A variable neighbourhood search algorithm for a demand-responsive bus system with capacitated vehicles

Dilay Aktas*[1], Kenneth Sörensen[2], and Pieter Vansteenwegen[1]

[1]KU Leuven, Institute for Mobility - CIB, Department of Mechanical Engineering, KU Leuven, 3001 Leuven, Belgium

[2]ANT/OR - Operations Research Group, Department of Engineering Management, University of Antwerp, 2000 Antwerp, Belgium

SHORT SUMMARY

In this study, we design a metaheuristic algorithm for a demand-responsive public bus system operating during peak hours with capacitated vehicles. Morning peak hours are considered where the passenger flows towards a city center are typically much larger than the flows in the opposite direction. A single-line system with express services away from the city center is optimized. Based on the expected demand, it is decided whether a bus should visit all the stops ahead or take the express route away from the city center to increase the frequency of the service *towards* the city center. Due to problem complexity, only small-sized instances can be solved optimally. Therefore, a metaheuristic algorithm is proposed based on Variable Neighborhood Search. The results show that the demand-responsive system can improve the average passenger travel time up to 25% compared to the conventional system especially if limited vehicle capacities need to be considered.

Keywords: demand-responsive bus services, semi-flexible bus services, variable neighbourhood search.

1. INTRODUCTION

Most public bus systems can be considered under two broad categories: conventional systems and demand-responsive public bus systems. In a conventional system, typically a number of vehicles follows fixed routes based on a predefined timetable. These routes and timetables are decided based on historical demand and communicated to the passengers. When demand is high and consistent for a region and time period, these systems perform well due to the high degree of resource sharing. However, in the case of low and more variable demand, the resource sharing potential decreases drastically due to the lack of flexibility. As a result, many demand-responsive systems are recently introduced and studied, with different degrees of flexibility. Some of these systems alternate between a conventional and a demand-responsive service based on a demand threshold (e.g. Cayford & Yim (2004), Li & Quadrifoglio (2010), Kim & Schonfeld (2012), Li & Quadrifoglio (2010)). Other operate with partially fixed routes and a possibility to deviate from this route within a service area in case of demand (e.g. Kim & Schonfeld (2012), Quadrifoglio & Dessouky (2004), Quadrifoglio, Dessouky & Palmer (2007), Potts, Marshall, Crockett & Washington (2010), Malucelli, Nonato & Pallottino (1999)). The last systems are categorized as semi-flexible by Vansteenwegen et al. (2022) in a recent and comprehensive survey on demand-responsive public bus systems.

In this study, we consider a single line where the demand for transportation in one direction during peak hours is much larger than the demand in the opposite direction and the demand outside peak hours. In a conventional system, this would result in overcrowded buses in one direction and almost empty buses in the other direction. This imbalance in the demand will now be exploited to provide a better service for the larger flows. The proposed system aims to increase the frequency of service towards a city center during morning peak hours compared to a conventional system, by allowing some of the vehicles to perform *express services* away from the city center. For simplicity, only morning peak hours are considered, but the same system could also be applied to evening peak hours. As a result, the average passenger travel time is decreased and more passengers are served especially if limited vehicle capacities need to be considered.

2. METHODOLOGY

As opposed to a conventional bus system where a number of vehicles drives back and forth between a terminal and a city center following the same regular routes, in the demand-responsive system (DRS), it is decided beforehand whether a bus heading away from the city center, should take the regular route or the express route to increase the frequency towards the city center. Since the system no longer has a fixed headway or a periodic timetable, the departure time of each bus, from the terminal and the city center, should also be optimized. Figure 1 gives an example of a line, its regular route according to the conventional system and its express route. The express route is determined beforehand based on the location of the stops and the road network.

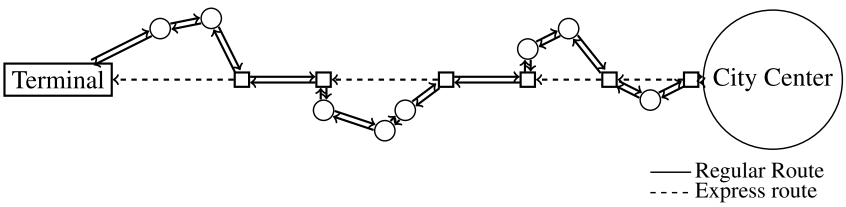

Figure 1: General Representation

When optimizing this system, a fixed fleet of identical and capacitated vehicles are used, each of which perform a number of services during peak hours. The demand-responsive system takes over from the conventional system at the beginning of peak hours and immediately after peak hours, the system returns back to the conventional system using other vehicles. Therefore, when peak hours start, the buses that are on the road continue their regular service at least until they reach the city center or the terminal for the first time. Then, a bus is allowed to wait before departing but an *express service* can only be performed by following the *express route* heading away from the city center. All passenger demand must be served. Based on the expected demand (distributed uniformly) from and to each stop, the objective is to minimize the total (expected) travel time of passengers traveling during peak hours measured by the sum of their in-vehicle time and waiting time.

Unlike the conventional system, the headway between any two consecutive services is no longer constant as a result of the service type and dwell time decisions. This means that the number of passengers arriving at a stop between two consecutive services is also no longer constant and each has the expected waiting time of half of that headway (Osuna & Newell 1972), if they can all board. Thus, the total waiting time becomes a quadratic function of the headways. It is also assumed that passengers get on the buses in a first-come, first-served basis as long as there is room in the bus. Therefore, when the bus capacity is insufficient to take all the passengers waiting at a stop, the expected waiting time of the passengers that could not board increases by the additional time they need to wait for the next service. Overall, except for the passengers using an express service to arrive their destinations, the total in-vehicle time is the same as the conventional system.

In calculating the total travel time, lastly the passengers already waiting at different stops when the peak hours start should be considered. This is because the system takes over from the conventional system at the beginning of peak hours and the decisions that are made on the first services performed during peak hours will therefore affect these passengers as well. Thus, the total waiting time and the total in-vehicle time both consider the passengers that have arrived at their origin stops before the peak hours start but served by the first service during peak hours.

Since the total waiting time is a quadratic function of the headways and express buses might pass the regular buses on the way, this problem should be modeled as a mixed integer quadratic program. However, only small-sized instances can be solved optimally. Thus, a variable neighbourhood search algorithm for real-sized instances is designed and the performance of the algorithm is evaluated by comparing its results with the solutions obtained by the mathematical model solved by CPLEX for smaller instances.

Variable Neighbourhood Search Algorithm

The Variable Neighbourhood Search (VNS) algorithm starts with the initial solution where all service types are set to regular and all waiting times are set to zero. This in fact corresponds to the conventional system. Then, better solutions in terms of the service type of a service and departure time from the city center and the terminal are explored. First switching the service type of a service and then adjusting the departure time of that service from the city center and the terminal, a neighbourhood of the incumbent solution is explored. A neighbour of the incumbent solution in this neighbourhood then corresponds to a service switch of a bus and several departure time changes of the same bus with adjusted waiting times at the city center and the terminal for that service. Note that when there is no more room in the bus, some passengers might be left behind. During morning peak hours, this only happens towards the city center due to much larger flow of demand. Therefore, if a service has insufficient capacity to take all the passengers waiting at a stop, changing the service type from regular to express and decreasing dwell times at the city center and the terminal of the service would improve the total travel time by shifting the service to an earlier time and increasing the frequency of service towards the city center. Then, based on the total travel time, the best neighbouring solution is found. If the best neighbouring solution is at least as good as the incumbent solution, the incumbent solution is updated to enter the next iteration. If none of the neighbouring solutions provide a better objective function value, in this case, a larger neighbourhood of the incumbent solution is explored by adjusting the waiting time of any bus in any of its services at the city center or the terminal. If the incumbent solution remains the same since the beginning of the iteration, diversification is applied to update the incumbent solution before entering the next iteration. The diversification switches the service type of a random number of services, while maintaining the current number of express services in the solution. Lastly, the algorithm runs as long as the computation time allows and it re-starts with a random solution in terms of service type decisions with all the waiting times set to zero after 200 iterations without improving the best found solution. The best found solution so far is checked and updated whenever the incumbent solution is updated. These steps are summarized in Algorithm 1.

Algorithm 1: Variable Neighbourhood Search

```
Generate an initial solution: conventional system
while computation time allows do
    Construct a neighbourhood of the incumbent solution
    for each bus and each service do
        Switch the service type, express → regular or regular → express
        Adjust the departure time of that service from the city center and the terminal
    end
    Find the neighbouring solution that gives the best improvement
    if There is improvement then
        Update the incumbent solution
    else
        Adjust the departure time of other services from the city center and the terminal,
        if There is improvement then
            Update the incumbent solution
        else
            Apply diversification keeping the number of express services constant
            Update the incumbent solution
        end
    end
    if The best solution is not updated for a number of iterations then
        Re-start with a new initial solution with randomly chosen express services
    end
end
```

3. RESULTS AND DISCUSSION

In order to evaluate the performance of the algorithm, three different lines are considered in the first experiment: S (small), M (medium), L (large) lines with 2, 8, and 14 stops and a city center. More details on instances can be found in Table 1.

Table 1: Instance Characteristics

Instance,	# of Stops		One-way driving time (min)		Peak Hours	Fleet
	Regular	Express	Regular	Express	π, (mins)	Size, m
S	1	1	10	5	30	2
M	4	4	20	10	40	2
L	7	7	30	15	60	2

A demand scenario where the mean demand to travel towards the city center is 20 times the mean demand to travel from the city center during peak hours and the mean demand in both directions outside peak hours is considered. Then, it is scaled such that the maximum number of passengers in a bus according to the conventional system is less than 85 passengers. The ratio of demand used in the experiments is given in Table 2 below.

Table 2: Ratio of Demand

Peak Hours						Outside Peak Hours	
Towards the city center (n)		Towards the terminal				Towards the city center	Towards the terminal
$j \to n$	$j \to s \neq n$	$n \to s$		$j \neq n \to s$		$j \to n$	$n \to s$
		s_R	s_E	s_R	s_E		
4	0.10	0.10	0.10	0.05	0.05	0.10	0.10

Stop j: Origin, s: Destination, n: City Center, s_R: NOT on the express route, s_E: ON the express route.

For each line, three different types of vehicles with capacities of 85, 70 and 60 are considered. The mathematical models (MM) are solved by the solver CPLEX version 12.7 with GAMS IDE and the variable neighbourhood search algorithm (VNS) is implemented in Matlab 2019b on a computer equipped with a processor AMD Ryzen 7 PRO 2700U w/Radeon Vega Mobile Gfx 2.20 GHz. Then, the conventional system (CS) is simulated for each instance with the given demand scenario and the average travel time (ATT) is compared with the average travel time of the demand-responsive system.

Table 3: Overall Results

				Demand-Responsive System								
	Conventional System			Mathematical Model					Metaheuristic			
Line	C (people)	ATT-CS (min)	NP-CS (people)	ATT-MM (min)	Imp-MM %	CPU-MM (s)	GAP %	NP-MM (people)	ATT-VNS (min)	Imp-VNS %	CPU-VNS (s)	NP-VNS (people)
S	85	12.5	80	11.1	11%	334	0%	64	11.1	11%	10	64
S	70	13.7	70	11.1	18%	178	0%	64	11.1	18%	10	64
S	60	14.8	60	11.2	24%	818	0%	60	11.2	24%	10	60
Average	-	13.7	-	11.2	18%	443	0%	-	11.2	18%	10	-
M	85	21.5	80	20.1	6%	3600	20%	65	19.8	8%	10	76
M	70	22.6	70	20.1	11%	3600	18%	65	19.9	12%	10	68
M	60	23.8	60	20.1	15%	3600	13%	60	20.1	15%	10	60
Average	-	22.6	-	20.1	11%	3600	17%	-	19.9	12%	10	-
L	85	31.2	84	29.5	5%	10800	35%	68	29.3	6%	10	84
L	70	33.5	70	29.5	12%	10800	30%	68	29.3	12%	10	68
L	60	35.1	60	32.8	7%	10800	34%	60	29.6	16%	10	59
Average	-	33.3	-	30.6	8%	10800	33%	-	29.4	11%	10	-

C: Capacity, NP-CS(MM, VNS): Maximum number of passengers on a bus in CS(according to MM, according to VNS), Imp-MM(VNS): Improvement of ATT over CS obtained by MM(VNS), GAP: Optimality gap by MM

According to the results given in Table 3, VNS algorithm finds the optimal solutions to the instances of the small line within ten seconds. For the instances with medium and large lines, the VNS solutions are always at least as good as the best solutions found by the mathematical model within the given computation time for the instances of medium and large lines. Moreover, for those instances where the bus capacity is set to 85 people, the results of the conventional system correspond to a capacity that is large enough to accommodate all passengers arriving at a stop. For the other cases where the capacity is smaller, the average travel time per passenger increases since the bus capacity is insufficient for the conventional system and passengers accumulate at their origin stops. Overall, compared to the conventional system, the demand-responsive system improves the average passenger travel time by about 12% and 14% by the mathematical model and the VNS algorithm, respectively. It should also be noted that for larger instances, the computation time for the mathematical model increases drastically. Actually, that is the reason why the instances used above are limited in size, in order to be able to still obtain good solutions from the mathematical model and to evaluate the performance of the VNS algorithm. Yet, the mathematical model cannot find the optimal solutions for instances M and L within the given computation time. The solutions obtained have on average 17% and 33% optimality gap for M and L, respectively.

In order to analyze the performance of the system, the VNS algorithm is applied to a larger instance in the second experiment. This time, however, a more realistic peak hour duration of 120, 150 and 180 minutes are considered. The algorithm now runs for 10 and 100 seconds. The results are given in Table 4.

Table 4: Large-sized Line Results

	Conventional System			VNS-10s			VNS-100s		
Line	C (people)	ATT-CS (min)	NP-CS (people)	ATT-VNS (min)	Imp-VNS %	NP-VNS (people)	ATT-VNS (min)	Imp-VNS %	NP-VNS (people)
L_{120}	85	31.2	84	29.0	7%	73	29.0	7%	73
L_{120}	70	38.2	70	29.0	24%	68	29.0	24%	68
L_{120}	60	43.3	60	30.6	29%	60	30.6	29%	60
Average	-	37.6	-	29.5	20%	-	29.5	20%	-
L_{150}	85	31.2	84	29.5	6%	84	29.5	6%	84
L_{150}	70	40.7	70	30.6	25%	68	29.1	28%	65
L_{150}	60	47.4	60	33.6	29%	59	31.4	34%	60
Average	-	39.8	-	31.2	20%	-	30.0	23%	-
L_{180}	85	31.2	84	29.6	5%	84	29.3	6%	73
L_{180}	70	43.0	70	30.2	30%	70	29.3	32%	68
L_{180}	60	51.4	60	35.2	31%	60	32.2	37%	60
Average	-	41.9	-	31.7	22%	-	30.3	25%	-

According to the results given in Table 4 and similar to the results given in Table 3, the demand-responsive system improves the average travel time more when vehicle capacity is not sufficient for the conventional system. This is expected since passengers accumulate less at their origin stops as a result of the increased frequency by the demand-responsive system. For the same reason, higher improvement values are observed when longer peak hours are considered. Although the VNS algorithm finds high quality solutions within ten seconds for the instances of L_{120}, with the longer peak hour duration in L_{150} and L_{180}, the problem complexity increases. In order to benefit more from the demand-responsive system, longer computation time is required. Overall, the demand-responsive system improves the average travel time by 20%, 23%, and 25% on average for the large-sized instance and peak hour duration of 120, 150, and 180 minutes according to the best solutions found by the VNS within 100 seconds.

4. CONCLUSIONS

In this study, we develop a Variable Neighbourhood Search algorithm in order to optimize the operations of a new demand-responsive system which can be operated during peak hours with capacitated vehicles. Morning peak hours are considered where the passenger flows towards a city center are much larger than the flows in the opposite direction. Based on the expected demand and for a single line, it is decided which services can skip a number of predefined stops moving away from the city center. This way, average passenger travel time improves as the frequency of service towards the city center is increased and and less passengers are rejected due to the vehicle capacities.

The experimental results show that the metaheuristic algorithm finds solutions that are at least as good as the best solutions obtained by the mathematical model, and with a much shorter computation time. However, for the large-sized instances where the line includes more stops and also the duration of peak hours is longer, the problem complexity increases drastically. Therefore, the algorithm requires more computational effort and finds better solutions with increased computation time. Overall, the demand-responsive system potentially improves the average passenger travel time up to 25% for large instances. Lastly, higher improvement values in average travel time are observed when the vehicle capacity is more restricting and the peak hour duration is longer. This is the result of the increased frequency of service towards the city center, of the demand-responsive system.

ACKNOWLEDGMENT

This project was supported by the FWO (Research Foundation Flanders), project G.0759.19N.

REFERENCES

Cayford, R. & Yim, Y. (2004), 'Personalized demand-responsive transit service'.

Kim, M. & Schonfeld, P. (2012), 'Conventional, flexible, and variable-type bus services', *Journal of Transportation Engineering* **138**(3), 263–273.

Li, X. & Quadrifoglio, L. (2010), 'Feeder transit services: choosing between fixed and demand responsive policy', *Transportation Research Part C: Emerging Technologies* **18**(5), 770–780.

Malucelli, F., Nonato, M. & Pallottino, S. (1999), Demand adaptive systems: some proposals on flexible transit, *in* 'Operational research in industry', Springer, pp. 157–182.

Osuna, E. & Newell, G. F. (1972), 'Control strategies for an idealized public transportation system', *Transportation Science* **6**(1), 52–72.

Potts, J. F., Marshall, M. A., Crockett, E. C. & Washington, J. (2010), 'A guide for planning and operating flexible public transportation services'.

Quadrifoglio, L. & Dessouky, M. M. (2004), Mobility allowance shuttle transit (mast) services: formulation and simulation comparison with conventional fixed route bus services, *in* 'Modelling, simulation, and optimization Á Proceedings of the 4th IASTED international conference. Kauai, HI, USA, 17Á19 August. Calgary: Acta Press, 6pp'.

Quadrifoglio, L., Dessouky, M. M. & Palmer, K. (2007), 'An insertion heuristic for scheduling mobility allowance shuttle transit (mast) services', *Journal of Scheduling* **10**(1), 25–40.

Vansteenwegen, P., Melis, L., Aktas, D., Montenegro, B. D., Vieira, F. & Sörensen, K. (2022), 'A survey on demand-responsive bus systems.', *Transportation Research Part C: Emerging Technologies, accepted for publication* .