A bi-level optimization model for the EV charging stations location problem using a tripartite graph representation

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SHORT SUMMARY

We propose a bi-level mixed integer program to model the optimal location and sizing of Electric Vehicle (EV) charging stations deployment in an inter-urban area, aiming to accommodate in-route charging demand. Our key step is a network simplification, allowing to model the charging behavior through a tripartite graph of origins, charging stations, and destinations. We apply a heuristic to find a high-quality solution, where parametric versions of the two levels are independently solved. While the upper level is a linear integer program, the lower level is suitably solved through a user equilibrium model, highly benefiting from the compact network. Our methodology is validated in the Quebec network and shown to achieve a robust solution in a short running time. Charging stations were located between the primary population centers, serving the main share of the demand. Layout alternatives and parameters changes can be easily evaluated due to the model simplicity.

Keywords: bi-level, compact network, electric vehicles, facility location, variables relaxation

1. INTRODUCTION

The growing concern of climate change has been drawing wide attention to the attempts to reduce greenhouse gas emissions. EVs are one of a set of environmental solutions, reducing emissions produced by road transportation, inter-urban air pollution, and our dependency on fossil energy sources.

A major obstacle to mass adoption of EVs is their limited travel range, together with the limited availability of charging infrastructure. The willingness of customers to purchase an EV is widely dependent on the degree of discomfort involved in battery charging. This motivates the proper planning of location and sizing of charging stations.

The structure of the charging station location problem is inherently bi-level, as the relationship between the decision makers, the operator and EV users, is hierarchical. The siting of new charging stations (leader's decision) must respond to the anticipated EVs demand and vehicle distribution in the network (follower's decision), while the demand and path choice depends on the locations and sizing of the charging stations. Nevertheless, most models to date handle the problem in a single layer (e.g., Zhang et al. 2017, Xie et al. 2018, Anjos et al. 2020), where EVs assignment is embedded within the charging station location model. This is performed by simply assigning the demand to a single (or several) path(s) connecting an origin-destination (O-D) pair, if the travel range enables it. Most models consider only the shortest path between each O-D. This approach has notable drawbacks further discussed next. First, it restricts the assignment by ignoring other feasible paths. Second, charging stations are assumed to be used only if they are located along the considered shortest path. In practice, multiple routes are used, and drivers are often willing to conduct some detour to charge their vehicle.

The optimal location of new charging stations has received growing attention in the last two decades. Most problems presented in the literature extend the general facility location problem by introducing EVs travel range constraints. The spatial distribution of demand is usually modelled as node-based or flow-based demand. In the node-based approach, users are assigned to one or more fixed locations, usually user's residence or workplace. Such models are suitable when deployment decisions are limited to an urban or suburban area. However, when considering long-haul travels, the limited travel range of EVs requires a different modelling approach; EVs charging is required along the journey and the station location decision often considers the attractive paths between different O-Ds. In the flow-based approach, trips are assigned to the network only if the charging infrastructure along their path allows them to complete their journey. Anjos et al. (2020) suggested a model that combines the two approaches to model an aggregated intra-urban and inter-urban areas. Recent contributions focused on multi-period planning of charging stations location, privileging long-term planning modelling (e.g., Xie et al. 2018, Anjos et al. 2020).

A significant limitation of common flow-based models is their simplistic assignment; most models arbitrarily assign the demand to one (or few) shortest path(s) if the travel range and the located charging stations along the route enable its completion. A journey between remote O-D pairs can be feasible by adding a minor detour to the shortest path so it passes through a charging station. Single-layer models have difficulty to accommodate such journeys. Moreover, as a single-layer model does not include path choice, the impact of congestion, both on travel times and waiting times at stations, is ignored. This issue poses a significant limitation on the estimation of satisfied demand rate and stations usage.

Another significant hurdle is the size of these network models. Even the (simple to describe) facility location problem is NP-hard. Due to the charging station location problem complexity, any exact optimization method is likely to require immense computational effort, and applications are commonly demonstrated and limited to small networks.

The objective of this study is to develop a more realistic model for the location and size of new charging stations, maximizing stations usage for in-route charging. We aim to express the interaction of decision-makers, the leader (charging station operator) and the followers (EV drivers), while presenting an efficient approach, applicable to realistic networks.

2. PROBLEM FORMULATION

The problem is formulated as a bi-level mixed integer programming problem. We define a graph $G(N,A)$, where N comprises 3 types of vertices: N_o - origins, N_d - destinations, N_c - charging stations $(N_o, N_d, N_c \subseteq N)$, and A comprises 3 types of arcs: A_d - direct O-D arcs, A_1 - origins to stations , A_2 - stations to destinations $(A_1, A_2, A_d \subseteq A)$. Problem parameters and decision variables are given in Table 1 and the two layers of the problem are defined bellow. As we focus on in-route charging, we consider only fast charging stations, namely level 3 chargers that provide an 80% state of charge in 20-30 minutes.

Table 1: Parameters and variables

Parameters	
B	Budget
C_{ij}	cost of traveling in arc *(i,j)*
I	set of the possible number of outlets to be installed in a station
T	modelling period
ST_j	average service time at location *j*
a_i, b_i	coefficients of the waiting time function at a station with $i \in I$ outlets
C_j^s	cost of installing a charging station at location $j \in N_c$
C_j^i	cost of installing $i \in I$ outlets at location $j \in N_c$
D_i	Demand from origin i
μ_i	Maximum utilization coefficient for a station with $i \in I$ outlets
Decision variables	
X_{ij}	the number of EVs traveling on arc *(i,j)*
X_j	the number of EVs entering node $j \in N_c$
U_i	Unsatisfied demand from origin $i \in N_o$
Y_j	1 if a charging station is located in $j \in N_c$, 0 otherwise
S_j^i	1 if $i \in I$ charging units are installed at station $j \in N_c$, 0 otherwise

Upper Level

$$\underset{Y,S}{Max} \sum_{j \in N_c} X_j \tag{1}$$

$$s.t \sum_{j \in N_c} (\mathrm{C}_j^s \cdot \mathrm{Y}_j + \sum_{i \in I} \mathrm{i} \cdot \mathrm{C}_j^i \cdot S_j^i) \leq B \tag{2}$$

$$\sum_{i \in I} S_j^i \leq Y_j \quad \forall j \in N_c \tag{3}$$

$$Y_j, S_j^i \in \{0,1\}, \forall j \in N_c, \forall i \in I \tag{4}$$

X_{ij} solves the lower-level problem with S_j^i fixed.

Lower Level

$$\underset{X,U}{Min} \sum_{ij \in A_d, A_2} C_{ij} \cdot X_{ij} + \sum_{j \in N_c} \sum_{i \in I} S_j^i \int_0^{X_j} a_i \cdot b_i^{(\frac{ST_j}{i \cdot T} X_j)} dX_j + \sum_{i \in N_o} M \cdot U_i \quad (5)$$

$$s.t \sum_j X_{ij} + U_i \geq D_i \ \ \forall i \in N_o \quad (6)$$

$$\sum_i X_{ij} - \sum_k X_{jk} = 0 \ \forall j \in N_c \quad (7)$$

$$X_j = \sum_i X_{ij} \ \ \forall j \in N_c \quad (8)$$

$$X_j \leq \frac{T}{ST_j} \sum_{i \in I} \mu_i \cdot i \cdot S_j^i \ \ \forall j \in N_c \quad (9)$$

$$X_j, X_{ij}, U_i \geq 0 \ \ \forall ij \in A \quad (10)$$

The upper level problem maximizes the utilization of charging stations, expressed through the total number of EVs entering charging stations. Eq. 2 expresses the budget constraint and eq. 3 relates the binary variables, ensuring that charging units can be installed only if a charging station is installed.

The lower level features the user perspective, minimizing the generalized cost of the travel (eq. 5), whereas first component is the travel time, second is the waiting time at stations and third is a penalty for any unsatisfied demand. Waiting time at stations, expressed through the second component, is an empirical function estimated using real data measurements of average waiting times in Quebec charging stations. The exponent is the utilization factor, meaning the proportion of time in which charging outlet(s) in a station is (are) occupied out of the modelling period. Eq. 6 is the demand assignment. Eq. 7 are the flow balance constraints. Eq. 8 relates flow variables. Eq. 9 expresses station capacity, relating the variables of the two levels.

3. METHODOLOGY

The following sub-sections describe the main components of the model: network simplification and solution approach. The network simplification includes estimation of travel times, consideration of EVs travel range, paths finding, and determination of a tripartite graph. The solution method uses relaxation variables to relate the upper and lower level variables and iteratively solves each of the layers (levels) to optimality.

We assume that due to EVs low market share, they have a marginal effect on traffic condition and travel times. Accordingly, the assessment of travel times can be extraneous to the optimization model.

Network Simplification

This module includes the formation of an alternative tripartite graph, that is used in the optimization model instead of the road network. The graph comprises origins, charging stations and destinations and its generation is conducted as a pre-processing step.

First, we run a user equilibrium model over the road network, using the non EVs demand only. The obtained assignment produces the estimated travel times at equilibrium.

Then, we form the tripartite graph, where 3 types of arcs are formed: direct (N_o to N_d), origins to charging stations (N_o to N_c), and stations to destinations (N_c to N_d). Note that in a widely spread network, where multiple in-route charges are required, layer(s) of charging stations can be added to the graph. Arcs are formed between all vertices of each group if the path between them meets EVs travel range constraint. The travel time between the two vertices is assigned as the arc cost. Note that this cost represents travel time at equilibrium and not in a specific path. This is a simple directed graph in which all paths are feasible for EVs. To exemplify this procedure, the tripartite representation of the Sioux-Falls benchmark (LeBlanc et al., 1975) is presented in Figure 1.

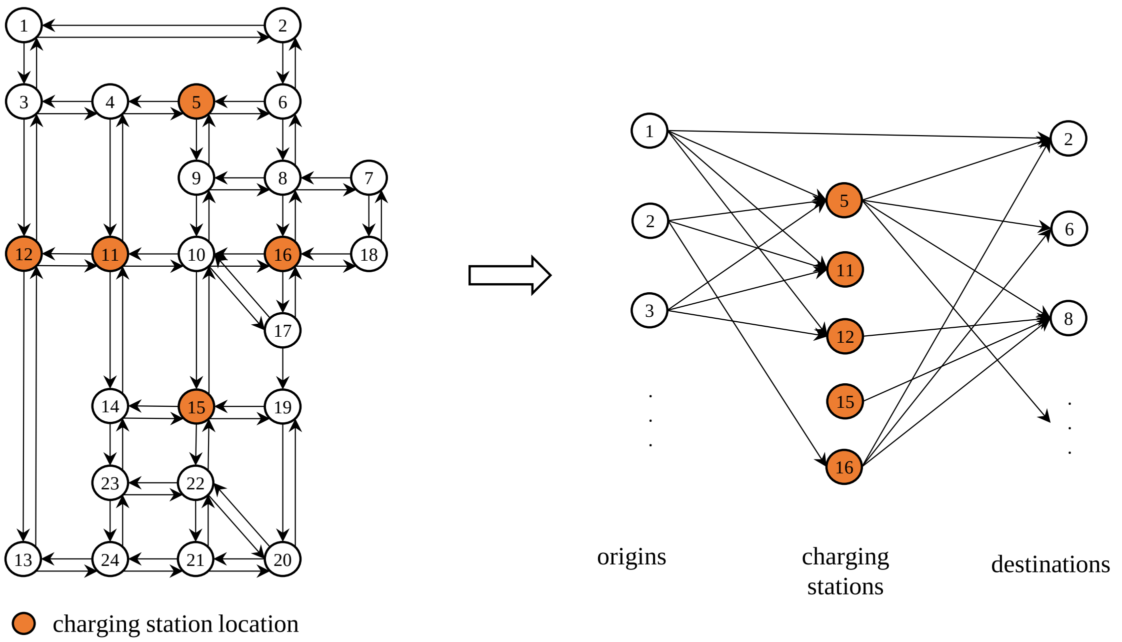

Figure 1: Graph representation

Note that the size of the graph depends on the number of O-Ds and charging stations, regardless of the road network size. This modelling does not necessarily reduce the size of the network but transforms the road network into a concise in-route charging graph.

After forming the graph, we create a path set of all attractive paths for each O-D pair. A path is considered attractive if it is longer by no more than some threshold compared to the shortest path (200% in this paper). Paths are found by serially excluding arcs of paths that are already exist in the set.

Solution Approach

The main idea behind the heuristic algorithm we use is the relationship between the upper and lower level variables (Sun et al. 2008). Relaxation variables replace the lower level variables in the upper level problem, and in each iteration, a temporary value is assigned to them, enabling to solve the two layers independently. We use Eq. 9 to define the relaxation variables y_j^*:

$$y_j^* = \frac{T}{ST} \sum_{i \in I} \mu_i \cdot i \cdot S_j^i - X_j \quad \forall j \in N_c \tag{11}$$

The heuristic is as follows:

1. Find an initial feasible solution for the discrete upper level problem.
2. Solve the lower level problem for the current upper level solution.
3. Calculate relaxation variables.
4. Replace lower-level variables in the upper-level problem with the relaxation variables and solve it.
5. Calculate the difference between two consecutive upper level objectives, and if lower than a certain tolerance stop, otherwise go to step 2.

The upper level problem is a binary linear program where the number of variables depends on the number of candidate locations and the number of station size categories. This problem can be efficiently solved with a variety of existent solvers.

The lower level is solved as an EVs user equilibrium model using a Frank-Wolfe algorithm, a well-established method that has proven to comply optimality conditions. In each iteration, waiting times at stations are updated according to the expected in-flow, and path choices are updated accordingly. In each solution of the lower-level problem we use a subset of the path set, comprising only edges and vertices associated with stations that exist in the solution.

The station capacity constraint (eq. 9) is applied indirectly by reaching an equilibrium state. Congestion in stations results in longer waiting times, leading to lower in-flows in the next iteration and shifting demand to other stations. In the absence of charging alternatives, surplus demand is considered as unsatisfied. The penalty for unsatisfied demand is expressed through the third component of the objective function (eq. 5). The big-M requires careful calibration; a high value may result in a violation of the capacity constraint, prioritizing long waiting times over unsatisfied demand penalty; a low value may result in over-estimation of the unsatisfied demand rate. We calibrated the big-M to comply with the waiting time in a fully-capacitated station, thereby, up to that in-flow rate, demand is assigned to the station, as the waiting times are shorter than the penalty, and any additional demand is assigned to the unsatisfied demand rate.

The compact structure of the graph enables to consider all attractive paths rather than a subset in the path set. This ensures an optimal solution of the lower-level problem without applying an iterative path finding procedure which is highly time consuming. This is a notable contribution of the suggested modelling approach.

4. RESULTS AND DISCUSSION

To test our methodology, we used the Quebec province network (Anjos et al., 2020). The network comprises 961 links, 896 nodes and 547 demand zones. Assuming an EV share of 10%, its overall demand is 41360 trips. O-D pairs with a demand of less than 1 trip were neglected. The current network has 344 fast charging stations. New charging stations can be located every 20km and include 2, 4 or 8 outlets. Considering the locations of existing stations, the network comprised 333 candidate locations. The average EVs travel range was set as 250km according to the current EV models in the Quebec market, average charging time to 30 minutes, range anxiety to 75 km (30% of battery range), initial battery state to 72%, budget to $10M and installation costs to $22.5k and $45k for a new outlet and station, respectively.

All implementations are coded in Python 3.9 and the upper level problem is solved with Gurobi 9.0.3. Running time on a simple laptop (i5-8250U, 1.8GHz CPU, 8GB RAM) was 129 seconds. The tripartite graph comprised 11615 arcs. The attractive path set included 1586 paths.

The solution, presented in Figure 2 was found after 3 iterations, included 22 new charging stations, all comprising the maximal number of 8 outlets, to be installed. The total station flow was 704.25 vehicles, and the unsatisfied demand rate was 0.77%. Locating stations in all candidate locations results in the same objective value (704.25 vehicles). Thus, the solution was proven to be optimal.

All new charging stations were located in between the main population centers. We see two explanations: (1) as the model deals with average flows, the solution serves the main share of the demand, and (2) the basic model limits the assignment to a single in-route charging and thus, remote O-D pairs are infeasible to serve.

EVs travel range is widely reflected in the model, determining the feasibility of paths. Thus, we tested the algorithm performance under different range values. The solution showed robustness to changes in the travel range, locating charging stations in the same place with the same number of outlets. The unsatisfied demand rate and the size of the tripartite graph are shown in Table 2. As expected, the satisfied demand rate and network size, representing the number of feasible paths, both increased with the increase in travel range.

Table 2: EVs Travel range impacts

Travel Range (km)	Unsatisfied demand %	Arcs in tripartite graph
200	1.24	3667
225	1.03	4341
250	0.77	5004
275	0.58	5632
300	0.35	6254

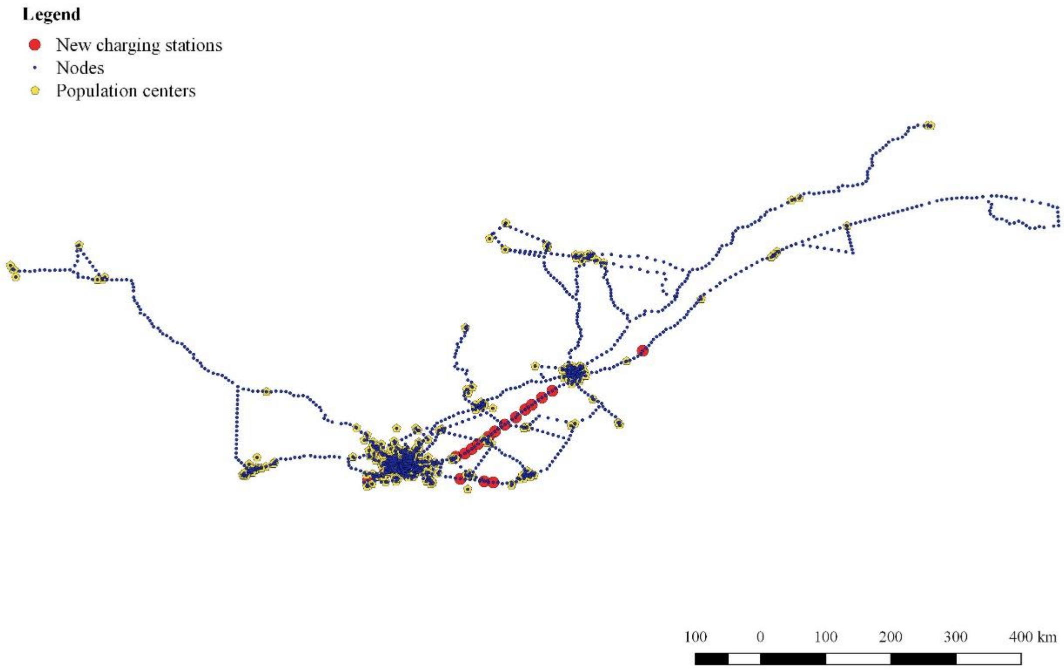

Figure 2: Quebec network and new charging station locations

5. CONCLUSIONS

This paper describes the development of an optimization model for the location and sizing of charging stations for EVs. The main idea of our approach is to model charging patterns using a tripartite graph that comprises origins, charging stations and destinations. The compact network structure enables to easily apply equilibrium algorithms to compare in-route charging alternatives.

The bi-level optimization model maximizes the utilization of new charging stations to be deployed. We applied a simple heuristic that utilizes relaxation variables to relate the variables of the upper and lower level problems. The solution scheme allowed solving the bi-level to optimality in a short running time. The model was validated on the Quebec network and shown to achieve a robust optimal solution. The efficiency of the model enables to easily evaluate layout alternatives and analyze the impact of changes in parameters such as EVs travel range and budget, thereby it can provide a useful planning tool.

ACKNOWLEDGMENT

This research is conducted under the generous support of the Natural Sciences and Engineering Research Council of Canada through its Collaborative Research and Development Grants.
The authors wish to thank to Electrification des Transports team at Hydro-Québec for sharing with us their expertise on EVs, and to Emma Frejinger, Bernard Gendron and Miguel Anjos for preliminary discussions of the model.

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