Assignment matrix free algorithms for on-line estimation of Dynamic Origin-Destination matrices

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1 Introduction

Dynamic Traffic Assignment (*DTA*) models represent fundamental tools for planning, designing and managing transportation networks (Chiu et al., 2011; Bliemer et al., 2017). These models are used by planners and policy makers to forecast traffic flows on the network and assess new transport policies. In real-time applications, they are used to evaluate traffic strategies such as route guidance and traffic control operations.

As biased models lead to incorrect predictions, which can cause inaccurate evaluations and huge societal costs (Hao, Hatzopoulou, & Miller, 2010), the calibration of *DTA* models is now a quite establish and active research field (Tympakianaki, Koutsopoulos, & Jenelius, 2015; Krishnakumari, van Lint, Djukic, & Cats, 2019). When it comes to estimating Origin-Destination (OD) demand flows, perhaps the most important input for *DTA* models, one algorithm proved to outperform all the others for real-time applications: the Kalman Filter (Ashok, 1996; Zhou & Mahmassani, 2007).

Kalman filter uses a series of measures obtained over time to estimates the most likely status of an unknown variable. In the case of on-line estimation of demand (simply called On-line Dynamic Demand Estimation - or *ODDE* - in the rest of this paper), time-dependent OD flows are the unknown variables and observed traffic conditions are the data. However, despite more than twenty years of intense research effort, the ODDE problem has not yet been quite solved. The main reason is that, even for relatively small networks, the problem is prone to exhibit underdeterminedness due to the high number of unknown variables (Marzano, Papola, & Simonelli, 2009). Additionally, the relation between demand and traffic conditions is highly nonlinear, meaning that a change into the input demand does not reflect into a proportional change of the *DTA* outputs. Researchers distinguish thus between three main sources of error:

1. **Number of variables:** The ODDE procedure generally returns effective results when the number of observations (traffic data) is similar to the number of unknowns (OD flows). However, as this usually does not occur in practice, dimensionality reduction techniques should be deployed to avoid this issue (Marzano et al., 2009; Djukic, Van Lint, & Hoogendoorn, 2012).

2. **Nonlinear relationships between variables:** There are two main ways of considering non–linearity within the ODDE, which ideally should be jointly considered. First, to include different data sources in order to increase the observability of the system. For instance, jointly considering speeds, densities, and counts can help to better understand traffic phenomena (Balakrishna, Antoniou, Ben-Akiva, Koutsopoulos, & Wen, 2007; Frederix, Viti, Corthout, & Tampère, 2011; Yang, Lu, & Hao, 2017). Second, describing nonlinear systems entails deploying nonlinear models. As the conventional Kalman Filter is a simple linear model, several nonlinear extensions have been proposed (Antoniou, Ben-Akiva, & Koutsopoulos, 2007).

3. **Demand structure:** Mobility demand derives from the demand for activities and, as such, it has a structure. Different models should be used to target different components of the

demand, including random fluctuations, structural and seasonal trends, and regular trends (Zhou & Mahmassani, 2007; Cantelmo, Qurashi, Prakash, Antoniou, & Viti, 2019).

This paper introduces a nonlinear Kalman Filter framework for on-line dynamic OD estimation that reduces the number of variables and can easily incorporate heterogeneous data sources to better explain the nonlinear relationship between traffic data and time-dependent OD-flows. The remainder of the paper is structured as follows. The model, an extension of the Local Ensemble Transformed Kalman Filter (LETKF) proposed in (Carrese, Cipriani, Mannini, & Nigro, 2017), is shortly presented in Section 2. Section 3 shows the numerical results on a synthetic experiment and, finally, Section 4 provides some concluding remarks.

2 Methodology

In this section, we shortly summarize the Kalman Filter (KF), the Ensemble Kalman Filter (EnKF), the Local Ensemble Transformed Kalman Filter (LETKF), then moving to our extension, which combines Principle Component Analysis with the Local Ensemble Transformed Kalman Filter (PCA-LETKF). We refer the interested reader to (Antoniou et al., 2007; Carrese et al., 2017) for more details.

2.1 Kalman Filter

The Kalman Filter for ODDE problem formulated as a state-space model is composed of two types of equations:

1. The *transition equation*, which describes the evolution of the system over time;
2. The *measurement equation*, which maps the available traffic data to the state vector.

In essence, the Kalman filtering approach assumes that at each step we have two Gaussian distributions: the predicted state, which follows a Gaussian distribution fully described by the *transition equation*, and the observed data, which is described by the *measurement equation*. The combination of these two provides the most likely state for the unknown variable.

2.2 Ensemble & Local Ensemble Transformed Kalman Filter

As mentioned in Section 1, the Kalman Filter is probably the most popular algorithm to tackle the ODDE problem. However, both the *transition equation* and the *measurement equation* assume a linear relation between variables. In the case of the *transition equation*, this relation is usually represented in the form of an auto regressive process (Ashok, 1996), while the *assignment matrix* is usually used to feed the *measurement equation*. As these formulation poorly represent traffic dynamics, non linear models need to be deployed in real cases (Antoniou et al., 2007). The Ensemble Kalman Filter (**EnKF**) is one of these options (Hunt, Kostelich, & Szunyogh, 2007). The EnKF chooses an ensemble of initial conditions around the current estimate and propagates each ensemble member based on a nonlinear model. Thus, the uncertainty of the estimation is propagated from one time interval to the other and the ensemble is used to parametrize the distribution of the state variables.

The Local Ensemble Transformed Kalman Filter **LETKF** adds two extension to the normal EnKF. First, allows to minimize the Kalman filter cost function in the ensemble space, thus reducing the dimension of the problem and reducing problem complexity (it is "transformed"). Second, it provides a framework for data assimilation that allows a system-dependent localization strategy, breaking down the problem into sub-problems to be solved in parallel fashion (it is "local"). An additional value is that the LETKF, differently from the EnKF, does not require the assignment matrix, which is difficult to obtain and imposes a linear relation between ODs and traffic data. Finally, ignoring the assignment matrix allows to easily include different data sources within the model. Again, for more details about the equations, we refer to (Hunt et al., 2007; Carrese et al., 2017).

2.3 PCA Local Ensemble Transformed Kalman Filter

As pointed out in (Hunt et al., 2007; Carrese et al., 2017), the LETKF provides a framework for data assimilation that allows a localization strategy, i.e. if different data can be referred to specific OD flows, the problem can be broken down into sub-problems to be solved in parallel. For the ODDE, the "local" approach means dividing the network into subnetworks and the demand matrix into submatrices, each submatrix containing the ODs that mostly affect the traffic measurements in the corresponding subnetwork. This is something that has been already tested for the off-line OD estimation problem (Cantelmo, Cipriani, Gemma, & Nigro, 2014; Antoniou, Azevedo, Lu, Pereira, & Ben-Akiva, 2015; Lu, Xu, Antoniou, & Ben-Akiva, 2015). However, this entails developing procedures to explicitly map the relative weight of the information (e.g. how ODs and link flows are correlated). An assumption-free procedure that already calculates how ODs are correlated is the Principal Component Analysis (PCA)(Prakash, Seshadri, Antoniou, Pereira, & Ben-Akiva, 2018; Qurashi, Ma, Chaniotakis, & Antoniou, 2019). As suggested in (Djukic et al., 2012; Prakash et al., 2018), we replace the OD demand with its principal components. As showed by the previous authors, this reduces problem complexity making the ODDE problem simpler. Additionally, the PCA identify variables that are highly correlated, providing a good classification for the localization framework. This means that, with respect to the conventional LETKF, less ensembles are needed to achieve similar or better estimates.

3 Results and discussion

We tested the EnKF, LETKF, and PCA-LETKF on a synthetic network with 3249 OD pairs and 395 detectors. The synthetic model uses the following (nonlinear) relationship between ODs and traffic counts.

$$Y_i = \sum_{n \in ODs} w_1 X_n + w_2 X_n^2 \tag{1}$$

Where Y_i represents the general detector i and X_n represents the demand for OD pair n. The two weights w_1 and w_2 are random weights that relates link and demand flow for each OD/detector. Results in terms of estimated link flows and OD flows at the end of the different model runs are shown in Figure 1, where the red dashed line represent the initial error.

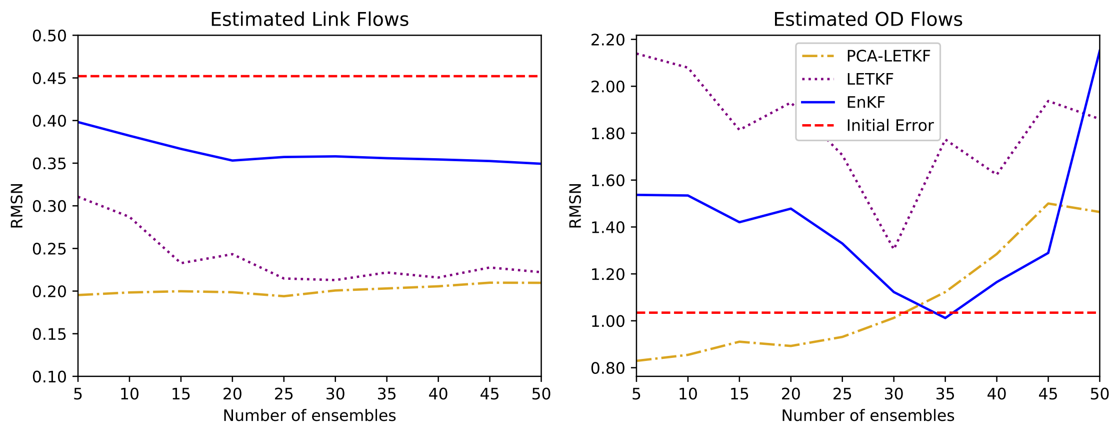

Figure 1: Error in terms of link flows (left) and deviation from the OD flows (right) for each model.

As expected, both the LETKF and PCA-LETKF outperform the traditional EnKF. Despite being a quite advanced model capable of handling non linearity, even using 50 ensembles it only reduces the error from 0.45 to 0.34 (blue thick line). It is also important to point out that - in order to capture nonlinear phenomena - each ensemble requires to perform an objective function evaluation. This entails running the DTA model 50 times for each time interval, one for each ensemble member. The main reason is that we only have 395 detectors to explain 3249 variables.

In similar conditions, the LETKF is definitely providing better results in terms of link flows. However, the PCA-LETKF (yellow dash-dotted line) performs better already with 5 ensembles, while the normal LETKF (purple dotted line) only provides good results ($RMSN \approx 0.2$) for more than 25 ensembles, where again more ensembles means more computational time. The reason is again that - despite its name - the LETKF is not exploiting any localization strategy. This means that more ensembles are needed to learn the structure of the data. Additionally, the PCA-LETKF also performs better in terms of OD flows for most of the cases.

4 Conclusions

This paper introduces a methodology that combines Principal Component Analysis and the Local Ensemble Transformed Kalman Filter (PCA-LETKF). We show that - for transport applications - the proposed PCA-LETKF outperforms conventional nonlinear models, including the LETKF model. The reason is that the PCA finds spatial correlations between variables empowering the LETKF to better exploit its "local" nature.

Next steps, which will be showed at the hEART2020 conference, will include a systematic analysis with respect to the sizes of the network. The simple (nonlinear) function showed in Equation 1 will be used to assess how performances change with respect to both network sizes and number of detectors. Then, the model will be tested on the real large-sized network of Vitoria, Spain. We will use Aimsun (AIMSUN, 2017) as mesoscopic simulation model. As the network of Vitoria has the same characteristics of our case study (3249 OD pairs and 395 detectors), this will validate applicability and usability of the model.

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