Multi-class Dynamic Traffic Assignment based on Link Transmission Model and Mathematical Programming

Xiao Lin*[1] and Lóránt Tavasszy[2]

[1]Dr., Dept. Transport and Planning, Delft University of Technology, The Netherlands

[2]Prof.dr.ir., Dept. Transport and Planning, Delft University of Technology, The Netherlands

SHORT SUMMARY

We develop a multi-class dynamic traffic assignment procedure based on the link transmission model. LTM has been used for DTA, but without the consideration with multiple vehicle classes. We extend LTM using the dynamic PCE principle that considers vehicle spacing under both free flow and congested conditions. The multi-class DTA can be formulated as linear programming. A small instance is implemented in Matlab. The results of the instance are illustrated and discussed.

Keywords: Dynamic traffic assignment, link transmission model, multi-class traffic, operations research, linear programming.

1. INTRODUCTION

Dynamic traffic assignment (DTA) is widely studied and used for traffic management infrastructure planning (Peeta & Ziliaskopoulos, 2001). A DTA process mainly consists 2 parts: route choice and network loading, as summarized in (Bliemer et al., 2017). The most common route choice principles are user equilibrium (UE) and system optimal (SO). These two principles are also referred to as the Wardrop first and second principle. Variations to these principles are reviewed by (Szeto & Wong, 2012).

The loading methods usually involves a traffic flow modeling that estimates travel time based on the loading of the network. Approaches include LWR (Carey & McCartney, 2004), METANET (Wang, Messmer, & Papageorgiou, 2001), cell transmission model (CTM) (Carey & Watling, 2012), and link transmission model (LTM) (Himpe, 2016; Long & Szeto, 2019; Chakraborty, Rey, Moylan, & Waller, 2018). Among these, we focus on LTM method in this research, as it captures spill back with relatively simple formulation and thus has the potential to be applied in larger networks.

Considering multiple user classes (such as different vehicle types), is an important extension to DTA. This requires DTA's underlying traffic loading model to handle the multi-class (MC) dimension. The extension has been addressed in different loading models (Logghe, 2003; Liu, De Schutter, & Hellendoorn, August, 2014). Attempts are also made to incorporate multi-class considerations in LTM (Rachim, Solekhudin, et al., 2017; Smits, Bliemer, & Van Arem, 2011). However, to the best of our knowledge, multi-class DTA using LTM has not yet been developed. We address this gap by developing a MC SO-DTA based on LTM. We explain the mathematical formulation of our approach, in particular, linear programming (LP). We then illustrate the DTA process in a small numerical example.

2. METHODOLOGY

This section formulates the MC DTA using LTM and mathematical programming (LP). We firstly extend LTM with its link model and node model by adding the MC elements. A linear programming is then formulated to describe the SO-DTA process.

The link transmission model

LTM is developed in (Yperman, 2007) and has wide applications including DTA. One advantage of LTM is that it only poses flow constraints at inflow and outflow of links. Assume a network $\mathscr{G}$, with directed links $\mathscr{E}$ between nodes $\mathscr{N}$. The origin-destination (OD) pairs are $\mathscr{W}$, and the paths connecting each of the OD pairs are $\mathscr{P}_w$. If path p uses link $i \in \mathscr{E}$, then the link-path instance $\delta_{ip} = 1$, otherwise $\delta_{ip} = 0$. In LTM, this is referred to as the link model and can be expressed by the following outflow (1,2) and inflow (3,4) constraints:

$$G_i^-(k+1) \leq N_i^-(k+1) - N_i^-(k) = N_i^+(k+1-t_i) - N_i^-(k), \tag{1}$$

$$G_i^-(k+1) \leq q_i^{\max}, \tag{2}$$

$$G_i^+(k+1) \leq N_i^+(k+1) - N_i^+(k) = N_i^-(k+1-\tau_i) + \rho_i^{\text{jam}} L_i - N_i^+(k), \tag{3}$$

$$G_i^+(k+1) \leq q_i^{\max}, \tag{4}$$

$$N_i^-(k) = \sum_{\tau=1}^{k} G_i^-(\tau), \tag{5}$$

$$N_i^+(k) = \sum_{\tau=1}^{k} G_i^+(\tau). \tag{6}$$

in which $G_i^+(k)$ and $G_i^-(k)$ refer to the number of vehicles flowing in/out of the link i at time step k, respectively. $N_i^+(k)$ and $N_i^-(k)$ refer to the to the cumulative in/out flow of the link i at time step k. Let Δt be the length of a time step, then t_i refers to the time required for a vehicle to travel the whole link i with free flow speed v^{ff} ($v_i^{\text{ff}} t_i \Delta t = L_i$); τ_i refers to the time required for the congestion state to propagate backwards (with speed of $\overleftarrow{v}_i$) from downstream to upstream on link i ($\overleftarrow{v}_i \tau_i \Delta t = L_i$). ρ_i^{jam} is the maximum density link i can take; L_i is the length of link i; q_i is the maximum flow capacity in vehicle/hour of link i. These parameters can be derived from the triangular fundamental diagram (FD).

LTM with multi-class extension

To perform multi-class DTA, the LTM needs to be extended to include multiple vehicle classes. Since road infrastructures are shared by different user classes, it would be most straightforward to aggregate the occupancy of different vehicle class in terms of both flow and density. Some attempts are seen in literature. A capacity normalization is applied in (Smits et al., 2011), setting the normalized maximum flow to 1 for each vehicle class. The passenger car equivalent (PCE) unit is applied in (Rachim et al., 2017) with the consideration of vehicle space in free flows. In this paper, we use a more reasonable approach similar to that of (Rachim et al., 2017). We use the standstill spacing to calculate the aggregated maximum density ρ^{jam} with the standstill PCE value, and the free flow spacing to calculate the aggregated maximum flow $q^{\max}$ with the free flow PCE value.

We now explain how PCE is applied here using a 2-class example considering cars and trucks. Let π_c^{ff}, π_c^{ss} be the PCE value of cars at free flow and stand still conditions (both equal to 1 by definition). Let π_t^{ff} and π_t^{ss} be the PCE of trucks in free flow and stand still situations, respectively.

We then have the following:

$$\pi_t^{\text{ff}} = \frac{d_t^{\min} + v_t^{\text{ff}} h_t}{d_c^{\min} + v_c^{\text{ff}} h_c}, \tag{7}$$

$$\pi_t^{\text{ss}} = \frac{d_t^{\min}}{d_c^{\min}}, \tag{8}$$

in which $d_m^{\min}$ is the minimum spacing of vehicle class m at stand still; v_m^{ff} is the free flow speed of vehicle class m; h_m is the minimum time headway of class m.

Link model for MC-LTM

We use a path-based formulation for DTA. By extending the original LTM link model (constraints 1 – 4) to include multi-class element, the cumulative inflow and outflow vehicle count should also be considered in a class-specific way, providing details on OD pairs of vehicle flows as well as vehicle classes. In a network $\mathscr{G} = \{\mathscr{N}, \mathscr{E}\}$, with the multi-class consideration, the link model then becomes:

$$\begin{aligned} &G_{mwpi}^{-}(k+1) \leq N_{mwpi}^{-}(k+1) - N_{mwpi}^{-}(k) \\ &= N_{mwpi}^{+}(k+1-t_i) - N_{mwpi}^{-}(k), \end{aligned} \qquad \forall m,w,p,i,k \tag{9}$$

$$\sum_m \sum_w \sum_p \pi_m^{\text{ff}} G_{mwpi}^{-}(k+1) \leq q_i^{\max}, \qquad \forall i,k \tag{10}$$

$$\begin{aligned} &\sum_m \pi_m^{\text{ss}} G_{mwpi}^{+}(k+1) \leq \sum_m \pi_m^{\text{ss}} \left(N_{mwpi}^{+}(k+1) - N_{mwpi}^{+}(k) \right) \\ &= \sum_m \pi_m^{\text{ss}} \left(N_{mwpi}^{-}(k+1-\tau_i) - N_{mwpi}^{+}(k) \right) + \rho_i^{\text{jam}} L_i, \end{aligned} \qquad \forall w,p,i,k \tag{11}$$

$$\sum_m \sum_w \sum_p \pi_m^{\text{ff}} G_{mwpi}^{+}(k+1) \leq q_i^{\max}, \qquad \forall i,k. \tag{12}$$

Note that the sending flow constraint (9) is class-specific and the receiving flow constraint (11) is class-aggregated. In the above constraints, t_i and τ_i are calculated through v_i^{ff} and $\overleftarrow{v}_i$, respectively, which are assumed independent from the loading of the network.

Since cumulative vehicle count n and the increment of vehicle count G come with a discrete time dynamics, we use linear interpolation to approximate any points between two time instances. Let $\hat{\tau}$ and $\check{\tau}$ be the closest larger / smaller integer values of τ. Applying this principle for instance to the cumulative inflow $N_{mwpi}^{+}(k-\tau_i)$ we have the following:

$$N_{mwpi}^{+}(k-\tau_i) = (\hat{\tau}_i - \tau_i) N_{mwpi}^{+}(k-\check{\tau}_i) + (\tau_i - \check{\tau}_i) N_{mwpi}^{+}(k-\hat{\tau}_i). \tag{13}$$

Node model for MC-LTM

In (Yperman, 2007) several node models are discussed. In this paper we do not explore the possible distribution of inflow/outflow capacities in nodes. Only basic constraints are applied:

$$\sum_{i \in P(n)} G_{mwpi}^{-}(k) = \sum_{i \in S(n)} G_{mwpi}^{+}(k), \qquad \forall n,m,w,p,k. \tag{14}$$

$$\sum_w \sum_{p_w} \sum_{\tau=1}^{k} G_{mwpi}^{+}(\tau) \leq \sum_w \sum_{\tau=1}^{k} D_{mi}(\tau), \qquad \forall m,i,k. \tag{15}$$

Eq.14 describes that an intermediate node (with inflow and outflow links) does not hold any vehicles. $P(n)$ and $S(n)$ refer to the collection of the predecessor and the successor links of node n Inequality 15 describes that the cumulative inflow vehicles at a starting node is no larger than the demand ($D_{mwi}(k)$) that (attempts to) enter the network. We assume that destination nodes have infinite capacity and thus no specific constraints are added.

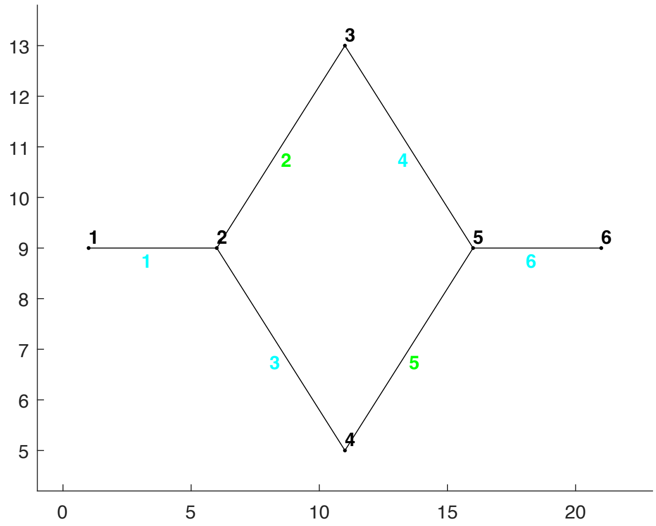

Figure 1: Test network

Multi-class DTA based on LTM and LP

An objective function is designed for the LP formulation which will later be solved by an off-the-shelf solver. The objective function is formulated as the summation of all outflow transition at all considered times and links multiplied by a coefficient that decreases over each time step: the system is thus "rewarded" more if vehicles exit links earlier. When this summation is maximized, the vehicles traversing the network is "propelled" to move forward as fast as possible (until they hit one or more constraints). The objective function is expressed in the following:

$$\max J|_{G^+,G^-} = \sum_{m\in\mathscr{M}} \sum_{w\in\mathscr{W}} \sum_{p\in\mathscr{P}_w} \sum_{i\in\mathscr{S}} \sum_{k\in\mathscr{K}} (K-k+1)\, G^-_{mwpi}(k). \tag{16}$$

Now the mathematical formulation of multi-class SO-DTA based on LTM is ready: the objective function is 16, with constraints 9 – 12, 14, and 15.

Since we only give the basic upper limit constraints for flows in node models. In this way we find the "optimal" turn fraction for all nodes, representing the lower bound of utilities of all user classes and all vehicles under the SO condition. The node models can then be calibrated by real world data to have more accurate results.

3. NUMERICAL EXAMPLE

We illustrate the proposed DTA procedure in a small network (see Figure1) with 1 pair of OD: from node 1 to node 6. Each time step Δt for simulation is 15 min. In total 15 time steps are included. The input parameters of the numerical simulation is listed in Table1. The LP formulation is coded and solved in Matlab 2020a on Windows 10 platform with Intel Core i5-8279U 2.40 GHz and 3 GB RAM. Solving the problem takes 0.15s.

The demand for both cars and trucks and the cumulative vehicle counts at inflow and outflow of links are illustrated in Figure2. In the figure, L1 – L6 refer to the 6 links of the considered network. From the figure we can see that cars make use of both paths (1–2–4–6 and 1–3–5–6). But trucks only use the 1–2–4–6 path (as the cumulative vehicle count for trucks are 0 at link 3 and link 5).

Table 1: **Parameters of the numerical experiment.**

Parameter	Value
L_i	$[50, 50, 80, 50, 50, 50]$ km
ρ_i	$[100, 37.5, 62.5, 37.5, 62.5, 87.5]$ veh/km
$\overleftarrow{v}$	30 km/h
v_c^{ff}	130 km/h
v_t^{ff}	80 km/h
d_c^{min}	6 m
d_t^{min}	25 m
h_c	1.2 s
h_t	1.2 s

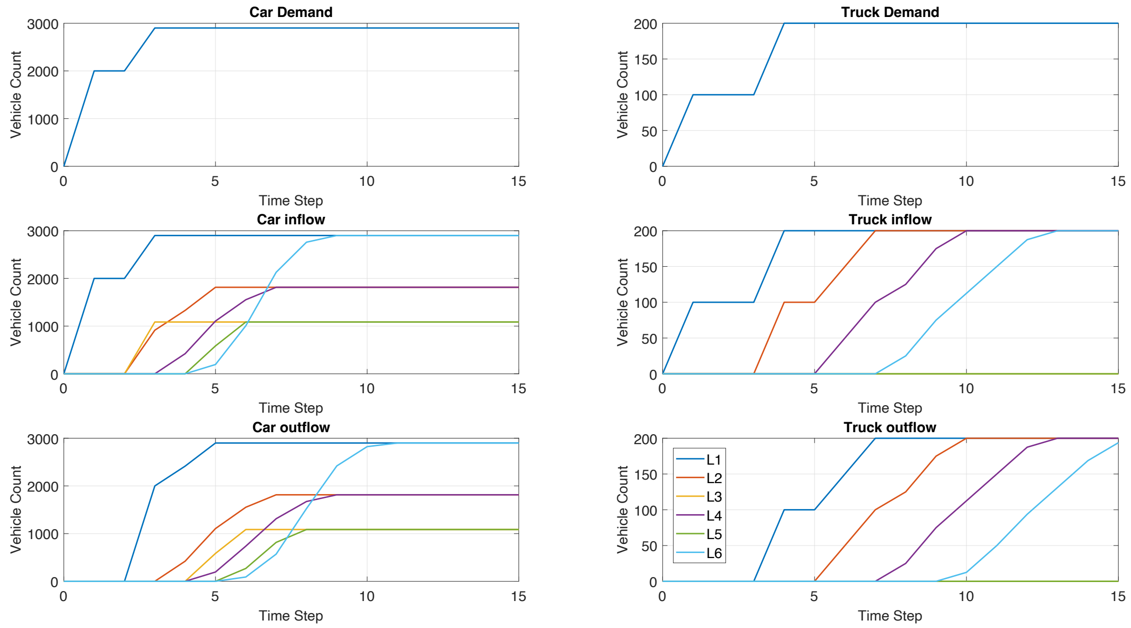

Figure 2: Cumulative vehicle count

4. CONCLUSIONS

This research develops and implements an SO-DTA with multi-class consideration. We use an MC LTM as network loading, with the dynamic PCE value for different vehicle classes at free flow and congested situations, respectively. The DTA procedure is formulated as an LP. A small test case is used to illustrate the simplicity and applicability of this method. Future research include developing multi-class DTA under the UE condition.

REFERENCES

Bliemer, M. C., Raadsen, M. P., Brederode, L. J., Bell, M. G., Wismans, L. J., & Smith, M. J. (2017). Genetics of traffic assignment models for strategic transport planning. *Transport reviews*, *37*(1), 56–78.

Carey, M., & McCartney, M. (2004). An exit-flow model used in dynamic traffic assignment. *Computers & Operations Research*, *31*(10), 1583–1602.

Carey, M., & Watling, D. (2012). Dynamic traffic assignment approximating the kinematic wave model: System optimum, marginal costs, externalities and tolls. *Transportation Research Part B: Methodological*, *46*(5), 634–648.

Chakraborty, S., Rey, D., Moylan, E., & Waller, S. T. (2018). Link transmission model-based linear programming formulation for network design. *Transportation Research Record*, *2672*(48), 139–147.

Himpe, W. (2016). *Integrated algorithms for repeated dynamic traffic assignments the iterative link transmission model with equilibrium assignment procedure.* Catholic University of Leuven, Belgium. (PhD Thesis)

Lin, X., Leclercq, L., Tavasszy, L., & van Lint, H. (2022). *Formulating multi-class user equilibrium using mixed-integer linear programming.* (Submitted to a journal)

Liu, S., De Schutter, B., & Hellendoorn, H. (August, 2014). Model predictive traffic control based on a new multi-class metanet model. In *19th ifac world congress* (pp. 8781–8786). Cape Town, South Africa: Elsevier.

Logghe, S. (2003). *Dynamic modeling of heterogeneous vehicular traffic.* Catholic University of Leuven, Belgium. (PhD Thesis)

Long, J., & Szeto, W. Y. (2019). Link-based system optimum dynamic traffic assignment problems in general networks. *Operations Research*, *67*(1), 167–182.

Peeta, S., & Ziliaskopoulos, A. K. (2001). Foundations of dynamic traffic assignment: The past, the present and the future. *Networks and spatial economics*, *1*(3-4), 233–265.

Rachim, D. K., Solekhudin, I., et al. (2017). Link transmission model involving multi class vehicle. In *2017 5th international conference on instrumentation, control, and automation (ica)* (pp. 158–162).

Smits, E.-S., Bliemer, M., & Van Arem, B. (2011). Dynamic network loading of multiple user-classes with the link transmission model. In *2nd international conference on models and technologies for intelligent transportation systems* (Vol. 57, p. 73).

Szeto, W., & Wong, S. (2012). Dynamic traffic assignment: model classifications and recent advances in travel choice principles. *Central European Journal of Engineering*, *2*(1), 1–18.

Wang, Y., Messmer, A., & Papageorgiou, M. (2001). Freeway network simulation and dynamic traffic assignment with metanet tools. *Transportation Research Record*, *1776*(1), 178–188.

Yperman, I. (2007). *The link transmission model for dynamic network loading.* Catholic University of Leuven, Belgium. (PhD Thesis)