

An analysis of power peaks in stochastic models of railway traffic

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SHORT SUMMARY

Railway traffic flow can be modeled by a string of consecutive trains, each subject to random speed variations that are described by a stochastic process. Despite analogies with car-follower models, railways include specific features and a safety system that forces vehicles to decelerate towards a fixed lower speed if an absolute safety distance with the vehicle ahead is not respected. By simulating this dynamic system, we compute performance indicators focusing on energy consumption and the power peaks arising when multiple trains accelerate simultaneously. We study how different conditions of the system and assumptions on the stochastic processes, e.g., describing human drivers vs automated train operations (ATO), affect energy consumption and power peaks. Our results show that an ATO controller aware of precise distance and speed information can be effective at reducing energy consumption and smoothing the peaks, which are a major concern of operators.

Keywords: Automated train operations; energy consumption; railway modeling; simulation; stochastic processes; traffic flow theory.

1. INTRODUCTION

Transport accounts globally for a large share of Greenhouse Gases emissions, and power consumption. One key direction to improve sustainability and energy efficiency of transportation is to switch to collective transport of larger vehicles, rather than private vehicles or small shipments by trucks. Despite railway is a rather energy efficient way of transportation, efforts to reduce its energy footprint are put into place by many transport operators and authorities (UIC, 2017).

In fact, energy consumption accounts for a large portion of railway operations costs. The amount of energy needed for moving depends on speed and resistances, and results in requirements for energy production. By carefully planning the speed and acceleration and braking processes of trains, energy can be saved. (Hansen & Pachl, 2014). In electrified railway systems, energy distribution and usage is centralized, i.e., all vehicles draw energy at the same time from a distribution network. The total energy required by the system is then the sum of all energies required by all vehicles.

A different stream of research tackled instead the peak value of the power needed. Power peaks may arise when multiple vehicles require large amount of power, for instance for going at sustained high speed, or during acceleration. Those peaks threaten grid stability and are a large concern for operators. A power distribution network which is designed for lower peaks might fail, when the power demand is very high, causing reduction of delivered energy or in the worst cases, a blackout. Traffic volume is growing, speed is increasing, vehicles are heavier, resulting in higher chances of high power peaks; upgrading the power distribution requires large costs and long time. In addition to grid stability issues, the energy bill paid by rail operators usually depends on both total and maximum consumption (i.e. the highest peak) over the billing period (Albrecht, 2010).

In practical railway operations, a train is subject to random speed variations (e.g., due to driver behaviors and changes in line voltage and track resistance). [Corman, Trivella, and Keyvan-Ekbatani \(2021\)](#) provide empirical evidence of this effect using data from the Swiss railways, and introduce a stochastic process to model a system with two trains. The dynamic characteristics of a follower are studied given a trajectory of a leader, to quantify the emerging properties of the system.

We consider in this paper a stochastic model of railway traffic flow for a string of N consecutive trains, and use simulation to quantify the emerging properties of the system, from the point of traffic regularity and especially energy and power peaks. The strict safety system ensures that vehicles maintain a minimum safety distance. When two trains get too close, a yellow signal is triggered and the follower has to decelerate towards a fixed lower speed. This results in extra time lost, in a braking and a successive re-acceleration to a cruising speed.

When vehicles can be individually controlled, an interesting problem of traffic flow theory is to study under which conditions, a series of successive coordinated vehicles has impact towards traffic flow stability, i.e., studying string stability. This has been mostly focusing on vehicles with homogeneous, simplified characteristics ([M. Wang et al., 2017](#)); in the case of private vehicles, there is no external safety system which constrains their speed. The study of vehicular traffic does not normally study energy peaks as the energy consumption of all vehicles draws from separated energy carriers (such as Internal Combustion Engines, or battery-fed electric motors). There is nevertheless the interest in minimizing system properties such as emissions ([Qin, Hu, He, & Li, 2020](#)), which have some of their sources in speed and acceleration.

This work builds on the stochastic modelling of railway operations, which extends vehicular traffic flow theory to railway systems ([Corman et al., 2021](#)). To this end, it brings a fresh views to the existing models which incorporated some sources of stochasticity into energy modelling. For instance, [P. Wang, Trivella, Goverde, and Corman \(2020\)](#) discuss the case of randomly varying speed profile due to uncertain resistance parameters. Most approaches resort to micro-simulation to compute the energy consumption from a single train, and then by a traditional Monte Carlo approach, consider shifting the fixed speed profile to vary departure and arrival times.

Our contributions are the following: (i) We extend the model of [Corman et al. \(2021\)](#) by simulating and studying a string of consecutive trains, (ii) We outline a technique for detecting power peaks in such a system, (3) We provide insights on the impact of different processes (e.g., describing an ATO controller vs a human driver) towards traffic regularity, energy use, and power peaks.

In the rest of this paper, we present our methodology in Section 2, discuss the numerical results and findings in Section 3, and conclude in Section 4.

2. METHODOLOGY

We present first two stochastic process models for a string of trains, and then outline our method for detecting peaks in power consumption.

Models and simulation

Following the model for two trains by [Corman et al. \(2021\)](#), we define a general stochastic system of N consecutive trains. Denote with $v_n(t)$ and $s_n(t)$, respectively, the speed and space covered by train n at time $t \geq 0$. Also denote by $W(t)$ a standard Wiener process and by v_{CRUISE} a reference cruising speed for all trains. The first model we consider is based on the Ornstein-Uhlenbeck (OU) stochastic process, that is expressed for train n as:

$$\text{[OU]:} \quad \begin{cases} dv_n(t) = \beta_n(v_{\text{CRUISE}} - v_n(t))dt + \sigma_n dW(t), \\ ds_n(t) = v_n(t)dt. \end{cases} \quad (1)$$

In the OU process, the train speed is mean reverting towards v_{CRUISE} . This model may describe a human train driver that continuously adjusts the speed to keep it close to the target value. The mean reversion parameter β_n captures the reaction of the driver and of the speed control system.

A more sophisticated model is based on a doubly-mean-reverting (DMR) process, describing an ATO system where trains are aware of the location of the traffic ahead. Under this model, train n accelerates or decelerates to maintain both a target speed and a target headway with train $n - 1$:

$$\text{[DMR]:} \quad \begin{cases} dv_n(t) = [\beta_n(v_{\text{CRUISE}} - v_n(t)) + \alpha_n(s_{n-1}(t) - s_n(t))]dt + \widehat{\sigma}_n(v_n(t))dW(t), \\ ds_n(t) = v_n(t)dt, \end{cases} \quad (2)$$

where $\widehat{\sigma}_n(v) := \sigma_n \sqrt{[v \cdot (v_{\text{MAX}} - v)]/[v_{\text{CRUISE}} \cdot (v_{\text{MAX}} - v_{\text{CRUISE}})]}$ and $v_{\text{MAX}} > v_{\text{CRUISE}}$ is an upper bound on speed. The full dynamics we consider couple either (1) or (2) with a deterministic deceleration phase (at rate a_{DET}) that is triggered when the distance between two trains decreases below a minimum safety distance d_{MIN} , until the approach speed $v_{\text{APPROACH}} < v_{\text{CRUISE}}$ is reached. We assume that at $t = 0$ the trains have all speed v_{CRUISE} and distance $d_0 > d_{\text{MIN}}$ from the immediate follower. In contrast to [Corman et al. \(2021\)](#), we assume the first train is also subject to stochastic dynamics.

We can compute performance indicators of the dynamic system related to the energy consumption and peaks by drawing trajectories (or sample paths) of the stochastic part of the model in Monte Carlo Simulation. To do so, we discretize time into steps $\mathcal{T} = \{1, \dots, T\}$, equally spaced by Δt , and apply a standard forward Euler scheme to (1) or (2) to obtain speed and space trajectories. Finally, we employ the common dynamic equations for the motion of a railway vehicle to derive the train acceleration a_t , force f_t , and energy consumption e_t at all discrete time steps \mathcal{T} . Specifically, the force is $f_t = a_t \cdot m \cdot \rho + [\gamma_1 + \gamma_2 v_t + \gamma_3 v_t^2]$, where $a_t = (v_t - v_{t-1})/\Delta t$ is the acceleration, γ_i are the train resistance parameters, m the mass, and ρ the rotating mass factor. The energy is then $e_t = \max\{f_t, 0\} \cdot (s_t - s_{t-\Delta t})$. See [Hansen and Pachl \(2014\)](#) for more details on train dynamics.

Power peak detection

Since thousand of simulations are needed to obtain statistically relevant sample averages, a method to detect peaks in a simulated system energy trajectory is needed, where a *system energy trajectory* is defined as the cumulative energy consumption of the N trains as a function of time, measured over regular time intervals (e.g., of 30 seconds).

The method we propose is outlined in Algorithm 1. In a nutshell, we initially define as peaks the points marked as outliers according to a metric based on mean and standard deviation of the overall trajectory (Steps 2–3). Doing this requires applying first a Gaussian-weighted moving average filter (Step 1) to obtain a smoothed trajectory, so that we can exclude points with high value due to the background stochastic speed fluctuations. In other words, the effect of this filtering step is to smooth the fluctuations resulting from the stochastic process while preserving the major peaks that arise when multiple trains accelerate. Finally, we reconstruct the entire peak by iteratively incorporating neighboring peak points that fulfill a relaxed outlier condition (Step 4).

Note that whilst the detected peaks are based on the Gaussian-smoothed trajectory, the energy statistics we will present in the following section are based on the original trajectory.

Algorithm 1: Peak detection

Input: System energy trajectory $E = \{e_t : t \in \mathcal{T}\}$; Parameters $(w, \alpha_1, \beta_1, \alpha_2, \beta_2)$, with $\alpha_1 \geq \alpha_2$ and $\beta_1 \geq \beta_2$; Set of peak points $P \leftarrow \emptyset$; Variable $FullPeakFound \leftarrow FALSE$

Step 1. Apply Gaussian filter to E with window w to get smoothed trajectory \hat{E}

Step 2. Compute mean (μ) and standard deviation (σ) of \hat{E}

Step 3. Set initial peak points $P \leftarrow \{e_t \in \hat{E} : e_t \geq \alpha_1 \mu + \beta_1 \sigma\}$

while $FullPeakFound = FALSE$ **do**

Step 4. $P^N \leftarrow \{e_t \in \hat{E} \setminus P : e_t \geq \alpha_2 \mu + \beta_2 \sigma \wedge (e_{t-1} \in P \vee e_{t+1} \in P)\}$

if $P^N = \emptyset$ **then**

 | $FullPeakFound \leftarrow TRUE$

else

 | $P \leftarrow P \cup P^N$

Output: Peak points $P \subset \hat{E}$

3. RESULTS AND DISCUSSION

We present our results and insights in this session, starting by briefly describing the computational setup. The parameters employed are summarized in Table 1 and largely follow on Corman et al. (2021) for the stochastic process models and Trivella, Wang, and Corman (2021) for the train dynamics (e.g., resistance parameters). We consider a system of $N = 6$ trains assuming they are identical, i.e., they are subject to the same dynamics. We simulated 5000 trajectories of this system for a time horizon $T = 2000$ s discretized with $\Delta t = 1$ s. In this setting, the total simulation time was roughly 40 and 72 seconds for the OU and DMR model, respectively, when using Matlab R2021b on a laptop with a processor i7-10610U and 16 GB RAM.

Table 1: Parameters of trains and processes.

Name	Value	Unit	Name	Value	Unit	Name	Value	Unit
m	500	t	v_{CRUISE}	35	m/s	a_{DET}	-0.55	m/s ²
ρ	1.06	-	v_{MAX}	40	m/s	α_n	$2 \cdot 10^{-5}$	-
γ_1	5.8	kN	$v_{APPROACH}$	20	m/s	β_n	0.02	-
γ_2	0.072	kN s/m	d_0	3.2	km	σ_n	0.05	-
γ_3	0.013	kN (s/m) ²	d_{MIN}	3.0	km			

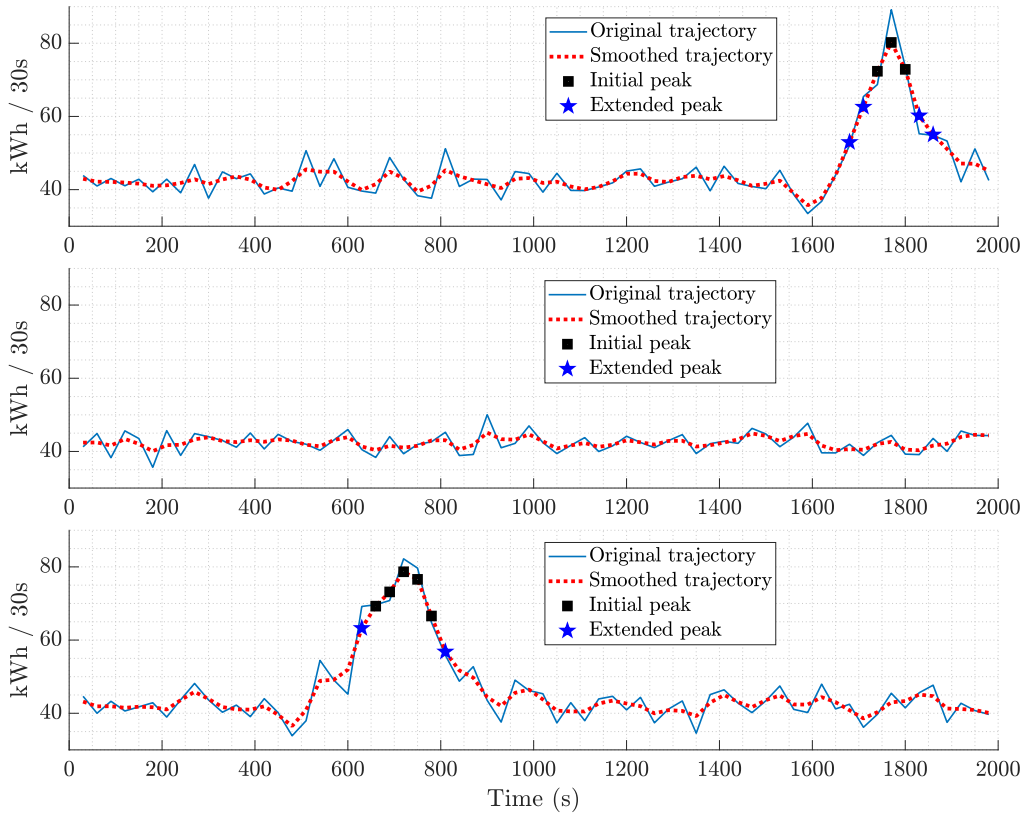
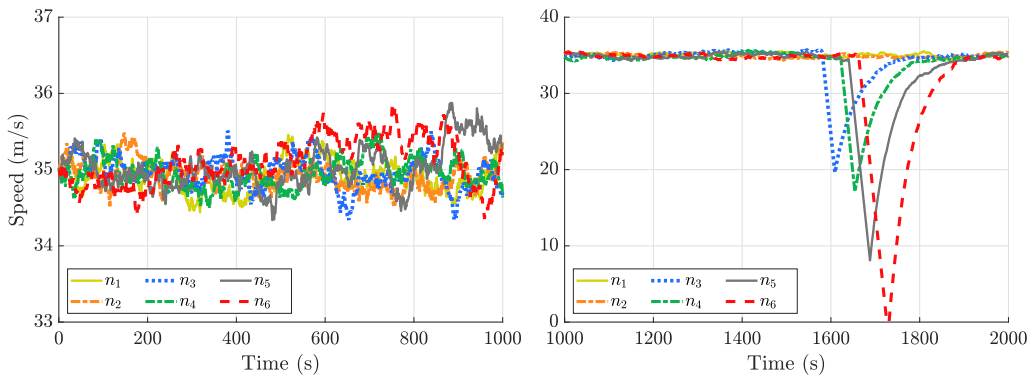
We analyze first the system from a traffic regularity perspective in Table 2. We focus on two indicators: the percentage of trajectories where a yellow signal is triggered, and the average *first time to yellow* (FTTY), which records the first time in which a train triggers a yellow signal (trajectories without yellow signal count as $T = 2000$ s in the average). Two main insights from this table are:

- The OU model incurs many more triggers than the DMR model, and when this happens, it is much earlier for any of the followers as shown by the FTTY statistics. These results suggest that an ATO model aware of the location of the traffic (in addition to the speed) leads to a higher level of traffic regularity that enables better exploiting the capacity of the network.
- The percentage of yellow signals increases substantially when moving from the first follower (train 2) to the last (train 6). This implies that yellow signals propagate backwards, that is, a train n decelerating will very likely trigger train $n + 1$ to decelerate or brake too. This is a major cause of peaks in energy consumption, which prompts our analysis next.

Table 2: Performance indicators: Regularity.

Train (1 is the leader)		1	2	3	4	5	6
Triggers (%)	OU	0	15.2	29.0	41.7	52.0	60.1
	DMR	0	2.8	4.7	6.0	7.6	8.7
FTTY (s)	OU	2000	1891	1795	1702	1625	1560
	DMR	2000	1981	1970	1961	1950	1944

We now focus on energy consumption and show some examples of our peak detection procedure in Figure 1. We verified that our method is accurate as the identified peaks correspond indeed, in almost all cases, to multiple trains accelerating after a yellow signal. For example, the peak in the top panel is due to four trains accelerating as illustrated in the time-speed profiles in Figure 2. Note that in this figure we vary the scale of the y-axis in the first half (left) and second half (right) of the horizon to show more clearly the stochastic variations and yellow signal effect.

**Figure 1:** Examples of simulated system energy trajectories and detected peaks.**Figure 2:** Example of time-speed profiles of a six-trains system.

We computed three metrics related to the energy use of the full six-train system, where energy consumption is measured over intervals of 30 seconds. The results in Table 3 are averaged over 5000 trajectories and show that the total consumption under OU is 2.2% higher than under DMR. This effect is due to trains in the latter model less frequently having to decelerate, brake, and accelerate again, following a yellow signal. Moreover, OU trajectories exhibit a highest consumption point that is about 25% higher than DMR trajectories, which is a substantial difference. Finally, our method detected peaks in over 60% of the OU trajectories, while the same number is less than 10% in DMR trajectories. The main implication from these results is that an ATO system (represented here by the DMR process) has the potential not only to reduce the overall energy use but also to avoid yellow signal propagation and the consequent generation of critical power peaks.

Table 3: Performance indicators: Energy.

	Model	OU	DMR
Total energy consumption (kWh)		2876	2814
Max consumption in trajectory (kWh)		64.8	52.0
Trajectories with detected peaks (%)		60.6	9.3

4. CONCLUSIONS

This work is the first to analyze a stochastic railway traffic flow model for a string of leader-follower trains. By simulating different stochastic processes, we observed that the propagation of yellow signals in such a system and the consequent synchronized acceleration of trains may generate significant peaks in energy use, which is a main concern of railway operators. We thus examined the performance of the system based on a peak detection method we develop, and derived insights on the potential benefits of ATO for both smoothing peaks and improving regularity.

Future work involves performing sensitivity analysis on relevant train and process parameters, and developing operational measures to actively avoid the propagation of yellow signals, such as (i) accounting for regenerative energy, (ii) optimizing the acceleration decisions of individual trains, and (iii) adopting waiting policies for trains that have triggered a yellow signal.

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