On the overlap between regional paths in multi-region macroscopic fundamental diagram traffic equilibrium models

Lawrence Christopher DUNCAN*[1], Thomas Kjær RASMUSSEN[2]

[1] Postdoc, [2] Associate Professor,

Department of Technology, Management, and Economics, Technical University of Denmark, Denmark

SHORT SUMMARY

Multi-region Macroscopic Fundamental Diagram (MFD) traffic equilibrium models have been developed as a computationally efficient alternative to classical route choice traffic assignment models. There remains a gap in the research, however, for adapting and extending existing state-of-the-art route choice models to formulate analogous (but context specific) regional path choice models. In this study, we focus on adapting/extending the most commonly used Path Size Logit (PSL) route choice model. A key difference between route choice and regional path choice is that for the latter the travel time experienced traversing through a region depends on the regional path being taken. Therefore, the degree to which two regional paths overlap depends not only on the regions that are shared but also the travel time that is shared within those shared regions. Accounting for this, we formulate a new Intersectional PSL regional path choice model and discuss/demonstrate its theoretical properties compared to standard PSL.

Keywords: discrete choice modelling, macroscopic fundamental diagram, multi-region system, regional path choice, regional path correlation, transport network modelling

1. INTRODUCTION

An issue that is commonly experienced by government transport professionals across the globe, is that current road network travel behaviour models – which are typically based on static route choice traffic assignment – are highly detailed and can thus be computationally expensive to solve. This presents a challenge for e.g. policy analysis where lengthy computation times can limit how rigorous analyses can be. For example, limiting a procedure for setting toll prices to trail-and-error rather than a proper optimisation process.

Multi-region Macroscopic Fundamental Diagram (MFD) traffic equilibrium models have been developed as a computationally efficient alternative to classical route choice traffic assignment models. The idea is to partition road networks into a set of regions, where the traffic state in each region is described by a speed-accumulation MFD function (see e.g. Geroliminis & Daganzo, 2008), and then model how traffic exchanges between the regions. Thus, rather than dealing with thousands of links as in detailed network models, multi-region MFD traffic equilibrium models provide a possibility to work with a much smaller number of regions, which should dramatically reduce computation times.

Yildirimoglu & Geroliminis (2014) developed the first such model, where a Multinomial Logit (MNL) regional path choice model is adopted for traffic flow equilibration along with a stochastic network loading procedure to estimate time-dependent regional trip lengths. Batista & Leclercq (2019) later developed a 'stochastic' approach for the traffic equilibrium based on monte carlo simulations of trip length / MFD distributions to account for correlations between regional path utilities. Mariotte et al (2020) adopt a Wardrop user equilibrium (Wardop, 1952) variant to dictate the regional path choice, and then also optimise the regional path choices to fit data. Extending these works, Batista et al (2021) recently developed a heuristic approach for updating the traffic-dependent trip lengths / regional paths during the dynamic traffic assignment, while numerous methods have been proposed for dynamically modelling the transfer of traffic flow at region borders (e.g. Yildirimoglu & Geroliminis, 2014; Mariotte & Leclercq, 2019; Mariotte et al, 2020).

So far, as mentioned above, the two main regional path choice models that have been adopted thus far are a standard MNL choice model and a specially designed Monte Carlo Simulation-based (MCS) choice model. The deficiency of the MNL model, in its inability to capture correlations between overlapping routes / regional paths, is well-known (see e.g. Ben-Akiva & Bierlaire, (1998), Cascetta et al (1996)) Furthermore, the MCS model is not generic since it utilises features specific to one dynamic modelling approach, and potentially has the typical drawbacks of monte carlo simulation approaches, i.e. computationally expensive / less well-defined convergence properties, which become more problematic at larger scales.

Addressing these issues, in this study, we adapt and extend existing state-of-the-art route choice models to formulate analogous (but context specific) regional path choice models. Due to their computational convenience, well-behaved convergence properties, and ease of calibration, Path Size Logit (PSL) correction-term models (Ben-Akiva & Bierlaire, 1998) (or similar) are the most common type of correlation-based model used in practice. PSL models capture route correlations by including correction terms within the probability relations to penalise routes for 'overlapping' (i.e. sharing links) with other routes, see Duncan et al (2020) for a recent review of PSL models.

As we explore in this study, however, the overlapping of regional paths has an extra dimension. One of the key differences between route choice and regional path choice is that for the latter the travel time experienced traversing through a region depends on the regional path being taken. Therefore, the degree to which two regional paths overlap depends not only on the regions that are shared but also the travel time that is shared within those shared regions. To account for this, in this study, we formulate a new Intersectional PSL regional path choice model and discuss/demonstrate its theoretical properties compared to MNL and Standard PSL.

2. GENERIC REGIONAL PATH CHOICE SETUP

Here, we briefly set up a generic regional path choice model system. An area is partitioned into a set of R regions. Without loss of generality, we focus on the regional path choice for a single regional OD movement between two different regions. A regional path (also referred to as r-path) is defined as a sequence of regions traversed when travelling the OD movement. P is the choice set of regional paths, P_r is the set of r-paths that traverse through region $r \in R$, and R_p is the set of regions in r-path $p \in P$. As mentioned above, a key feature of regional path choice modelling is that the distance a driver travels through a region (and therefore travel time) depends on the r-path the driver is taking. Suppose then that, in some generic situation, a driver is presented with a set of region travel times that are r-path-specific, (and assume that drivers only consider travel time in their r-path utilities). Denote therefore $t_{p,r}$ as the travel time of region $r \in R_p$ when traversing r-path $p \in P$. Supposing that the travel time for a r-path can be attained through summing up the travel times of its regions, then the total travel time of r-path $p \in P$ is: $T_p = \sum_{r \in R_p} t_{p,r}$. Lastly, the probability that r-path $p \in P$ is chosen is denoted by Q_p.

3. MULTINOMIAL LOGIT

For pedagogical purposes, we shall begin by first presenting the MNL regional path choice model that Yildirimoglu & Geroliminis (2013) adopt in the first application of r-path choice models. The well-known MNL probability relation for r-path $p \in P$ is:

$$Q_p = \frac{e^{-\theta T_p}}{\sum_{k \in P} e^{-\theta T_k}},$$

where $\theta > 0$ is the Logit scaling parameter.

To demonstrate the inability of MNL to capture r-path correlations, consider in Fig. 1A-B a similar version for r-path choice modelling of the famous 'loop-hole' network (also known as the red-bus/blue-bus network) presented in Cascetta et al (1996). R-path 1 is R1→R2→R3→R6, r-path 2 is R1→R2→R4→R6, and r-path 3 is R1→R5→R6. For all demonstrations a parameter value of $\theta = 1$ is adopted. It is assumed here that the travel time for each region is the same regardless of which r-path is being travelled, i.e. $\alpha_1 = \alpha_2$ and $\alpha_3 = \alpha_4$. And, for this demonstration, $\alpha_1 = \alpha_2 = 0.001$ (i.e. the relative travel times of the origin/destination regions are neglible) and $\alpha_3, \alpha_4 \in (0,1)$ is the variable. For all α_1, α_2 (where $\alpha_1 = \alpha_2$), the r-path travel times are equal: $T_1 = T_2 = 2\alpha_1 + 1 = T_3 = 2\alpha_2 + 1$. Under the MNL model, the r-path choice probabilities are therefore $Q_1 = Q_2 = Q_3 = \frac{1}{3}$, regardless of α_3, α_4. However, r-path 1 and r-path 2 are correlated due to sharing region 2, where the degree to which the r-paths are correlated depends on α_3, α_4. For α_3, α_4 close to zero, r-path 1&2 share very little travel time and thus the choice probabilities should be $\frac{1}{3}$ (as there are in effect three distinct r-paths with equal travel times). However, as α_3, α_4 tend towards 1 r-paths 1&2 become more correlated to the point where they basically become the same r-path. At this point, there become in effect just two r-paths to choose from and the probabilities of r-paths 1&2 should equal the probability of r-path 3, where $Q_1 \cong Q_2 \cong \frac{1}{4}$, $Q_3 \cong \frac{1}{2}$. As well-known and demonstrated above, MNL fails to represent this.

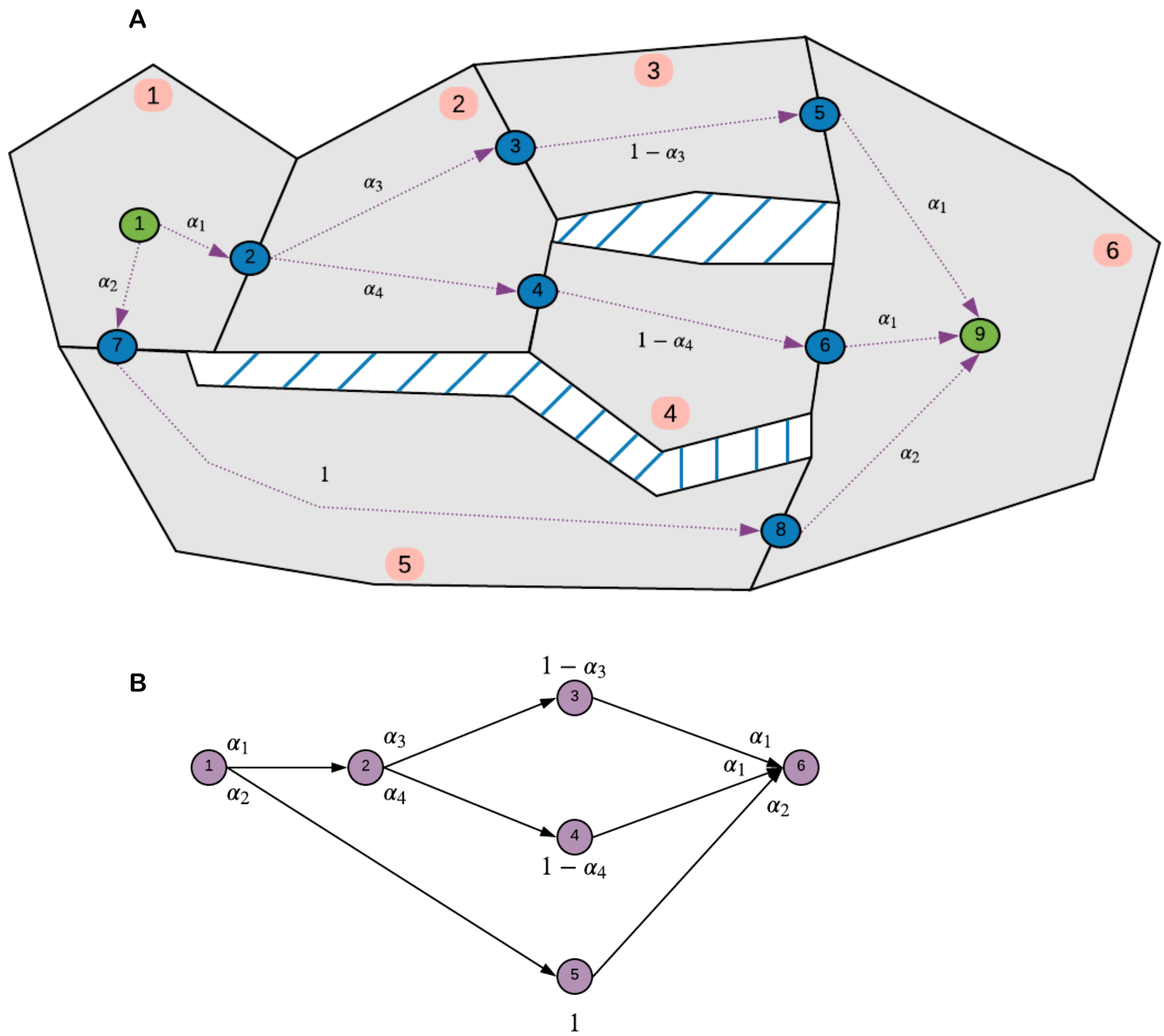

Fig. 1. Loop-hole multi-region system. A: Region representation. B: Regional network.

4. PATH SIZE LOGIT MODELS

Path Size Logit route choice models include correction terms in the probability relation to adjust probabilities to capture route correlations. Here, we adapt the approach for the context of regional path choice modelling. The choice probability relation for r-path $p \in P$ is:

$$Q_p = \frac{e^{-\theta T_p + \beta \ln(\gamma_p)}}{\sum_{k \in P} e^{-\theta T_k + \beta \ln(\gamma_k)}},$$

where $\beta \ln(\gamma_p)$ is the path size correction term for r-path $p \in P$, $\beta \geq 0$ is the path size scaling parameter, and $\gamma_p \in (0,1]$ is the path size term for r-path $p \in P$. As we discuss further below, it is not possible for a r-path to be completely distinct due to unavoidable sharing of region travel time in the originating and destinating regions. If a r-path were to be distinct, however, it would have a path size term equal to 1, resulting in no penalisation. Less distinct r-paths have smaller path size terms and incur greater penalisation.

Standard Path Size Logit

By adapting the definition of the PSL model path size term proposed by Ben-Akiva & Bierlaire (1998), a Standard PSL (SPSL) r-path choice model can be formulated. The SPSL path size term for r-path $p \in P$ is defined as follows:

$$\gamma_p^{SPS} = \sum\nolimits_{r \in R_p} \frac{t_{p,r}}{T_p} \frac{1}{\sum_{k \in P_r} 1}$$

To dissect the SPSL path size term: each region r in r-path p is penalised (in terms of decreasing the path size term and hence the utility of the r-path) according to the number of r-paths in the choice set that also use that region $\left(\sum_{k \in P_r} 1\right)$, and the significance of the penalisation is also weighted according to how prominent region r is in r-path p, i.e. the travel time of region r for r-path p in relation to the total travel time of r-path p $\left(\frac{t_{p,r}}{T_p}\right)$.

To demonstrate how SPSL can capture r-path correlations, consider again the loop-hole network in Fig. 1. For all path size demonstrations, a parameter value of $\beta = 0.8$ is adopted. For the same conditions as described above for the MNL demonstration (i.e. $\alpha_3 = \alpha_4$, $\alpha_1 = \alpha_2 = 0.001$), Fig. 2A displays the SPSL r-path choice probabilities for α_3, α_4 in the range $(0,1)$. As shown, for low α_3, α_4, the prominence of the shared travel time region (region 2) is small, and therefore the r-path 1&2 path size terms are close to 1 and the probabilities are not adjusted for correlation (i.e. $Q_1 = Q_2 = Q_3 = \frac{1}{3}$). As α_3, α_4 increase towards 1, however, the prominence of the shared travel time region increases, the path size terms thus decrease, and the r-path 1&2 probabilities are adjusted for the correlation (i.e. $Q_1 = Q_2 = \frac{1}{4}$, $Q_3 = \frac{1}{2}$).

R-path choice modelling presents unique challenges. The most notable of which is that the region travel times are not the same for all r-paths that travel through it. This is in contrast to traditional route choice where the link travel times are assumed the same for all travellers. This can be problematic for the SPSL model, since r-path k contributes to the path size term of r-path p in region r the same (i.e. 1) regardless of how much r-paths k and p actually overlap in region r. To demonstrate, consider again the loop-hole network in Fig. 1, but where this time α_3 is varied with α_4 fixed, set to $\alpha_4 = 0.999$. For α_3 close to 0, the three r-paths are non-overlapping in terms of shared travel time (since the shared travel time in region 2 between r-paths 1&2 is minimal), and the r-path probabilities should thus be $Q_1 = Q_2 = Q_3 = \frac{1}{3}$. On-the-other-hand, for α_3 close to 1, the shared travel time in region 2 is significant and r-paths 1&2 become in-effect the same, and the probabilities should thus be $Q_1 = Q_2 = \frac{1}{4}$, $Q_3 = \frac{1}{2}$. Fig. 2B displays the SPSL r-path choice probabilities for α_3 in the range $(0,1)$. For α_3 close to 1, the probabilities are indeed close to $Q_1 = Q_2 = \frac{1}{4}$, $Q_3 = \frac{1}{2}$, however for α_3 close to 0 the probabilities are from $Q_1 = Q_2 = Q_3 = \frac{1}{3}$. This is because r-path 1 contributes fully to the path size term of r-path 2 in region 2 regardless of the travel time they actually share in the region.

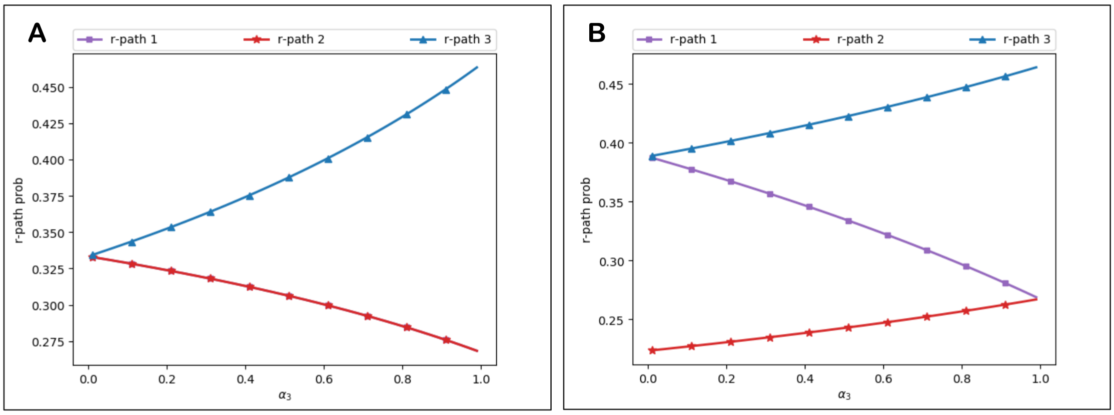

Fig. 2. Loop-hole multi-region system: SPSL r-path choice probabilities for varying α_3 ($\alpha_1 = \alpha_2 = 0.001$). A: $\alpha_4 = \alpha_3$. B: $\alpha_4 = 0.999$.

Intersectional Path Size Logit

As demonstrated above, the SPSL model has a deficiency in that path size terms only consider whether r-paths share regions, without actually considering how much the r-paths overlap in the region in terms of shared travel time. Addressing this problem, we thus formulate the Intersectional PSL (IPSL) r-path choice model. The IPSL path size term for r-path $p \in P$ is defined as follows:

$$\gamma_p^{IPS} = \sum\nolimits_{r \in R_p} \frac{t_{p,r}}{T_p} \frac{1}{\sum_{k \in P_r} \left(\frac{\min\left(t_{p,r}, t_{k,r}\right)}{t_{p,r}} \right)}$$

The IPSL path size term includes a path size contribution factor to weight the contributions of r-paths to path size terms. The contribution factors aim to reduce the contributions for r-paths that barely overlap (in terms of shared travel time) in regions. To dissect the IPSL contribution factor $\left(\frac{\min(t_{p,r}, t_{k,r})}{t_{p,r}}\right)$, $\min\left(t_{p,r}, t_{k,r}\right)$ is the travel time in region r that is shared by both r-path p and r-path k, i.e. the intersection of travel time in the region. If $t_{k,r} \geq t_{p,r}$, then the contribution of r-path k to the path size term of r-path p is 1, since all of the travel time of r-path p is shared with r-path k in region r. If $t_{k,r} < t_{p,r}$, however, then the contribution of r-path k is smaller than 1, as it is not the case that all of the travel time of r-path p in region r is shared with r-path k. The lesser the shared travel time of r-paths p and k in region r, the less the contribution of r-path k.

Considering again the loop-hole multi-region system in Fig. 1, Fig. 3A displays the SPSL r-path choice probabilities for α_3 in the range $(0,1)$, with $\alpha_1 = \alpha_2 = 0.001$, $\alpha_4 = 0.999$. As shown, overcoming the weakness of SPSL, the IPSL probabilities successfully achieve $Q_1 = Q_2 = Q_3 = \frac{1}{3}$ for α_3 close to 0. This is due to the path size contribution factor diminishing the contribution of r-path 1 to the path size term of r-path 2 in region 2. Another interesting feature is that r-path 1 has a greater probability than r-path 2 between $\alpha_3 = 0$ and $\alpha_3 = 1$, which is due to r-path 2 being less distinct, i.e. a greater proportion of its journey is shared (almost all of it).

An interesting question is what should happen when the origin/destination regions become over-prominent. In the case of the loop-hole multi-region system, supposing that $\alpha_1 = \alpha_2$, what should happen when α_1 becomes so large such that it is then arbitrary which r-path is chosen, as the shared travel time is in the compulsory origin/destination regions. Fig. 3B displays the IPSL r-path choice probabilities for increasing α_1, with $\alpha_2 = \alpha_1$, $\alpha_3 = \alpha_4 = 0.999$. As shown, for small α_1, the probabilities are close to $Q_1 = Q_2 = \frac{1}{4}$, $Q_3 = \frac{1}{2}$, since regions 1&6 (the origin/destination regions) are not prominent. However, as α_1 increases and regions 1&6 become more prominent, the probabilities tend towards $Q_1 = Q_2 = Q_3 = \frac{1}{3}$: all r-paths become less distinct as α_1 increases and in-effect become the same r-path consisting of just the origin/destination regions. In this case, the travel time in regions 1&6 is the same for all r-paths which is why the probabilities tend to $Q_1 = Q_2 = Q_3 = \frac{1}{3}$.

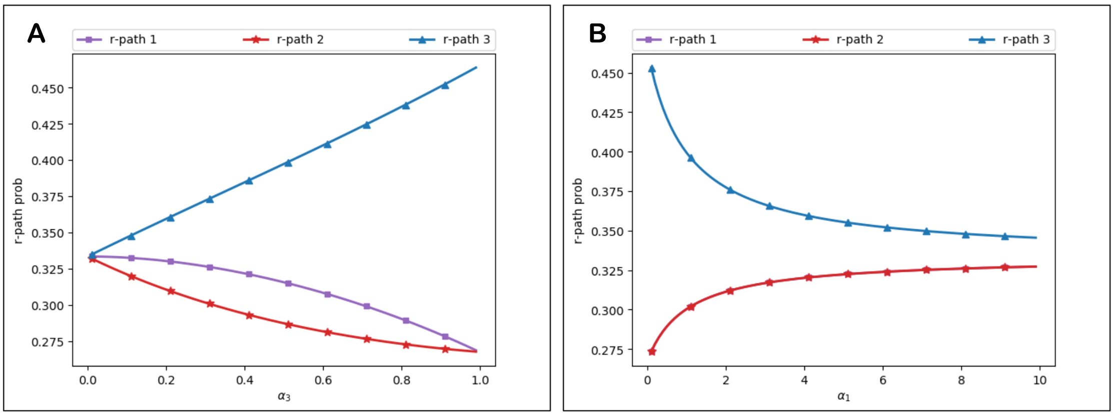

Fig. 3. Loop-hole multi-region system: IPSL r-path choice probabilities. A: for varying α_3 ($\alpha_1 = \alpha_2 = 0.001$, $\alpha_4 = 0.999$). B: for varying α_1 ($\alpha_2 = \alpha_1$, $\alpha_3 = \alpha_4 = 0.999$).

5. CONCLUSIONS

As shown in this study, the overlapping of regional paths has an extra dimension compared to traditional route choice. A key difference between route choice and regional path choice is that for the latter the travel time experienced traversing through a region depends on the regional path being taken. Therefore, the degree to which two regional paths overlap depends not only on the regions that are shared but also the travel time that is shared within those shared regions. Accounting for this, in this study, we formulated a new Intersectional PSL regional path choice model and discussed/demonstrated its theoretical properties compared to MNL and standard PSL. Future research includes embedding these new regional path models within a multi-region MFD traffic equilibrium framework, and calibrating the models with tracked regional path observation data.

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