Max-Pressure Traffic Signal Control for Mixed Traffic Flow based on Capacity Estimation

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SHORT SUMMARY

Intersection control plays a vital role in addressing the issue of transportation efficiency in urban areas. Connected and Automated Vehicle (CAV) technology enable instant traffic information to be shared through vehicular networks, which emerges as a promising way to save vehicle travelling time and improve intersection capacity. Meanwhile, with the development of CAVs, the mixed traffic environment composed of traffic participants with differing intelligent levels will become a long term important stage of the intelligent transportation system. Considering the changes in the mixed traffic environment, this paper proposed a modified max-pressure traffic signal control method for mixed traffic environment to improve traffic efficiency. The real time traffic penetration rate is considered in the calculation of the saturation flow rate. And the pressure in the max-pressure also depends on the traffic penetration rate. By comparing the proposed MPMF method with the classic max-pressure control and exiting fixed time control method, the proposed MPMF method can effectively improve the performance of intersections and be suitable for multi-intersections road network.

Keywords: connected and automated vehicle, mixed traffic flow, max-pressure controller, traffic signal control

1. Introduction

With the increasing traffic demand, traffic congestion has been a major challenge in urban areas around the world. An analysis of the traffic conditions in 416 major cities around the world in 2019 shows that more than half of cities have seen an increase in congestion (TomTom, 2019). Many researchers have pointed out that it is very necessary to control urban traffic in a more effective way to solve these problems. At the same time, the coexistence of vehicles with differing intelligent levels, such as human-driven vehicles (HDVs) and Connected Automated Vehicles (CAVs), is constituting a new type of mixed traffic environment (Zhu & Zhang, 2017), which is an important next step for intelligent transportation systems to transit from fully manual to fully automated driving.

In the transportation field, adaptive signal control methods have been extensively studied and proven to help address the congestion problem, such as the well-known practical traffic signal control systems SCOOT and SCATS (Hunt, Robertson, Bretherton, & Winton, 1981; Lowrie, 1982). Significant works have been proposed to overcome the non-linearity and randomness of traffic systems, such as fuzzy logic (Rahman & Ratrout, 2009), game theory (Alvarez, Poznyak, & Malo, 2007), and agent-based learning methods (Yang, Tan, & Menendez, 2017; Tan et al., 2020; Du, ShangGuan, Rong, & Chai, 2019). The centralized control has a control center, which needs to communicate with sensors and traffic lights deployed in the network. In theory, it can achieve system optimally due to its system-wide view of the network topology and demand distribution. However, in practice, the complexity of the urban network system makes centralized control computationally non-scalable and limited in applicability. The limited expansion of centralized methods and potential problems with high costs have prompted the emergence of decentralized control methods, which distribute computations to the local traffic controller, and hence are scalable to large networks. More analysis and comparison of centralized and decentralized solutions for optimized traffic control have been provided by Chow et al. (Chow, Sha, & Li, 2020).

Max-pressure (hereinafter referred to as MP) is a local decentralized controller originally applied to scheduling packets in wireless communication networks. It was first involved in the transportation system by Varaiya (2013a,b) to improve the efficiency of signalized intersection (Varaiya, 2013). The MP traffic controller has attracted a lot of attention due to its simple calculation and stable performance. In the original version presented by Varaiya et al. knowledge on the traffic queues, turn probabilities, and saturation flow rates are required as inputs, and the decision of which phase to activate is calculated periodically. Many variants of MP have been proposed, focusing on how to use the known queue length to calculate pressure in a more concise and precise way (Li & Jabari, 2019; Gregoire, Frazzoli, de La Fortelle, & Wongpiromsarn, 2014). Kouvelas et al. investigated different modifications of max-pressure control and their ability to stabilize the system queues via simulation experiments (Kouvelas, Lioris, Fayazi, & Varaiya, 2014). Pedro Mercard proposed to use travel time instead of queue length as input to calculate pressure, thereby improving the practical applicability of the MP controller (Mercader, Uwayid, & Haddad, 2020). This modified version solved the problem of spillbacks and implemented the MP algorithm at a signalized intersection in Jerusalem.

However, most of the existing studies consider the pure traffic flow, and do not consider the impact of random mixing of HDVs and CAVs on the intersection control method in the mixed traffic environment. The coexistence of multiple vehicles with differing intelligent levels makes the problem of cooperative intersection control more complicated (Yao et al., 2019; Navas & Milanés, 2019; Talebpour & Mahmassani, 2016). On the one hand, due to the difference in vehicle driving models, given the same green time, the numbers of CAVs and HDVs that can pass through the intersection are different (Ghiasi, Hussain, Qian, & Li, 2017). On the other hand, the uncertainty of human drivers makes it difficult to implement consistent vehicle operation rules at the road network level (Wang, Zheng, Xu, Wang, & Li, 2020). Therefore, the optimization method under pure traffic flow is not suitable for the mixed traffic environment.

Inspired by the above points, this paper proposed a modified version of max-pressure traffic signal control method for the mixed traffic flow. This study makes the following contributions: (1) The capacity of the mixed traffic flow is analyzed, mainly based on vehicle headway and CAV penetration rate. (2) The max-pressure controller for mixed flow (hereinafter referred to as MPMF) is proposed to optimize the control effect of traffic signal lights.

2. Max-pressure for mixed traffic flow

Capacity of mixed traffic flow at signalized intersection

In this paper, we assume that the mixed traffic flow consists of two types of vehicles, CAVs and HDVs. The set of vehicle types can be presented as $A = \{0, 1\}$, where 0 and 1 represent HDV and CAV respectively. The type of headway in the mixed traffic flow can be denoted as h_{ab}, where a and b are the type of the preceding vehicle and the following vehicle respectively, $a, b \in A = \{0, 1\}$. In general, CAVs are expected to have a faster response time than HDVs. Thus, h_{00} has the largest value and h_{11} has the smallest. Assuming that the penetration rate of mixed traffic flow is p, which means that the probability that a vehicle in the traffic flow is a CAV is p. Then the proportion of four types of headway can be calculated in Eq.(1)

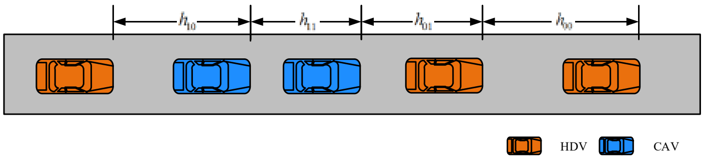

Figure 1: Four types of headway in the mixed traffic flow.

$$P(h_{00}) = (1-p)^2,\ P(h_{01}) = (1-p)p,\ P(h_{10}) = p(1-p),\ P(h_{11}) = p^2 \tag{1}$$

The average time headway of the traffic flow denoted as $\bar{h}$ can be expressed as Eq.(2).

$$\begin{aligned}\bar{h} \approx E(h) =& \sum P(h_{ab}) \times h_{ab},\ (a,b \in A = \{0,1\}) \\ =& (1-p)^2 \times h_{00} + (1-p)p \times h_{01} + p(1-p) \times h_{10} + p^2 \times h_{11}\end{aligned} \tag{2}$$

Then the saturation flow rate of mixed traffic flow can be calculated as Eq.(3), which is related to the penetration rate of the traffic flow.

$$cm = \frac{1}{\bar{h}} = \frac{1}{(1-p)^2 \times h_{00} + (1-p)p \times h_{01} + p(1-p) \times h_{10} + p^2 \times h_{11}} \tag{3}$$

To estimate the capacity of mixed traffic flow at signalized intersection, it should be noticed that the vehicle departure headway also related to the position of vehicle at the queue. For brevity, we use an average headway during the entire green time to represent the four kind of headway. Tab.(1) shows the values obtained in the SUMO simulation software.

Table 1: The value of headway in mixed traffic flow tested by the simulation software.

$h_{00}(s)$	$h_{01}(s)$	$h_{10}(s)$	$h_{11}(s)$
1.5	1.9	2.0	2.3

Combining the value tested from the simulation and the calculation of the saturation flow rate, the relationship of the *cm* and *pr* can be obtained as Fig.(2). It can be seen that the saturation flow rate of the mixed traffic flow increases as a quadratic function of the PR.

Max-pressure traffic signal controller

The max-pressure algorithm can be modeled as a store-and-forward queuing network model. The queue update equation can be formulated as Eq.(4). $x_{l,m}(t)$ is the queue length of the road with origin link l and destination link m at time t. $c_{l,m}$ is the saturation flow rate. $\sum_k a_{k,l}(t)$ is the all arrivals from the other intersection and $d_{l,m}(t+1)$ is the external arrivals from the outside relating to the traffic demand. Specifically, $a_{k,l}(t)$ can be calculated by Eq.(5) where $r_{l,m}(t+1)$ is the

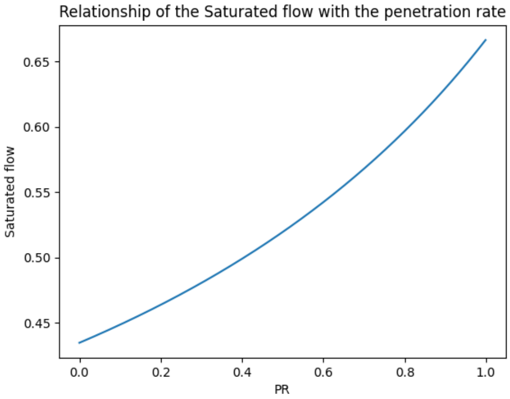

Figure 2: Relationship of traffic penetration rate and saturation flow rate.

turning ratio, representing the proportion of vehicles waiting at l will turn to link m. For the entry link, there is no vehicle coming from the other intersection, then $\sum a_{k,l}(t) = 0$. Also for the exit link, there is no external arrivals and $d_{l,m}(t+1) = 0$.

$$x_{l,m}(t+1) = x_{l,m}(t) - c_{l,m}(t+1) \cdot s_{l,m}(t) \wedge x_{l,m}(t) + \sum_k a_{k,l}(t) + d_{l,m}(t+1) \tag{4}$$

$$a_{k,l}(t) = [c_{k,l}(t+1) \cdot s_{k,l}(t) \wedge x_{k,l}(t)] \cdot r_{l,m}(t+1) \tag{5}$$

The weights used to indicate the importance of a path are calculated by Eq.(6), which is the difference between the upstream queue length and the average downstream queue lengths.

$$w_{l,m}(t) = x_{l,m}(t) - \sum_{p \in O_n} r_{m,p}(t) \cdot x_{m,p}(t),\ \forall l \in I_n \tag{6}$$

The original version proposed by Varaiya (Varaiya, 2013) is shown in Eq.(7). The pressure for a phase is the sum of the multiplication of the saturation flow rate and weights of all paths, which are controlled by the same phase.

$$p_{f,n}(t) = \max(0, \sum_{(l,m) \in M_{j,n}} w_{l,m}(t) \cdot c_{l,m}(t)), \forall f \in F_n \tag{7}$$

For the mixed traffic flow, the pressure for each phase is modified to Eq.(8), where the saturation flow rate for each path is related to the penetration rate as mentioned in Eq.(3).

$$pm_{f,n}(t) = \max(0, \sum_{(l,m) \in M_{j,n}} w_{l,m}(t) \cdot cm_{l,m}(t)), \forall f \in F_n \tag{8}$$

The green time for each phase is T_g which also is the period for the phase selection. At the end of the current phase f^{ct}, the pressure for all phases F_n will be calculated and the f^* with the highest pressure will be chosen and activated, where n is the index of the intersection. If f^{ct} is different from the chosen phase f^*, the yellow light will be activated first (set to 3s) before starting turn to the phase f^*.

$$f^* = arg \max_{f \in F_n} pm_{f,n}(t) \tag{9}$$

3. Case study and analysis

We conducted two cases with open source traffic simulation software SUMO (Behrisch, Bieker, Erdmann, & Krajzewicz, 2011) to verify the proposed method, including a single intersection scenario and a four intersections scenario. The fixed timing and original version of MP are also implemented to compare with the proposed mixed flow MP (MPMF) approach. In these experiments, the activation time for each phase is set to $T_g = 20s$.

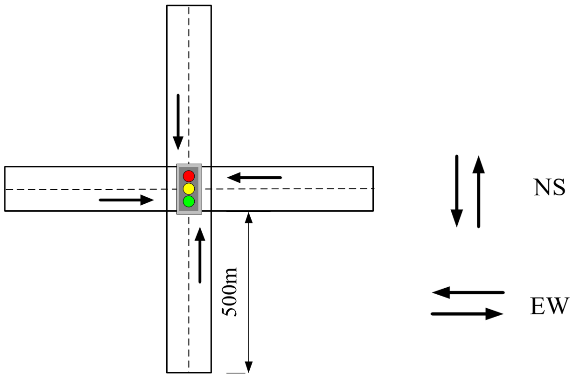

Figure 3: Schematic diagram of the single intersection scenario.

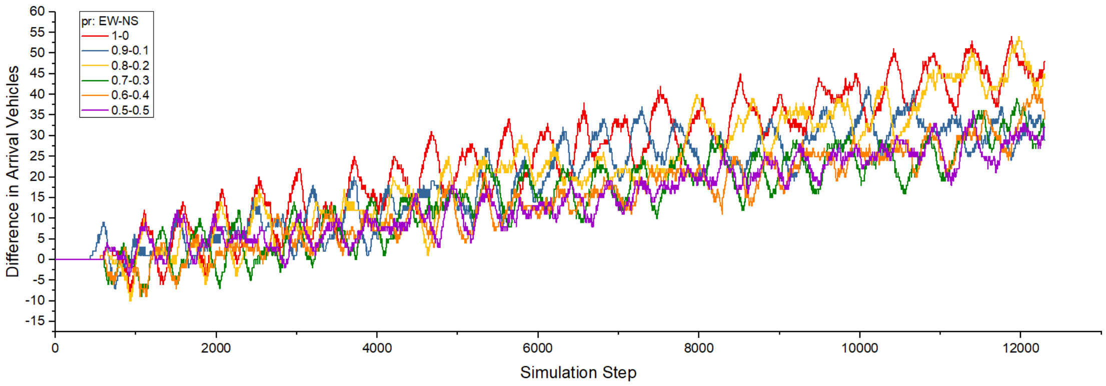

Figure 4: The difference between MPMF and MP in the number of arrival vehicles.

Single intersection scenario

Firstly, we conducted a single intersection case with a two phase-single intersection as shown in Fig.(3). In this case, the mixed vehicles enter the intersection from four separate entrance road, phase 1 represents the east-west direction and phase 2 represents the north-south direction. The vehicle arrival rates are the same in both directions. The penetration rate (PR) in East-West and North-South is set separately. For example, $'EW-NS=0.9-0.1'$ means that the PR in the road East-West (phase 1) is $PR_{EW}=0.9$ and the PR in the road North-South (phase 2) is $PR_{NS}=0.1$. MP and MFMP are implemented and the gains of MFMP compared with MP under different value of $'EW-NS'$ are studied.

Fig.(4) shows the variation of the difference in the number of arrival vehicles with the simulation time (each step is $0.1s$), which is the number of arrivals of the MPMF method minus the number of arrivals of the MP method. It can be seen that the MPMF method has more vehicle arrivals as the simulation proceeds, which means that the road capacity is higher under the control of MPMF than the MP method. Also, the improvement of the MPMF method is more obvious when the PR difference between the two directions is large. When $'EW-NS=1-0'$ (red line) and $'EW-NS=0.8-0.2'$ (yellow line), after 12,000 simulation steps (20 minutes), the advantage of MPMF reached 50 vehicles.

Fig.(5) and Fig.(6) shows the performance gains of MPMF compared to MP in the average vehicle speed and average waiting time respectively. In terms of vehicle speed, when the ratio in PR between the two directions is larger than 7:3, the proposed MPMF method can improve the vehicle speed up to $7.5\%-8.5\%$, otherwise MPMF can also has the improvement more than 4%. From Fig.(6), it can be seen that the proposed MPMF can reduce the average waiting time of all vehicles about $5\%-37\%$, which means that the MPMF can effectively reduce the travel time of vehicles

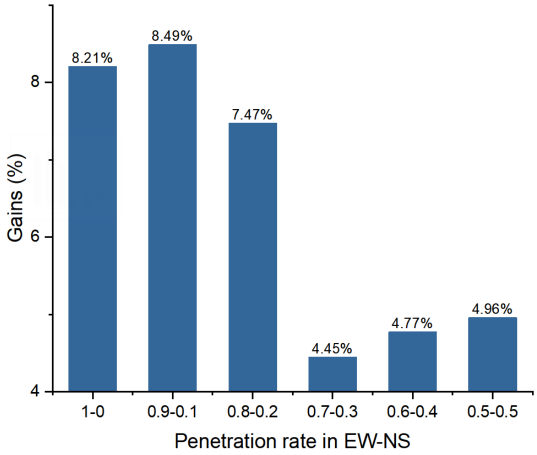

Figure 5: Gains of MPMF relative to MP in average vehicle speed.

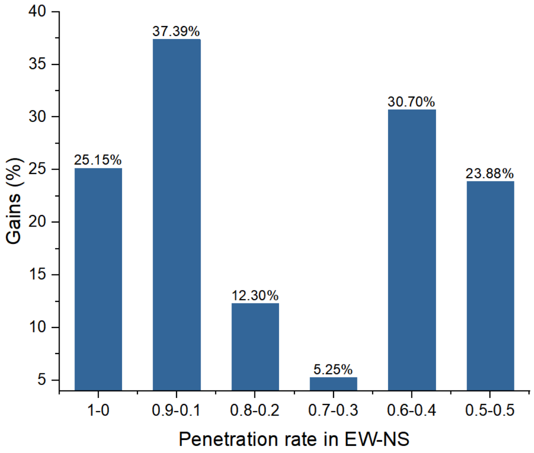

Figure 6: Gains of MPMF relative to MP in average waiting time.

on the road.

Four intersections scenario

We compared the performance of the proposed method with MP, and existing fixed timing method on a small-scale road network with four intersections, as shown in Fig.(7). In this scenario, a typical 4-leg, 3-lane (in each direction) signalized intersection is considered as the basic intersection unit that constitutes the road network. Four phases are defined for each intersection: west-east-through (WET), west-east-left turning (WEL), north-south-through (NST) and north-south-left turning (NSL). And each phase f_i $(i \in [0,4))$ controls two paths. Right turn direction is always allowed. Fixed timing controller activates four phases in sequence, and each phase also has a duration of $20s$. The PR in this scenario is a global value for the entire road network regardless of the direction.

Fig.(8) shows the gains of MPMF and MP compared with the fixed time controller in average vehicle speed. The results show that both of the MP method and the proposed MPMF methods can significantly increase the average vehicle speed. For MP method, the average speed can be increased by $15\% - 28\%$. Further, the gain of the proposed MPMF method can reach $17\% - 30\%$. The proposed MPMF method can further improve traffic efficiency, especially when the PR is between 0.2 and 0.8. The reason is that intersections are more prone to PR imbalance when the traffic flow is in a more mix state, which means the number of CAVs and HDVs are similar. At this time, the dynamic estimation of the traffic capacity plays a more obvious influence.

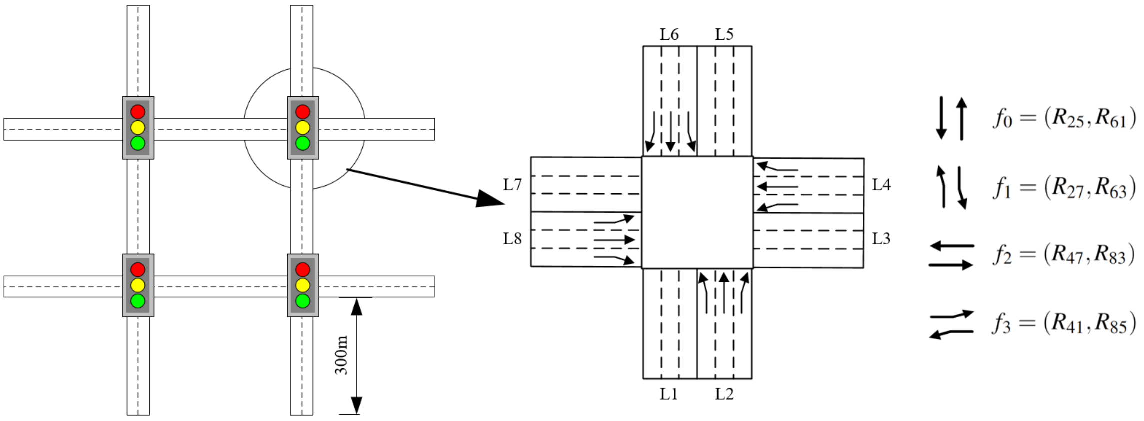

Figure 7: Schematic diagram of the four intersections scenario.

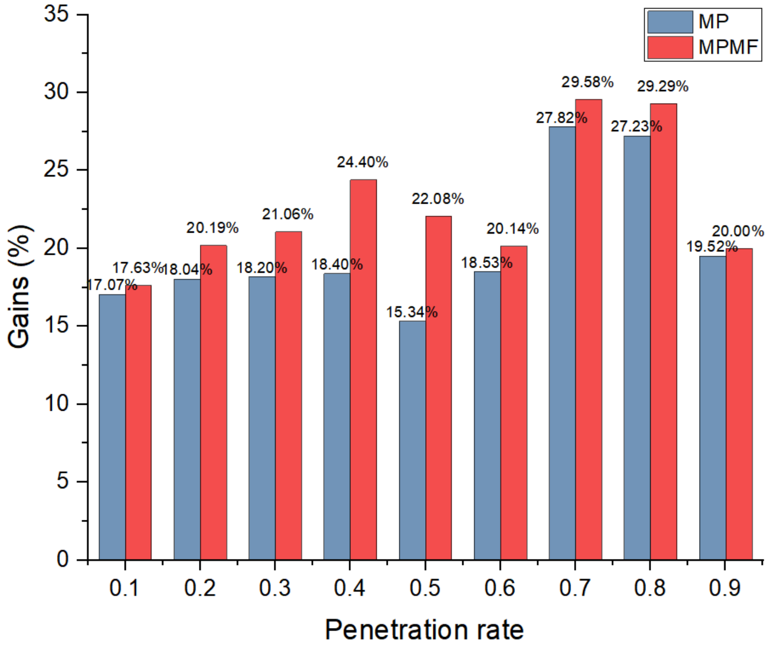

Figure 8: Gains of MPMF and MP relative to fixed time controller on average vehicle speed with different PR.

4. Conclusions

This paper proposed a modified max-pressure traffic signal controller used in a mixed traffic environment. First, traffic saturation flow rate of mixed traffic flow related to the traffic penetration rate is analysed. Then the modified saturation flow rate is involved in the original version of max-pressure traffic controller. The proposed method is tested in the simulation software SUMO and a single intersection scenario and a four intersections scenario are applied in the verification. The performance of MP, MPMF and fixed time traffic controller are implemented and compared. The results show that the proposed MPMF method can effectively improve the overall performance of traffic flow in terms of vehicle speed and vehicle waiting time. In addition, it can perform well under varying traffic flow conditions especially when there are big difference in PR between different road or the number of CAVs and HDVs changes dynamically.

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