Assortment Optimization for Boundedly Rational Customers

Mahsa Farhani*[1], Caspar. G. Chorus[2], and Yousef Maknoon[3]

[1]Ph.D. candidate, Faculty of Technology, Policy and Management, Delft University of Technology, Netherlands

[2]Full Professor, Faculty of Technology, Policy and Management, Delft University of Technology, Netherlands

[3]Assistant Professor, Faculty of Technology, Policy and Management, Delft University of Technology, Delft University of Technology, Netherlands

Abstract

This paper presents an assortment optimization model for boundedly rational customers. The problem has application in designing the travel menu for on-demand mobility services. We present the customer behavior using the Random Regret Minimization (RRM) choice model, considering the reference-dependency and choice-set dependency of preferences as strong violations of perfect rationality premises. We propose an efficient algorithm to find the optimal assortment when customers' behavior follows RRM. We have tested our algorithm for micromobility services. The results show that our proposed algorithm can find the optimal solution for all studied instances. Moreover, we compare the planned assortments against the widely used multinomial logit model (based on the premise of full rationality) to examine the effects of reference-dependency and choice set-dependency on the assortment decisions. Our results indicate that these behavioral phenomena have significant impacts on the optimal choice set, so they need to be taken into account by those who want to offer a menu of options to their customers.

Keywords: Assortment optimization, bounded rationality, Mobility on-demand services.

1. INTRODUCTION

In the last decade, app-based mobility services have received considerable attention as an interface between the service provider and the customer. The foundation of the app-based system is based on the real-time interaction between the booking system and the user's choice. Upon customers' arrival, a request is submitted to the platform. The platform then offers a travel menu (i.e., assortment) to the user to choose from. Inferring customer preferences and responding accordingly plays a vital role in app-based mobility services. One of the central decisions is which alternative to include in the list of offered alternatives to each arriving passenger in the list of assortment.

Assortment planning relies on behavioral models to estimate individuals' behavior. There are two main approaches to model individuals' behavior: non-parametric and parametric. In the non-parametric approach, the preferences between alternatives are known. In this approach, one can sort all alternatives based on customer preferences based on which an assortment can be obtained (see (Rusmevichientong, Van Roy, & Glynn, 2006), (Farias, Jagabathula, & Shah, 2013), (van Ryzin & Vulcano, 2015), and (Bertsimas & Mišic,

2015) for example). The main drawback of the non-parametric approach is its inability to explain the factors that affect the choice of individuals. Moreover, sorting the alternatives based on their attractiveness is challenging in practice. In the parametric approach, individuals' behavior is modeled as a function of explainable attributes. Assuming that individuals are rational agents, van Ryazin and Mahajan show that a greedy algorithm can determine the optimal assortment if individual behavior is modeled by multinomial logit (MNL) model (Ryzin & Mahajan, 1999). Their approach was further developed for other choice models such as mixture of MNLs (MMNL) (Rusmevichientong, Shmoys, & Topaloglu, 2010), Nested Logit (NL) (Davis, Gallego, & Topaloglu, 2014), and Generalized Attraction Model (GAM) (Gallego, Ratliff, & Shebalov, 2015). Similar assumptions are also considered in the context of on-demand transportation systems. (Atasoy, Ikeda, Song, & Ben-Akiva, 2015) propose a two-step approach to form a travel menu for each customer. First, feasible alternatives are generated by solving the supply-side operational problem. Then, the optimal assortment of alternatives is selected from the generated alternatives based on the classical MNL model. (Song, Danaf, Atasoy, & Ben-Akiva, 2018) employ a mixed logit model to capture heterogeneous travelers' behaviors and offer a personalized travel menu to each customer.

In all studies mentioned above, it is assumed that customers are rational agents. However, studies in marketing, economics, psychology (e.g., (Simon, 1957) and (Hauser, 1978)), and transportation ((Di & Liu, 2016)) have revealed that individuals' decisions may deviate from perfect rationality due to their cognitive limitations and biases. Such behavioral elements can often result in surprising outcomes which are not consistent with the premise of perfect rationality. Customer behavior needs to be modeled based on bounded rationality assumptions to replicate humans' cognitive limitations and biases.

In literature, few studies focus on assortment planning, taking into account the bounded rational behavior of the customers. The first group of studies uses the notion of consideration sets, proposed by (Simon, 1957). Consideration-based choice models assume that consumers form a consideration set and then choose from the alternatives in the consideration set. Considering sets show the humans' cognitive limitation to acquire or process information. This limited ability leads to a lack of attention or intentional inattention. Li develops a one-step consideration-based assortment planning framework by applying the Random Consideration Set model in which each individual forms a consideration set by independently considering each alternative with a given probability (Li, 2018). Jagabathula et al. propose a two-step framework that incorporates consideration-set generation by applying a price threshold from the customers' perspective. The choice probabilities are estimated using a non-parametric choice model (Jagabathula & Rusmevichientong, 2017). Aouad et al. develop a two-step consider-then-choose framework by reducing the number of feasible alternatives' permutations in a rank-based choice model (Aouad, Farias, & Levi, 2015). Wang et al. have developed a two-stage consider-then-choose choice model by incorporating search cost. That is, customers maximize the expected utility net of search cost. The second step follows the MNL model to choose from the consideration-set (Wang & Sahin, 2018). Another stream of consideration-based assortment optimization studies incorporate *framing effects* to form consideration sets ((Gallego, Li, Truong, & Wang, 2020) and (Aouad & Segev, 2021)). Framing effects suggest that consumer choice is influenced by how the products are framed or displayed.

Finally, (Wang, 2021) developed a consideration-based MNL model called the two-step Threshold Multinomial Logit (TMNL) model. Under the TMNL, it is assumed that bounded rationality influences customers' consideration-set formation; however, the customers are assumed to behave rationally within the consideration-set. In the first step, individuals form their consideration sets by defining a cut-off level on utilities. This cut-off is generated by considering a tolerance level for deviation from the largest utility in the choice set. Then, in the second step, they examine all alternatives in the consideration-set and choose the one with the highest realized utility. The second step is modeled by classical linear in parameter MNL model.

Reference-dependency of preferences is another behavioral bias scarcely incorporated in assortment optimization problems. The theory of reference-dependent preferences originates in the seminal paper of (Tversky & Kahneman, 1991). Generally, reference-dependent choice models denote that a reference point influences individuals' preferences between given alternatives. This reference dependency of preferences can explain cognitive biases in many choice situations, like transportation ((Van de Kaa, 2010)). In the assortment optimization literature, (Wang, 2018) developed a choice model that combines MNL preferences with reference prices. In this study, the utility of each alternative depends on other offered options through a choice-set dependent reference price. Loss aversion behavior is also incorporated into the utility specification. Thus, the proposed approach captures the price-specific reference-dependency and choice-set dependency of preferences. It has been shown that incorporating reference prices can improve the prediction accuracy of the model.

Besides, there is ample evidence that customer behavior may deviate from perfect rationality assumptions due to the impacts of the offered choice set (the composition of the choice set) on customer's preferences ((Rooderkerk, Van Heerde, & Bijmolt, 2011)). That is, customer choice behavior is influenced by the composition of choice set in a manner inconsistent with the perfect rationality assumption. This choice-set dependency of preferences is referred to as *context effect* (e.g., *attraction effect*, *compromise effect* and *decoy effect*). Incorporating this phenomenon into the choice models is not straightforward. The reason is that the context effects arise from the structure of choice situation faced by individuals.

Despite the evolution of assortment optimization studies, bounded rationality is still largely overlooked in this research area. Broadly, bounded rationality can be explained by the lack of information or attention, ignoring some available alternatives or attributes (studied under the consideration-based choice models), reference-dependency of preferences, choice reversals originated from the choice-set composition (i.e. context effects), and not well-defined preference order (e.g., loss aversion behavior). Reference-dependency of preferences, loss aversion behavior, and context effects are scarcely integrated into assortment planning frameworks. The proposed approach in our research incorporates the mentioned behavioral biases and can be employed in various application areas ranging from retailing to on-demand transportation services.

2. METHODOLOGY

In this research, we employ the Generalized Random Regret Minimization (G-RRM) model proposed by (Chorus, 2014) to model customer choice behavior. Regret refers to when one or more non-chosen alternatives outperform the chosen one in terms of one or more attributes. The RRM model postulates that decision-makers aim to choose the option which minimizes anticipated random regret. This choice model enables us to capture humans' bounded rationality caused by the reference-dependency of preferences, the context effects (compromise and decoy effects (Guevara & Fukushi, 2016)), semi-compensatory and loss-aversion behavior. It must be noted that the compromise effect has been established in a wide range of decision contexts, like travel behavior (Chorus & Bierlaire, 2013). Thus, using the RRM model which captures the popularity of compromise alternatives helps us model customer behavior in a more realistic way.

Problem description

The main goal of this research is to select the optimal assortment of alternatives from the given universal set Ω including N alternatives. The objective of assortment optimization is to maximize the expected profit per customer. In our problem setting, alternative i is defined by the bundle of M attributes $(x_1^i, \ldots, x_M^i)$, including the associated price.

The random regret of alternative i is composed out of a systematic regret R_i and an i.i.d. random error ε_i, $RR_i = R_i + \varepsilon_i$. The systematic regret of alternative i is defined as the sum of the binary regrets that are associated with bilaterally comparing the attributes of alternative i with each of the other alternatives in the choice set. Thus, the systematic regret of alternative i in assortment s is written as below:

$$R_i(s) = \sum_{j \in s,\ j \neq i} \sum_{m=1}^{M} \left(\ln\left(\gamma + \exp\left[\beta_m.(x_m^j - x_m^i)\right]\right) - \ln(1+\gamma) \right) \tag{1}$$

Where β_m and γ denote the weight of attribute m and the regret weight, respectively. It is worth noting that the regret weight adjusts the convexity of the regret function (1). Also, the choice probability associated with alternative i is defined by Equation (2), where V_0 denotes the constant attraction value of the no-purchase option.

$$\pi_i(s) = \frac{\exp\left(-R_i(s)\right)}{V_0 + \sum_{j \in s} \exp\left(-R_j(s)\right)} \tag{2}$$

Let p_i denote the associated profit of alternative i. Therefore, the assortment optimization problem is formulated as follows.

$$\max_{s \subseteq \Omega} \sum_{i \in s} p_i.\pi_i(s) \tag{3}$$

Behavioral properties

The RRM model (1) is a reference-dependent choice model in which each offered alternative acts as a reference point for other options. Moreover, this model relaxes the independence of irrelevant alternatives (IIA) property. These properties stem from the binary comparisons between the attributes of the offered alternatives. Besides, (1) captures the compromise effect and decoy effect as popular context effects. It must be noted that the decoy effect is a direct violation of the regularity assumption. It also represents semi-compensatory and loss-aversion behavior which indicate that deteriorating an attribute can not be completely compensated by improving another attribute to a similar extent. These properties are direct results of the convexity of the regret function; the regret decreases by improving an attribute, but that difference is always going to be smaller than the increased regret by the deterioration of an equally important attribute.

Solution method

Each optimization on Problem (3) could require the evaluation of all potential subsets of the universal set. Generally, assortment optimization is a combinatorial problem. It has been proven that the unconstrained assortment optimization under either MNL or NL choice model when dissimilarity parameters change in $(0\ 1]$ ((Davis et al., 2014)) can be solved in polynomial time. Otherwise, the assortment optimization is an NP-hard problem. To solve Problem (3), we propose a greedy algorithm described below.

INITIALIZATION. Generate all the subsets including two alternatives, $S = \{s \subseteq \Omega;\ |s| = 2\}$.

ITERATIVE STEP. For all $s \in S$, while $|s| \leq N$:

Enlarge s by adding alternative $w \in \Omega \backslash s$ which maximizes the expected profit of assortment $s \cup \{w\}$ defined by (3).

Stopping Condition. Stop when objective value decreases.

SOLUTION. Return s with the maximum objective value.

3. RESULTS AND DISCUSSION

The described methodology is coded with Python. As our running example, we consider an online vehicle-sharing service that provides two types of transportation services: (i) bike ($s = 1$) and (ii) scooter ($s = 2$). In this problem, alternative i is defined by three attributes: service type (s_i), the walking distance to the pick-up location (d_i), and associated price (p_i). We assume that there is a universal set including ten potential options as described in Table (1). The service provider offers each customer a set of alternatives so that the expected revenue per request is maximized.

Table 1: Universal set of potential alternatives.

No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Service type (s)	1	1	2	1	1	2	2	1	2	2
Walking distance (d)	100	110	120	130	135	140	145	150	155	300
Price (p)	25	24	25	18	17	17	16	12	19	20

To examine the proposed algorithm's performance and compare the RRM and MNL models, we generated multiple scenarios based on the parameters of the regret and utility functions in the RRM and MNL models. The regret weight (γ), weights of the service type (β_s), walking distance (β_d), and price (β_p) are defined in Table (3)). We employ Equation (1) for the RRM model. For the MNL model, we assume that the deterministic utility of alternative i is defined by $u_i = \beta_s.s_i + \beta_d.d_i + \beta_p.p_i$. We presume that the attraction value of the no-purchase option under both RRM and MNL models is 1.

Table (3) shows the optimal assortment under the RRM (S^*_{RRM}) and MNL (S^*_{MNL}) models as well as the proposed assortment by our algorithm (S^*_{Alg}). The expected revenue of the proposed assortments by the RRM (R^*_{RRM}), MNL (R^*_{MNL}), algorithm (R^*_{Alg}), and the expected revenue of the MNL's optimal assortment under the RRM model ($R^{RRM}_{S^*_{MNL}}$) are also shown in Table (3). Under all scenarios, our proposed algorithm finds the optimal assortment, which is obtained by the complete enumeration.

Table 2: Optimal assortments under different scenarios.

No.	γ	β_s	β_d	β_p	S^*_{RRM}& S^*_{Alg}	R^*_{RRM} & R^*_{Alg}	S^*_{MNL}	R^*_{MNL}	$R^{RRM}_{S^*_{MNL}}$
1	0	0.5	-0.9	-2.9	{1,2,3,10}	18.67	{1,2,3,4,5,6,9,10}	16.13	17.05
1.1	0	10	-0.9	-2.9	{1,2,3,4,5}	24.98	{1,2,3,4,9,10}	17.04	20.28
1.2	0	0.5	-10	-2.9	{1,10}	24.93	{1,2,3,9,10}	18.56	23.71
1.3	0	0.5	-0.9	-10	{1,2,3,10}	18.86	{1,2,3,4,5,6,9,10}	16.08	17.05
2	0.5	0.5	-0.9	-2.9	{1,2,3,10}	18.61	{1,2,3,4,5,6,9,10}	16.13	17.27
2.1	0.5	10	-0.9	-2.9	{1,2,3,4,5}	23.51	{1,2,3,4,9,10}	17.04	19.99
2.2	0.5	0.5	-10	-2.9	{1,2,3,10}	21.36	{1,2,3,9,10}	18.56	21.34
2.3	0.5	0.5	-0.9	-10	{1,2,3}	18.46	{1,2,3,4,5,6,9,10}	16.08	16.78
3	1	0.5	-0.9	-2.9	{1,2,3,10}	18.62	{1,2,3,4,5,6,9,10}	16.13	17.46
3.1	1	10	-0.9	-2.9	{1,2,3,4,5}	21.40	{1,2,3,4,9,10}	17.04	19.42
3.2	1	0.5	-10	-2.9	{1,2,3,9,10}	20.43	{1,2,3,9,10}	18.56	20.43
3.3	1	0.5	-0.9	-10	{1,2,3}	18.43	{1,2,3,4,5,6,9,10}	16.08	16.64

We use this example to show the behavioral differences between the RRM and MNL models. One can compare Scenario 1 and 3 to see the loss aversion behavior incorporated in the RRM model. The optimal assortment and expected revenue under the MNL model are the same for these two scenarios. Although the optimal assortments under the RRM are also the same, the optimal expected revenues are different for these scenarios. The reason is that the loss aversion behavior is embodied in Scenario 3 ($\gamma = 1$), but it is not included in Scenario 1 ($\gamma = 0$).

We also use this example to show that the RRM model can replicate the decoy effect which is the result of the choice-set dependency of preferences. To this end we compare two different assortments under Scenario 3. Let $s_1 = \{1,2,4,5,6,9,10\}$ and $s_2 = \{1,2,3,4,5,6,9,10\}$, Table (3) denotes the choice probabilities of the offered alternatives for s_1 and s_2. As it is shown by Table (3), the choice probabilities of alternatives 4, 5, and 6 increase after adding alternative 3 to Assortment s_1. Thus, alternative 3 performs as a decoy alternative for alternatives 4, 5, and 6. Moreover, this example indicates that the RRM model violates the regularity assumption.[1]

Table 3: Impacts of adding an alternative to an assortment under the RRM model

	1	**2**	**3**	**4**	**5**	**6**	**9**	**10**
s_1	0.0638	0.0702	-	0.0772	0.1336	0.1471	0.1859	0.2041
s_2	0.0635	0.0697	0.077	0.1323	0.1456	0.1842	0.1409	0.0803
$\theta_i = \frac{\pi_i(s_2)}{\pi_i(s_1)}$ 1	0.995	0.994	-	1.714	1.090	1.253	0.758	0.393

[1] If $\theta_i > 1$ then the regularity assumption is violated and alternative 3 is a decoy for alternative i (see (Guevara & Fukushi, 2016) for more details)

4. CONCLUSIONS

This study develops an assortment optimization framework that incorporates humans' bounded rationality. The proposed bounded rational framework leads to different assortment and purchase decisions made by the supply and demand sides, respectively. We explain these differences in light of the different behavioral assumptions of our model. We propose a greedy algorithm that can solve the problem. In the coming months, we will apply the algorithm to bigger choice sets and elaborate on mathematical proofs.

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