Departure-expanded network for electric demand responsive feeder bus coordination with timetabled transit

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SHORT SUMMARY

On-demand feeder service using electric vehicles provides user-centered mobility solutions to increase connectivity in the low-demand area and reduce their negative impacts on the environment. However, most studies neglect the synchronization of feeder service and timetabled transit to minimize customers' waiting time at transit stations. Moreover, existing studies on electric vehicle routing problems assume charging stations are uncapacitated. We propose an on-demand first-mile feeder service to coordinate its service with timetabled transit using electric buses/shuttles. The problem is modeled on a departure-expanded graph and formulated as a mixed-integer linear programming problem. Several new contributions are proposed in this study: considering flexible bus stops based on the meeting points of customers' origins, coordinating arrival time at transit stations, and coordinating electric bus charging scheduling to ensure charging station capacity constraints. We provide an illustrative example and conduct numerical studies on a set of instances to validate the proposed methodology.

Keywords: feeder service, demand responsive transport, electric vehicle routing, transit

1. INTRODUCTION

On-demand feeder service could provide user-centered mobility solutions to increase connectivity and accessibility in the low-demand area. Different methods have been proposed to minimize vehicles' routing costs while considering customers' inconvenience (Wang, 2017; Ma et al., 2019; Chen et al., 2020; Montenegro et al., 2020). However, most studies neglect the synchronization issue of feeder service and timetabled mass transit to minimize customers' waiting time at transit stations. On the other hand, with the climate change crisis, transport network companies start deploying electric vehicles (EVs) to reduce their CO_2 emission. This emerging trend brings new challenges to managing the charging scheduling of EVs. While existing studies on electric vehicle routing problems have developed optimization models for partial recharges, most assume charging stations are uncapacitated (e.g., Keskin and Çatay, 2016; Bongiovanni et al., 2019b). To address the above issues, we propose an on-demand first-mile feeder service to coordinate its service with timetabled transit using electric buses/shuttles. The problem is modeled on a departure-expanded (layered) graph and formulated as a mixed-integer linear programming (MILP) problem. Our new contributions with respect to the state-of-the-art methodologies include: considering flexible bus stops based on the meeting points (within a walking distance) of customers' origins, coordinating bus arrival times at transit stations to minimize customers' waiting time, and coordinating electric bus charging scheduling to ensure charging station capacity constraints. We provide an illustrative example and conduct numerical studies on a set of instances to validate the proposed methodology.

2. METHODOLOGY

Notation

Sets	
$0, N+1$	Instances of the depot
$\mathcal{L}$	Set of $\bar{\mathcal{L}}$ departures of trains (layers) with positive ride requests for the planning period, $\mathcal{L} = 1,2,3,\dots,\bar{\mathcal{L}}$
G	Set of dummy (artificial) bus nodes g for $\bar{m}$ optional bus stops, $G = \bigcup_{\ell\in\mathcal{L}} G_\ell$, where $G_\ell = \{g_1^\ell, g_2^\ell, \dots, g_{\bar{m}}^\ell\}$
D	Set of dummy (artificial) transit station (transfer) nodes d for $\bar{t}$ transit stations, $D = \bigcup_{\ell\in\mathcal{L}} D_\ell$, where $D_\ell = \{d_1^\ell, d_2^\ell, \dots, d_{\bar{t}}^\ell\}$
S	Set of chargers
R	Set of customers
K	Set of buses
V	Set of optional bus stops, transit stations (including their dummies) and chargers, i.e. $V = G \cup D \cup S$
$V_0, V_{N+1}, V_{0,N+1}$	$V_0 = V \cup \{0\}, V_{N+1} = V \cup \{N+1\}, V_{0,N+1} = V \cup \{0, N+1\}$
$\mathcal{A}_R$	Set of walking arcs of customers, *i.e.* $\mathcal{A}_R = \{(r,j) \mid r \in R, j \in G_{\ell(d^r)}\}$.
$\mathcal{A}$	Set of all arcs, i.e. , $\mathcal{A} = \{(i,j) \mid i \in 0, j \in G \cup D \cup N+1\} \cup \{(i,j) \mid i \in S \cup D, j \in N+1\} \cup \{(i,j) \mid i \in S, j \in G \cup N+1\} \cup \{(i,j) \mid i \in D_\ell, j \in S \cup G_\ell, \ell \in \mathcal{L}, \ell(i) < \ell(j)\} \cup \{(i,j) \mid i \in G_\ell, j \in G_\ell \cup D_\ell, \ell \in \mathcal{L}, \ell(i) = \ell(j)\} \cup \mathcal{A}_R$.
Parameters	
w_{rj}	Walking distance from customer r's origin location to optional bus stop j
c_{ij}	Distance from vertex i to vertex j
t_{ij}	Bus travel time from vertex i to vertex j
w_{max}	Maximum walking distance for customers
u	Service duration at optional bus stop
q_i	Change of load at transit stop i
Q_{max}	Capacity of bus
$[e^i, l^i]$	Desired time windows at node i
d^r	Customers' desired (transit) departure at his/her drop-off transit station, $\forall d^r \in D$
$E_{min}, E_{max}, E_{init}^k$	Minimum, maximum states of charge (SOC) of buses and the initial SOC of bus k
α_s	Charging rate of charger $s \in S$
β	Energy consumption rate per kilometer traveled
M	Big positive number
Decision variable	
y_{ri}	1 if customer r is assigned to the optional bus stop i, and 0 otherwise
x_{ij}^k	1 if arc (i,j) is traversed by bus k, and 0 otherwise
$h_{kk'}^s$	1 if bus k' arrives at charging station s later than bus k, and 0 otherwise
L_i^k	1 if k visit charger s, and 0 otherwise
A_i^k	Arrival time of bus k at vertex i
Q_i^k	Load of bus k at vertex i
H^r	Ride time of customer r
τ_s^k	Charging duration for bus k at charger s
E_i^k	State of charge of bus k at vertex i

The problem settings are as follows. A fleet of homogenous electric buses/shuttles is utilized in a service area to provide on-demand feeder service to connect to a set of transit stations for a planning period. Customers book their service in advance by informing their origin locations, desired drop-off transit stations, and desired departures of their trains. A set of optional bus stops are generated to ensure a maximum walking distance (e.g. 1 km) between customers' origins and optional bus stops. Electric buses start their service at depot(s), visit optional bus stops to pick up customers, and drop them off at transit stops within a maximum waiting time (e.g. 15 minutes) ahead of their desired departures of trains. All customers need to be served. The initial SOCs of buses are heterogeneous at the beginning of the planning period. Buses can visit charging station(s) for recharge only after dropping off all onboard customers. Chargers are heterogeneous and private-owned. We consider partial recharge with linear charging behavior, and the energy consumption is also proportional to the distance traveled. Note that we assume customers are informed in real-time about the arrival times of their buses and no waiting time at their pickup bus stops.

The considered problem is modeled on a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{A})$, where $\mathcal{V}$ denotes all vertices, and $\mathcal{A}$ the set of edges. The vertices contain the depot (and its dummy), a set of layered dummy nodes for optional bus stops, layered dummy nodes for transit stations, chargers, and customers' origin locations. The definition of vertices $\mathcal{V}$ and edges $\mathcal{A}$ is presented in Notation. The graph is a transit timetabled departure expanded network in which the number of layers is the same as the number of departures of a served transit line where there are positive customers' ride requests. Buses visit bus stops within the same layer. The problem is formulated as a MILP problem. The objective function (1) minimizes the weighted sum of total bus travel time and charging time, customers' in-vehicle ride time, and customers' waiting time at transit stations. Constraints (2) and (3) state that buses start and end their services at the depot. Constraints (4) and (5) state that buses can be recharged at most once and each bus stop node can be visited at most once. Constraint (6) is the flow conservation at nodes. Constraints (7) and (8) ensure each customer is connected by one bus stop with a walking distance of less than w_{max}. Constraint (9) states that a bus stop is visited when there are assigned customers. Constraints (10) and (11) ensure buses drop off customers at their desired departure transit nodes, and customers are picked up and dropped off by the same buses. Constraints (12)−(16) state the changes of (customer) load at bus stops and transit stops, and load capacity constraints. Constraint (18)-(19) calculates the customer's ride time. Constraint (20) states buses' time windows constraint at transit stations. Constraints (21)−(25) are buses' SOC capacity constraints, and SOC changes after discharging and charging. Bus arrival time consistency is ensured by constraints (17) and (26). Constraints (27)-(29) states when two buses visit the same charger at the same time, one must wait until the other finishes its charging. Constraints (30)−(33) define the range of all decision variables. Note that all arcs belong to the arc set $\mathcal{A}$.

$$\text{Min } \lambda_1 \sum_{k \in K} \Big(\sum_{(i,j) \in \mathcal{A} \setminus \mathcal{A}_R} t_{ij}\, x_{ij}^k + \sum_{s \in S} \tau_s^k \Big) + \lambda_2 \sum_{r \in R} H^r + \lambda_3 \sum_{i \in V} \sum_{j \in D} \sum_{k \in K} (l^j - A_j^k)\, x_{ij}^k \tag{1}$$

$$\sum_{j \in G \cup \{N+1\}} x_{0j}^k = 1, \qquad \forall k \in K \tag{2}$$

$$\sum_{i \in \{0\} \cup S \cup D} x_{i,N+1}^k = 1, \qquad \forall k \in K \tag{3}$$

$$\sum_{j \in G \cup \{N+1\}} x_{sj}^k \le 1, \qquad \forall k \in K, s \in S \tag{4}$$

$$\sum_{k \in K} \sum_{i \in V_0} x_{ij}^k \le 1, \qquad \forall k \in K, j \in G \tag{5}$$

$$\sum_{i \in V_0} x_{ij}^k - \sum_{i \in V_{N+1}} x_{ji}^k = 0, \qquad \forall k \in K, j \in V \tag{6}$$

$$\sum_{i \in G} y_{ri} = 1, \qquad \forall r \in R \tag{7}$$

$$\sum_{i\in G} w_{ri}y_{ri} \leq w_{max}, \qquad \forall r \in R \tag{8}$$

$$\sum_{r\in R} y_{rj} \leq M \sum_{i\in V_0} x_{ij}^k, \qquad \forall k \in K, j \in G \tag{9}$$

$$\sum_{i\in G} x_{id^r}^k = 1, \qquad \forall k \in K, r \in R \tag{10}$$

$$\sum_{j\in V_0} x_{ji}^k \geq \sum_{j\in G} x_{jd^r}^k - M(1 - y_{ri}), \qquad \forall k \in K, i \in G, r \in R \tag{11}$$

$$Q_j^k \geq Q_i^k + \sum_{r\in R} y_{rj} - M\left(1 - x_{ij}^k\right), \qquad \forall k \in K, i \in V_0, j \in G \tag{12}$$

$$Q_j^k \leq Q_i^k + \sum_{r\in R} y_{rj} + M\left(1 - x_{ij}^k\right), \qquad \forall k \in K, i \in V_0, j \in G \tag{13}$$

$$Q_j^k \geq Q_i^k - q_j - M\left(1 - x_{ij}^k\right), \qquad \forall k \in K, i \in G, j \in D \tag{14}$$

$$Q_j^k \leq Q_i^k - q_j + M\left(1 - x_{ij}^k\right), \qquad \forall k \in K, i \in G, j \in D \tag{15}$$

$$0 \leq Q_i^k \leq Q_{max}, \qquad \forall k \in K, i \in V_{0,N+1} \tag{16}$$

$$A_j^k \geq A_i^k + t_{ij} + u - M(1 - x_{ij}^k), \qquad \forall k \in K, i \in V \backslash S, j \in V_{N+1} \tag{17}$$

$$H^r \geq A_{d^r}^k - A_i^k - u - M(1 - y_{ri}), \qquad \forall k \in K, r \in R, i \in G \tag{18}$$

$$H^r \leq A_{d^r}^k - A_i^k - u + M(1 - y_{ri}), \qquad \forall k \in K, r \in R, i \in G \tag{19}$$

$$e^i \leq A_i^k \leq l^i, \qquad \forall k \in K, i \in D \tag{20}$$

$$E_0^k = E_{init}^k, \qquad \forall k \in K \tag{21}$$

$$E_{min} \leq E_i^k \leq E_{max}, \qquad \forall k \in K, i \in V \tag{22}$$

$$E_j^k \geq E_i^k - \beta c_{ij} - M\left(1 - x_{ij}^k\right), \qquad \forall k \in K, i \in V_0 \backslash S, j \in V_{N+1} \tag{23}$$

$$E_j^k \leq E_i^k - \beta c_{ij} + M\left(1 - x_{ij}^k\right), \qquad \forall k \in K, i \in V_0 \backslash S, j \in V_{N+1} \tag{24}$$

$$E_j^k \geq E_s^k + \alpha_s \tau_s^k - \beta c_{sj} - M\left(1 - x_{sj}^k\right), \qquad \forall k \in K, s \in S, j \in G \tag{25}$$

$$A_j^k \geq A_s^k + \tau_s^k + t_{sj} - M(1 - x_{sj}^k), \qquad \forall k \in K, s \in S, j \in G \tag{26}$$

$$L_s^k = \sum_{j\in G} x_{sj}^k, \qquad \forall k \in K, s \in S \tag{27}$$

$$A_s^k - M\left(2 - L_s^k - L_s^{k'}\right) - M\left(1 - h_{kk'}^s\right) \leq A_s^{k'}, \qquad \forall k, k' \in K, s \in S, k \neq k' \tag{28}$$

$$A_s^{k'} \geq A_s^k + \tau_s^k - M(1 - h_{kk'}^s), \qquad \forall k, k' \in K, s \in S, k \neq k' \tag{29}$$

$$x_{ij}^k \in \{0,1\}, \qquad \forall k \in K, i, j \in V_{0,N+1} \tag{30}$$

$$y_{ri} \in \{0,1\}, \qquad \forall k \in K, r \in R, i \in G \tag{31}$$

$$h_{kk'}^s \in \{0,1\}, L_s^k \in \{0,1\}, \qquad \forall k, k' \in K, s \in S, k \neq k' \tag{32}$$

$$\tau_s^k \geq 0, H^r \geq 0, A_i^k \geq 0, \qquad \forall k \in K, s \in S, r \in R, i \in V_{0,N+1} \tag{33}$$

3. RESULTS AND DISCUSSION

We illustrate the proposed modeling approach on a small example and validate it on a set of numerical test instances. We use the Gurobi solver to find the optimal solution using a laptop with Intel i5-7200U CPU, 2 Cores, and 8GB memory. The parameter setting is presented in Table 1.
Figure 1 shows the illustrative example. There are 8 customers, 3 bus stops, 2 electric buses with low initial SOCs, one slow charger (22kW, charger 1), one fast charger (50kW, charger 2), and one transit station. Three bus stops scatter around the depot, and there is at least one bus stop

within maximum walking distance near customers. Two chargers are at the same location as the depot while the transit stop is 500 meters from them. There are eight departures from 6 a.m. with a frequency of every 30 minutes, so we generate eight layers accordingly. Within each layer, a dummy node is created for each bus stop and transit stop. Before solving the model, we also did pre-processing by eliminating the layers without customer requests (layer 2, 3, 4 and 6) and removing unnecessary arcs accordingly. The result of this example shows that customers are assigned to the bus stops that minimize the objective function. Buses only visit those stops to pick up customers and drop them off at transit stops dummies with their preferred departure time. Both buses visit charger 2 (fast charger) one after another without overlapping.

Table 1: Parameter setting

Parameter	Value
w_{max}	1 kilometer
u	1 minute
Q_{max}	10 passengers/bus
α_1, α_2	22kW/h, 50kW/h
β	0.2387 kW/km
d^r	Randomly generated from 1 to 8
E^k_{init}	Randomly distributed between 50%-80% of maximum battery capacity (35.8 kWh)
E_{min}	3.58 kWh (10% of maximum battery capacity)
E_{max}	28.64 kWh (80% of maximum battery capacity)
Bus speed	50 km/hour
Maximum battery capacity	35.8 kWh
$\lambda_1, \lambda_2, \lambda_3$	1
M	1000

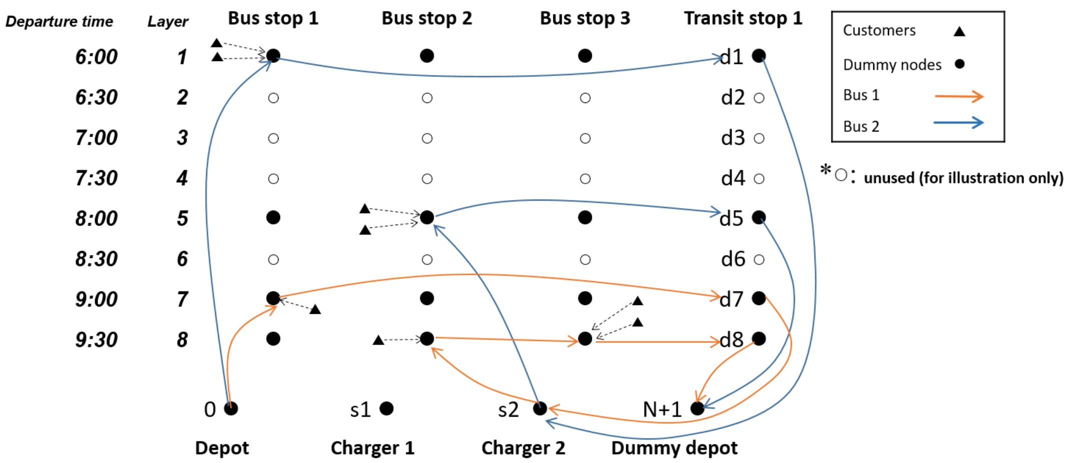

Figure 1. Illustrative example

To test the optimization model, we conduct 10 numerical tests as shown in Table 2. The location of bus stops is randomly generated around the depot within 6 km. We also randomly create customer requests with at least one bus stop near customers within the maximum walking distance. Table 2 shows that computational time to find optimal solutions increases dramatically with the size of the test instances.

Table 2: Numerical results

Number of customers	Number of optional bus stops	Number of vehicles	Optimal solution (minutes)	Computational time (sec.)
4	1	1	21.2	0.0
8	2	1	97.1	0.1
12	3	2	186.0	14.2
14	3	2	215.4	8.1
16	3	2	165.6	15.7
16	3	3	203.4	36.6
20	4	2	392.7	936.5
20	4	3	290.9	2381.4
20	5	3	305.7	10321.6
30	6	3	N.A.	>3 hours

4. CONCLUSIONS

We propose a new electric on-demand feeder bus service using meeting points to reduce operator's cost and consider bus arrival time synchronization with timetabled mass transit. The problem is modeled on a (transit timetabled) departure-expanded network to minimize the weighted sum of bus operational time (travel time and charging time), customer riding time, and early arrival with respect to customer's desired departure at transit stations. A MILP model is proposed by considering partial recharge with individual charger's capacity and occupation constraint, which is tested by a couple of instances. The computational results on a set of test instances show that customers arrive at the transit station before their preferred departure time, and buses visit chargers without overlapping. Further studies are ongoing, including solving a set of larger test instances by the commercial solver to obtain optimal solutions. We will propose new heuristics (e.g., guided local research or adaptive large neighborhood search) to solve large problems efficiently for its realistic applications. The results will be presented at the conference.

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