguyen and Dupuis (1984) network, as illustrated in Figure 2, for a time-dependent demand scenario. The analysis period is 10 min and the simulation step size is chosen as 20s. There are 6 OD pairs for this network (gray nodes). For each OD pair, vehicles enter the network at the constant rate of 0.2 vehicles/s during the first 3 minutes of the simulation period, then it increases to 0.4 vehicles/s for 4 minutes and afterward for the last 3 minutes it decreases to 0.1 vehicles/s.

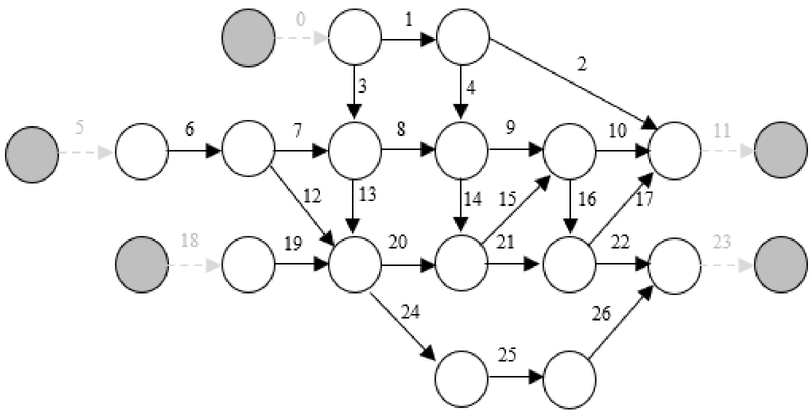


**Figure 2. Simulated network**

PWL MFD for each link is given by 5 breakpoints. For the demand function, the first and last breakpoints of link $i$ are $(0{,}0)$ and $(k_{jam}^i, Q_i)$ respectively. For the supply function, they are $(0, Q_i)$ and $(k_{jam}^i, 0)$. Characteristics of the links and other breakpoints on demand and supply function for each link are summarized in Table 2. Jam density is assumed to be 0.17 vehicles/m for all links.

**Table 2: Characteristics of the links in the simulated network**

| Link | Length (m) | Max flow (veh/s) | Speed limit (m/s) | Breakpoints on the MFD | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | Demand function | | | Supply function | | |
| 1 | 250 | 0.5 | 12.5 | (0.02,0.25) | (0.04,0.4) | (0.06,0.5) | (0.08,0.5) | (0.12,0.35) | (0.14,0.24) |
| 2 | 400 | 0.3 | 14 | (0.01,0.14) | (0.02,0.25) | (0.0265,0.3) | (0.11,0.3) | (0.13,0.23) | (0.15,0.13) |
| 3 | 250 | 0.5 | 12.5 | (0.02,0.25) | (0.04,0.4) | (0.06,0.5) | (0.08,0.5) | (0.12,0.35) | (0.14,0.24) |
| 4 | 250 | 0.5 | 12.5 | (0.02,0.25) | (0.04,0.4) | (0.06,0.5) | (0.08,0.5) | (0.12,0.35) | (0.14,0.24) |
| 6 | 250 | 0.5 | 12.5 | (0.02,0.25) | (0.04,0.4) | (0.06,0.5) | (0.08,0.5) | (0.12,0.35) | (0.14,0.24) |
| 7 | 250 | 0.5 | 12.5 | (0.02,0.25) | (0.04,0.4) | (0.06,0.5) | (0.08,0.5) | (0.12,0.35) | (0.14,0.24) |
| 8 | 250 | 0.3 | 12.5 | (0.01,0.125) | (0.02,0.2) | (0.04,0.3) | (0.06,0.3) | (0.1,0.23) | (0.14,0.12) |
| 9 | 250 | 0.5 | 12.5 | (0.02,0.25) | (0.04,0.4) | (0.06,0.5) | (0.08,0.5) | (0.12,0.35) | (0.14,0.24) |
| 10 | 250 | 0.5 | 12.5 | (0.02,0.25) | (0.04,0.4) | (0.06,0.5) | (0.08,0.5) | (0.12,0.35) | (0.14,0.24) |
| 12 | 300 | 0.5 | 15 | (0.01,0.15) | (0.03,0.38) | (0.05,0.5) | (0.07,0.5) | (0.1,0.4) | (0.14,0.2) |
| 13 | 250 | 0.5 | 12.5 | (0.02,0.25) | (0.04,0.4) | (0.06,0.5) | (0.08,0.5) | (0.12,0.35) | (0.14,0.24) |
| 14 | 250 | 0.5 | 12.5 | (0.02,0.25) | (0.04,0.4) | (0.06,0.5) | (0.08,0.5) | (0.12,0.35) | (0.14,0.24) |
| 15 | 300 | 0.5 | 15 | (0.01,0.15) | (0.03,0.38) | (0.05,0.5) | (0.07,0.5) | (0.1,0.4) | (0.14,0.2) |
| 16 | 250 | 0.5 | 12.5 | (0.02,0.25) | (0.04,0.4) | (0.06,0.5) | (0.08,0.5) | (0.12,0.35) | (0.14,0.24) |
| 17 | 300 | 0.5 | 15 | (0.01,0.15) | (0.03,0.38) | (0.05,0.5) | (0.07,0.5) | (0.1,0.4) | (0.14,0.2) |
| 19 | 250 | 0.5 | 12.5 | (0.02,0.25) | (0.04,0.4) | (0.06,0.5) | (0.08,0.5) | (0.12,0.35) | (0.14,0.24) |
| 20 | 250 | 0.5 | 12.5 | (0.02,0.25) | (0.04,0.4) | (0.06,0.5) | (0.08,0.5) | (0.12,0.35) | (0.14,0.24) |
| 21 | 250 | 0.3 | 12.5 | (0.01,0.125) | (0.02,0.2) | (0.04,0.3) | (0.06,0.3) | (0.1,0.23) | (0.14,0.12) |
| 22 | 250 | 0.5 | 12.5 | (0.02,0.25) | (0.04,0.4) | (0.06,0.5) | (0.08,0.5) | (0.12,0.35) | (0.14,0.24) |
| 24 | 300 | 0.5 | 15 | (0.01,0.15) | (0.03,0.38) | (0.05,0.5) | (0.07,0.5) | (0.1,0.4) | (0.14,0.2) |
| 25 | 250 | 0.5 | 12.5 | (0.02,0.25) | (0.04,0.4) | (0.06,0.5) | (0.08,0.5) | (0.12,0.35) | (0.14,0.24) |
| 26 | 300 | 0.5 | 15 | (0.01,0.15) | (0.03,0.38) | (0.05,0.5) | (0.07,0.5) | (0.1,0.4) | (0.14,0.2) |

## *Results*

The proposed framework is simulated considering two different scenarios. First, the baseline scenario in which the shortest path (SP) in distance is taken by all the vehicles for each OD pair at each time step. Second, the scenario in which the flow propagates according to SO-MFD-DTA. The baseline scenario corresponds to the usual user choice without traffic information. Such a route choice pattern is detrimental to the system-level efficiency and increases the TSTT by 37% compared to the optimal route choices given by the SO-MFD-DTA equilibrium, see Figure 3.

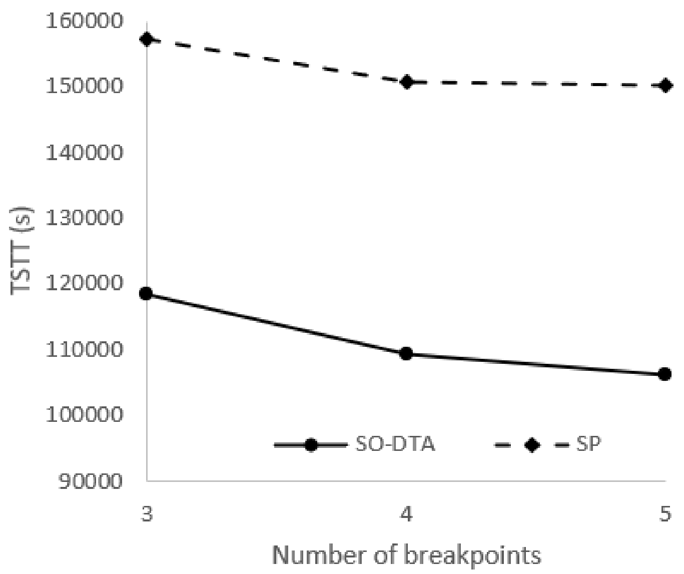


**Figure 3. Comparison between SO-MFD-DTA and the shortest path scenarios**

For both scenarios, we investigate how the accuracy of the link MFD estimation can influence the optimal route choices and the collective objective function. To do so, we reduce the number of breakpoints from 5 to 3, which corresponds to the classical but crude trapezoidal approximation.

Results from the optimization suggest that calculating the optimal route pattern when approximating MFDs with 3 breakpoints will lead to suboptimal route guidance in comparison to the framework where MFDs are approximated more accurately with 5 breakpoints. Take for example, in the simulated network, origin link 0 and destination link 11. Optimal paths suggested by the optimization framework with three-breakpoints MFDs are [0, 1, 2, 11] and [0, 1, 4, 9, 10, 11] which result in 118359s of TSTT. For the same OD, the optimal paths suggested by the optimization framework with five-breakpoints MFDs are different, i.e. [0, 1, 2, 11] and [0, 3, 8, 9, 10, 11], so is the TSTT 106062s. Most importantly, if we apply the optimal route distribution obtained from the three-breakpoints approximation and run the simulation with the five-breakpoints approximation, we see that the TSTT is equal to 108420s. It highlights the need for more accurate link MFD functions when calculating the system optimum and thus a more advanced optimization framework including convex combination with SOS2 approximations.

We are currently investigating environmental-related objective functions to be include in the presented framework. Such objective functions put more stress on the NVH constraints and require specific arrangements to reduce the computational burden. Those results will be added to the presentation for the Heart 2022 conference.

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