Revenue Maximizing Tariff Zone Planning for Public Transport Companies

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SHORT SUMMARY

This paper presents the tariff zone planning problem, which aims to maximize ridership and revenue for public service providers. We propose an optimization approach basis on partitioning the service area into zones and finding a price per zone such that the total expected revenue is maximized. It is assumed that the price per zone takes a discrete set of values. Public transport trips depend on the price system; public transport passengers always choose the time-shortest path. We propose new mixed-integer programming (MIP) that can optimally solve instances of sizes of more than 120 stops and enforce tariff zones to specific spatial patterns (rings and stripes, for example). The results demonstrate that expected revenue can be maximized without decreasing transit ridership. However, results show enforcing tariff zones to a specif spatial pattern reduces R and CPU time. This paper sheds light on urban public service providers enforcing contiguous tariff zones to maximize expected revenue.

Keywords: Contiguity, districting, mixed integer programming, public transportation, revenue management.

1. INTRODUCTION

Public transport companies are under pressure to reduce operational costs and increase the number of passengers and revenue. The design of the tariff system has been proven valuable to impact revenues in public transportation. There exist different tariff systems in public transport (Schöbel & Urban, 2020). The most widely accepted public transport tariff system is the counting zones tariff system. The literature on combining zoning and fare problems is scarce. The study of Hamacher and Schöbel (Hamacher & Schöbel, 2004) and Otto and Boysen (Otto & Boysen, 2017) are two examples of the literature that tackle a counting zones tariff system. They heuristically solve problems by minimizing the deviation from a given reference price. Their solution method is able to solve problems with up to 400 stops. While Otto and Boysen aim to maximize revenue by using a MIP model can solve optimally problems of up to 5 zones, 50 customers (demand points), and 10 stops. We contribute to this literature by a new approach yielding a counting zones tariff system that maximizes the total expected revenue (R). Therefore, we present a new mixed-integer programming (MIP) model and a MIP-based heuristic method to solve the revenue maximizing tariff zone problem (RMTZP).

We propose an optimization approach based on partitioning the service area into zones and finding a price per zone (counting zone tariff) that maximizes expected R. It is assumed that (i) the price per zone takes denumerable values, (ii) the number of public transport trips depends on the price (system), and (iii) public transport passengers always choose the time-shortest path. Our new model formulation is: (1) flexible to adjust to any objective function; (2) not limited to a predefined number of tariff zones; we impose contiguity of the tariff zones using the properties of primal and dual graphs, (3) coming with a new set of constraints that ensures contiguity and forces tariff zones to a desired spatial pattern (rings or stripes) without altering the model structure; (4) able to optimally solve instances of up to 120 districts (stops) within reasonable time using off-the-shelf solvers.

2. METHODOLOGY

We consider a public transport graph $\mathscr{G}^{\text{PT}} : (\mathscr{I}, \mathscr{A}, \tau_{ij})$ with nodes $i \in \mathscr{I}$, arcs $(i,j) \in \mathscr{A}$, and travel time τ_{ij}. The set of nodes $\mathscr{I}$ represents the public transport stops and the set of arcs $\mathscr{A}$ indicates the public transport connections between adjacent nodes (gray arcs and nodes, Figure 1). We impose contiguity of each tariff zone using the properties of primal and dual graphs (Validi, Buchanan, & Lykhovyd, 2021). As shown in Figure 1, each stop is located in a unique (artificial) district. Let $\mathscr{G}^{\text{BO}} : (\mathscr{N}, \mathscr{B})$ be the district border graph (blue arcs and nodes), with nodes $\mathscr{N}$ and arcs $\mathscr{B}$. Here, we consider $\mathscr{G}^{\text{BO}}$ as the dual to $\mathscr{G}^{\text{PT}}$.

The problem includes a set $\mathscr{S}_{ij}$ that contains the arcs that belong to the time-shortest path from $i \in \mathscr{I}$ to $j \in \mathscr{I}$. Due to the properties of planar dual graphs, for each arc $(i,j) \in \mathscr{A}$ there exist two intersecting arcs $((n,m), (m,n)) \in \mathscr{B}$. Let us define set $\mathscr{D}_{ij}$ denoting the border arcs (n,m) corresponding to $(i,j) \in \mathscr{A}$. Now, let us define the set $\mathscr{OD}_{ij}$ containing the border arcs along the time-shortest path from $i \in \mathscr{I}$ to $j \in \mathscr{I}$. For example, Figure 1 shows $\mathscr{G}^{\text{BO}}$, $\mathscr{G}^{\text{PT}}$, and the time-shortest path from node i1 to node i6, thus the set $\mathscr{D}_{i,j}$ is defined by blue arcs, $\mathscr{S}_{i1,i6}$={(i1,i2), (i2,i5), (i5,i6)}, and $\mathscr{OD}_{ij}$={(n1,n4), (n4,n1), (n4,n5), (n5,n4), (n5,n8), (n8,n5)}.

Following, we summarize the variables and main constraints of our MIP model **P1** to solve RMTZP:

Decision variables

- X_{ijt} = 1, if $t = 1, \ldots, T_{ij}$ tariff zones are visited on the shortest path from $i \in \mathscr{I}$ to $j \in \mathscr{I}$ (0, otherwise), with T_{ij} as the maximum number of tariff zones along the shortest path from i to j
- Y_{nm} = 1, if a tariff zone border is established along border arc $(n,m) \in \mathscr{B}$ (0, otherwise)
- W_{nm} = artificial flow along the border arc $(n,m) \in \mathscr{B}$

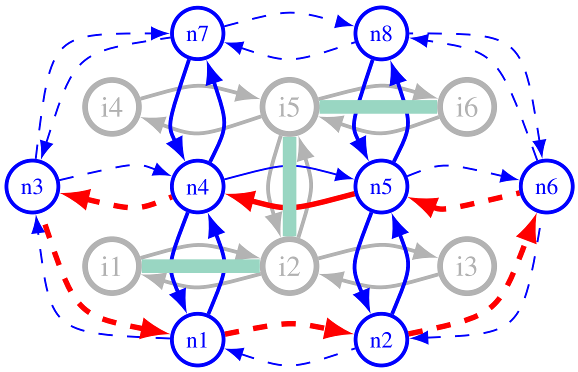
Constraints

- The number of tariff zones visited from i to j is equal to 1 plus the number of tariff zone borders crossed along the shortest path from node i to j (i.e., coupling Y_{nm} and X_{ijt}).
- Flow constraints that indicate flow out minus flow in along a border arc $(n,m) \in \mathscr{B}$ must be equal to b_n at border node n. b_n indicates the amount of artificial outflow or inflow at node $n \in \mathscr{N}$. Values of b_n depend on the desired spatial pattern of the tariff zone. The flow conservation constraints ensure contiguous tariff zones (Figure 1).

Relaxation of RMTZP:

The size of the RMTZP is mainly influenced by the number of O-D tuples. For this reason, we consider a subset, $\mathscr{C}$ of all O-D tuples. Set $\mathscr{C}$ contains $\gamma \cdot \mid \mathscr{I} \mid$ of O-D tuples with highest R. We

Figure 1: Graphs of the RMTZP. Note, $\mathcal{G}^{\text{BO}}$ contains more nodes than necessary for a dual graph. $\mathcal{G}^{\text{BO}}$ is shown in blue while $\mathcal{G}^{\text{PT}}$ is displayed in light gray. Shortest path from stop (district) i1 to stop (district) i6 is highlighted in green. Assume t=2 tariff zones are optimal for O-D tuple i1-i6 ($X_{i1,i6,2}$=1) and the tariff zone border is between node i2 and i5, then $Y_{n4,n5}$+$Y_{n5,n4}$=1. This in turn induces and artificial flow along the border arcs to ensure contiguous zones (red).



propose a MIP-based heuristic to find $\mathscr{C}$. Then, the model **P1** is solved by considering O-D tuples $(i, j) \in \mathscr{C}$ instead of all $i \in \mathscr{I}$ and $j \in \mathscr{I}$.

Price problem:

The zone problem **P1** depends on a given price system $p \in \mathscr{P}$. The trip price under a counting zone tariff system depends on the visited zones along the trip and the price per zone. Let $r_{ijt}(\pi_{p_t})$ represent the R on the shortest path from $i \in \mathscr{I}$ to $j \in \mathscr{I}$ given price system $p \in \mathscr{P}$, with π_{p_t} as the price per zone when visiting t zones under price system $p_t \in \mathscr{P}$. R in **P1** is given by:

$$\textbf{Maximize } R(p) = \sum_{i \in \mathscr{I}} \sum_{j \in \mathscr{I}} \sum_{t=1}^{T_{ij}} r_{ijt}(\pi_{p_t}) \cdot X_{ijt} \tag{1}$$

Therefore, we solve $|\mathscr{P}|$ independent zoning problems and select the solution with maximum R.

3. COMPUTATIONAL EXPERIMENTS AND RESULTS

In this section, we present the design of computational experiments and results. We evaluate the performance of the MIP model and our MIP-based heuristic under different problem sets. The performance is evaluated in terms of R and CPU time. We generate a set of artificial instances representing the service area of a city. Total demand depends on the number of inhabitants at each zone i, defined as a uniform [10,000; 20,000]. The total number of trips from i to j follows a gravity model, while public transport shares follow a multinomial logit model.

We solve all problem sets to optimality and compare the MIP model against the MIP-based heuristic, considering the constraints to enforce tariff zones to have a ring pattern. Problem sets consider different realistic sizes of $|\mathscr{I}|$= {49, 81, 121}, $|T_{ij}|$=7, and $|\mathscr{P}|$=10, and a network connectivity level given by {0.25, 0.5, 0.75, 1}. The larger the network connectivity level the larger is $|\mathscr{A}|$ for given $|\mathscr{I}|$.

Figure 2 shows R of the solution obtained with MIP model and MIP-based heuristic over 10 seeds,

Figure 2: Average R comparison of our MIP model and the MIP-based heuristic. All problems are solved to optimality.

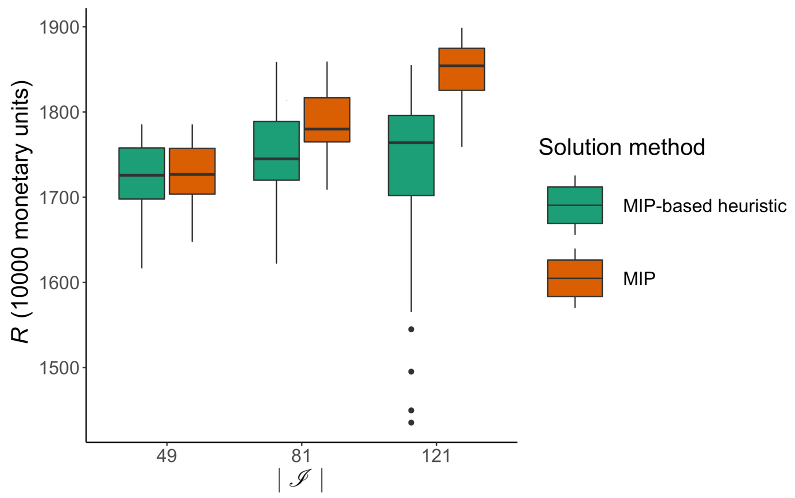


Table 1: Average R of the MIP model varying: constraints to enforce a ring pattern, and values of node degree (a_n) of the border nodes $n \in \mathcal{N}$

a_n	Ring pattern		No pattern	
	R (10000 monetary units)	CPU time (seconds)	R (10000 monetary units)	CPU time (seconds)
2	1965.36	125.76	1977.92	10559.98
3	1965.36	1639.02	1977.92	9051.35
4	1989.42	3308.62	1992.07	12980.14

and γ over the values { 2, 4, 8, 10, 15, 20, 25, 30}. Our first results show that the MIP model provides a solution with higher R than the MIP-based heuristic solution. On average, our MIP-based heuristic under estimates the optimal R by 2.54% but it is faster by 42.97% (not shown here).

The second set of experiments is devoted to determine the impact of enforced spatial patterns on R. This numerical study is focused on the MIP model **P1** and the instance with $|\mathcal{I}| = 49$. We determine the solution for different values of border node degree a_n given by $\{2, 3, 4\}$. The value of a_n indicates the maximum number of adjacent tariff zones that gather at a district border node $n \in \mathcal{N}$. Table 1 shows the average R and CPU times of solutions with and without a ring pattern. Results demonstrate that solutions with a lower R are obtained when constraints are included to enforce a desired spatial pattern compared to cases with no desired pattern. Solutions without any spatial pattern have an R that is on average 0.43% higher than solutions with a ring pattern. The CPU time for the solutions with no spatial pattern is 85.12% higher than for solutions with the ring pattern.

4. CONCLUSIONS

In this paper, we investigate how to design a counting zones tariff system to maximize the revenues of public transportation service companies. The price for a trip depends on the price per tariff zone and the number of visited zones. This approach is well-known and accepted by passengers and practitioners. We design an MIP model and an MIP-based heuristic to design an optimal counting zones tariff system and solve the price problem. Our approach is based on the properties of dual and primal graphs enabling contiguity of tariff zones and enforce spatial patterns of the zones. The proposed methods can solve instances of reasonable sizes. Our approach is flexible enough to enforce the counting zones tariff system to any spatial pattern. The results show that enforcing tariff zones to a specif spatial pattern reduces R and CPU time.

Currently, we are working on testing our approach with real data from the San Francisco Bay Area with 1,415 districts (i.e., stops). Due to the irregular shape of the San Francisco Bay Area, we are interested in studying how to enforce tariff zones to follow a desired spatial pattern. In addition, we plan to analyze price systems with discounts.

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