Modeling the effect of bike-and-ride mode on the competing public transit market in a linear monocentric city

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ABSTRACT

This paper investigates the interactions between transit operators when bike-train integration is implemented by a private transit operator. A Bertrand-Nash bi-level inter-modal equilibrium model for a deregulated transit market is introduced to describe the fare competition between two oligopolistic transit operators, bus and train, in a linear monocentric city. The transit market is assumed to be a profit-maximizing duopoly that competes non-cooperatively. Traveler behavior is assumed to be a deterministic user equilibrium transit assignment. As the operator offering bike-and-ride mode, the train operator can determine the locations to provide bike-and-ride facilities. Numerical examples are provided to show that this combined mode changes the pricing strategies for both operators and how the saved access time by cycling alters the market share and profit of both transit operators.

Keywords: Nash Equilibrium; Transit market; bike-and-ride; Bicycle-sharing system; Green transport policy

1. INTRODUCTION

Bike-and-ride (B&R) mode, a combination of cycling and public transport modes, has been revealed to encourage the use of both public transport modes and cycling (e.g., Martens, 2007) because it combines the advantages of bicycle and public transport mode by simultaneously solving the high access time (or the first/ last mile problem) encountered by public transport modes and the inability to provide distant trips encountered by cycling. In this mode, the traveler borrows a shared bicycle close to his origin, cycles to the public transport interchange, parks the shared bicycle there and gets on the public transport mode. Although B&R mode is expected to improve the competitiveness of both cycling and public transport modes and encourage the travelers to use both trips (Rixey, 2013), very few public transport operators consider operating bike-sharing systems (BSSs) to improve their competitiveness because of the concern that the increased patronage by B&R mode cannot recover the setup and operation costs of the BSS.

While the synergy and the possible implementation strategies of bicycle-transit integration has been investigated in many studies (e.g., Martin & Shaheen, 2014; Pucher et al., 2010; Cheng & Liu, 2012), only Li et al. (2015) modeled how the strategies associated with B&R mode influences the modal split in the transportation system. They modeled the effect of the pricing of a bike-sharing system (BSS) in a multi-modal network with four modes (i.e., auto, bus, combined bus and bicycle, and bicycle), but the combined mode can only be used in a bus-first-bicycle-second manner. Their results showed that (1) the bus travelers shift to the bicycle mode and the combined mode when bicycles are available; and (2) there exists an optimal rental price for the BSS to maximize total social welfare. However, in their studies, the BSS is owned by the public operator and the transport system only considered one public transit mode, so there is no clue how a BSS developed by a transit operator can change the modal split in public transit system with multiple modes. This study fills this gap by discovering whether and to what extent the BSS owned by a transit operator can change the competition in a *competing* public transit market, in which the operators can respond to the new B&R mode by changing their pricing strategies.

Considerable research has considered strategic interactions between private operators in competition. Zubieta (1998) presented a transit system that bus operators are competed for passengers to seek profit maximization. Szeto (2007) investigated the interactions and strategies between private information service providers and private toll road operators, whose objectives are profit-driven. Wichiensin et al. (2007) considered the fare setting in a duopoly transit market as the lower-level problem and investigated how the upper-level congestion charge impacts the lower-level competition. In these studies, the concept of oligopoly competition has been applied to determine the optimal strategies for the operators, where the operators are assumed to act independently without collusion or communication with each other. With respect to the types of the decision made, this paper introduces the decisions of the bike station location for one of the transit operators, which are not included in previous studies, in addition to the fare setting.

This paper proposes an analytical model to investigate the fare setting of both operators and determine the B&R service locations in a linear corridor highway with railway parallel aside. The continuum modeling approach is more effective for making a general conclusion on policy (e.g., Liu et al. 2009; Li et al. 2012), whereas the continuum network can reveal the spatial equilibrium travel pattern with multimodal choice in a citywide basis. To discover the multimodal travel pattern under the competition of the transit market, a one-dimensional continuum model can provide a clearer picture through analytical solutions. This model carries the previous works a significant step forward by modeling an intermodal equilibrium with three modes, where (1) there are pricing competitions between modes on highway and railway (different from Li et al. 2012), and (2) interactions between all four modes are revealed. These distinguish this paper to provide some insights on the fare competition and the introduction of B&R mode towards the multimodal transport system.

This paper makes two major contributions to the related literature:

1. It demonstrates and compares the interaction between private transit operators and the modal split before and after the implementation of B&R mode.
2. A Bertrand-Nash bi-level inter-modal continuum equilibrium model for a deregulated transit market in a linear monocentric city is proposed. Despite the fare setting, the locations of the bike-sharing stations are determined by the private operator for profit maximization. In addition, a simple example shows the impact of the access time reduced by B&R mode towards the fare setting and the modal share.

2. THE MODEL

The transportation system consists of two groups of parties: transit operators and travelers. The transit operators set the fares and the travelers choose the traveling mode. The network is a linear monocentric city that has two links, one for road and one for rail, connecting from the city boundary to the city center. Bus and auto modes share the road link and experience the same congestion. It is assumed that both bus and train systems have sufficient capacity to carry any demand that may arise without changing the waiting time. The access times and the waiting times of the bus and train stations at all points along the city are fixed. For clarity, this model formulation can be considered as a bi-level problem corresponding to the two groups of parties involved in the analysis. The upper-level, representing the transport operators, aims to profit maximization by setting their own fares and service frequencies. The lower-level, representing the travelers, focuses on the congested network equilibrium mode choice. The notation used in this paper is defined as follows:

m	Set of transport modes (B = bus, T = train, R = B&R, A = auto)
v_m	Total passenger flow of transport mode m
p_m	Fare of transport mode m (parking charge for auto mode)
f_m	Frequency of transport mode m
t_m^0	Free-flow travel time of transport mode m
w_m	Waiting time of transport mode m, i.e. $w_m = 60/f_m$
a_m^0	Access time of transport mode m
τ	Value of time
μ	Marginal travel time with respect to traffic volume
k	Factor to convert bus into equivalent passenger car unit
Q	Total passenger flow in the city
D	Distance between CBD and the city boundary

2.1 Model without B&R service

2.1.1 Lower-level modal choice

For users' mode choices, it is assumed that travelers minimize their generalized travel costs while all travel information is given. The generalized cost of the car mode is composed of the travel time cost, the parking charge, access time cost, and the operating cost. For a given frequency for bus, the bus flow on the road link can be determined explicitly. The generalized cost for car mode can be defined as follows:

$$G_A(x, f_B) = \tau\left(t_A^0 + \mu Q(x)\right) + \tau a_A^0 + p_A . \tag{1}$$

Similarly, the generalized cost for bus mode can be represented as:

$$G_B(x, f_B, p_B) = \tau\left(t_B^0 + \mu Q(x)\right) + \tau\left(a_B^0 + w_B\right) + p_B . \tag{2}$$

In both equations (1) and (2), the link flow of the roadway $Q(x)$ is the sum of the auto flow $q_A(x)$ and bus flow $q_B(x)$. The travel time cost is a function of link flow and distances from CBD x. Equation (2) shows the generalized cost of bus includes travel time cost, access and waiting time costs, and the bus fare. As the waiting time is a function of service frequency, a fixed frequency implies fixed waiting time for bus and train. The generalized cost of train is therefore

$$G_T(x, f_T, p_T) = \tau\left(t_T^0 x + a_T^0 + w_T\right) + p_T . \tag{3}$$

where the travel time is directly proportional to the distance from the CBD by assuming no congestion for train. It is noted that both train and bus adopt flat fare schemes throughout the whole city.

The mode choice is calculated by assuming a deterministic user equilibrium in which no user can reduce his/her travel cost by switching mode. Let $n_j(x)$ be the proportion of residents at location x who choose mode j, $j \in \{B,T,A\}$ and let $-j$ be the modes other than j. The mode choice user equilibrium is therefore defined as:

$$n_j(x)>0 \to G_j\left(x\right) \le G_{-j}\left(x\right),\ j=B,T,A,x\in\left[0,D\right], \tag{4}$$

where the generalized travel costs of all the modes are defined in equations (1) to (3). Equation (4) shows that all travelers at location x choose the mode with minimum travel cost. This implies that the market share of any mode is proportional to the region that its generalized cost is the minimum among all modes. To show the competition among auto and the public transport modes, there are a few assumptions on the generalized costs of the transport modes.

Assumption 1 The unit free-flow travel time of bus is the highest, followed by train and car, so that

$$t_B^0 > t_T^0 > t_A^0. \tag{5}$$

Assumption 2 The generalized cost of train under the free-flow condition at city boundary is lower than the bus (i.e., $G_T\left(D\right) < G_B\left(D\right)$) and greater than auto, so that

$$\tau\left[t_B^0 D + a_B^0 + w_B\right] + p_B > \tau\left[t_T^0 D + a_T^0 + w_T\right] + p_T > \tau\left[t_A^0 D + a_A^0\right] + p_A. \tag{6}$$

Assumption 3 The fixed cost of bus is lower than the train (i.e. $G_B\left(0\right) < G_T(0)$), and the fixed cost for auto is the highest, so that

$$\tau\left[a_B^0 + w_B\right] + p_B < \tau\left[a_T^0 + w_T\right] + p_T < \tau a_A^0 + p_A \tag{7}$$

Let $\bar{x}_{m_1 m_2}$ be the intersection point of generalized cost function by mode m_1 and m_2. By these three assumptions, it can be deduced that the intersection points of the three modes follow the sequence $\bar{x}_{BT} \le \bar{x}_{AB} \le \bar{x}_{AT}$. The two endpoints $\bar{x}_{BT}$ and $\bar{x}_{AT}$ split the travelers into three groups according to their distances from the CBD: bus, train, auto (from the closest to the most distant). However, to guarantee the market share of train to be non-zero here provides another assumption.

Assumption 4 The generalized cost of train should be smaller than or equal to the generalized costs of bus and auto at the intersection point $\bar{x}_{AB}$, so that

$$G_T\left(\bar{x}_{AB}\right) \le G_B\left(\bar{x}_{AB}\right) = G_A\left(\bar{x}_{AB}\right). \tag{8}$$

Equation (8) guarantees that none of the transport modes can be neglected in the traffic corridor, and neither the bus nor the train can completely dominate the transit market. Based on these four assumptions, by equating the generalized cost functions (1)-(3), the initial market share of train, bus, and auto can be determined.

2.1.2 Upper-level fare setting

As both bus and train operators solely aim to have profit maximization and set their prices non-cooperatively, their fare settings can be described as a Nash game. The operators' strategies directly influence the travelers' mode choices and the flow patterns on transit services. These interactions between strategies lead to a competition in which all the non-cooperative operators aim to maximize their own profit. The profit raised from fare for each operator can be expressed as

$$R_i = v_i p_i, i = \{B,T\}. \tag{9}$$

Equation (9) shows that the profit is the product of fare level and the passenger flow of the line, where the passenger flow is directly proportional to the size of the dominant region. Meanwhile, we can see that bus fare p_B influences the locations of $\bar{x}_{BT}$ while train fare p_T affects locations of $\bar{x}_{AT}$ and $\bar{x}_{BT}$. This implies the fare set by each operator can influence the profits of both operators. Suppose U_i is the best response fare function for mode *i*, this best response function of an operator is defined as a way that gives the operator the best profit for any choice of fare by its competitor. According to Bertrand's concept of equilibrium, we obtain equations (10) and(11):

$$R_T\left(U\left(p_B^*\right), p_B^*\right) \geq R_T\left(p_T, p_B^*\right), \tag{10}$$

$$R_B\left(p_T^*, U\left(p_T^*\right)\right) \geq R_B\left(p_T^*, p_B\right). \tag{11}$$

Considering these equations, the optimal fares of bus p_B^* and train p_T^* are $U_B\left(p_T^*\right)$ and $U_T\left(p_B^*\right)$ respectively.

2.2 Model with B&R service

In this transport system, the train operator is allowed to operate the B&R mode *for free* to reduce travelers' access time to the train station. The generalized travel cost of B&R mode is similar to the train mode:

$$G_R(x, f_T, p_T) = \tau\left(t_T^0 x + a_R^0 + w_T\right) + p_T \tag{12}$$

Providing that the generalized travel cost of B&R mode must be lower than the train mode, here introduces new assumptions for this model.

Assumption 5 The fixed cost of B&R mode is lower than train mode but higher than bus mode, so

$$G_B\left(0\right) < G_R\left(0\right) < G_T\left(0\right) \tag{13}$$

Assumption 6 The generalized cost of B&R mode under the free-flow condition at city boundary is lower than the bus (i.e. $G_R\left(D\right) < G_B\left(D\right)$) and greater than auto, so that

$$G_A\left(D\right) < G_R\left(D\right) < G_T\left(D\right) < G_B\left(D\right) \tag{14}$$

Figure 1 illustrates the interactions between the new mode with auto, bus, and train. The shaded region indicates the introduction of B&R expands the modal share of train and shrinks the modal shares for auto and bus. To maximize the profit, the train operator only needs to set up bike stations in regions that his generalized cost is not the lowest among the three modes, but the generalized cost of B&R is minimum. However, as a non-cooperative game, the bus operator can maximize its profit by reducing its fare to increase patronage. It is clear that the introduction of B&R becomes a significant determinant that impacts the competition between two transit operators.

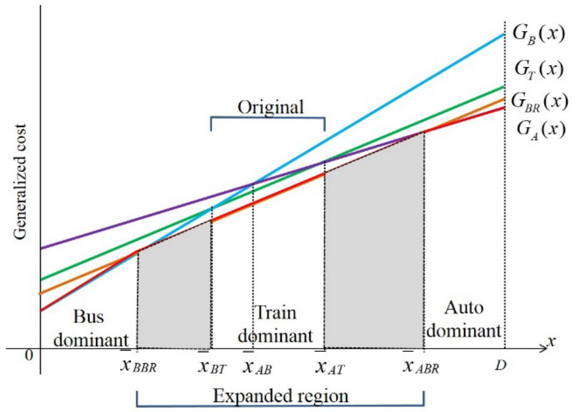


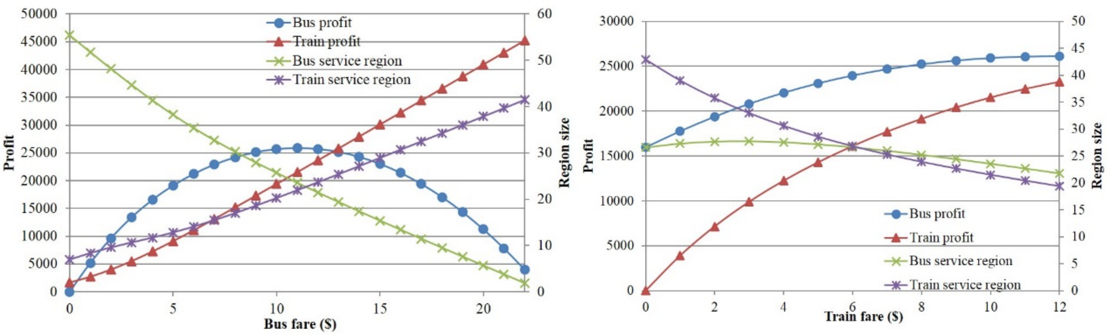
Figure 1 Interactions between the four modes in the transport system

3. NUMERICAL STUDIES

The numerical study is carried out to illustrate the effect of B&R on the fare levels, service regions, and profits of both transit operators. It compares the solutions under different reduced access times to show how the effectiveness of the B&R mode impacts the profit of the train operator. The parameters in this numerical example are stated: D = 100 km; Q = 10,000 veh/hr; $t_A^0 = 0.02; t_B^0 = 1/60; t_A^0 = 0.01;\ w_B = 10; w_T = 15;$ $a_A = 3$; $a_B = 8$; $a_T = 15$; $k = 4$; τ = \$1/min. As the service frequencies of bus and train are constant, the operating costs are constant and thus excluded. It is noted that the demand is distributed evenly along with the city.

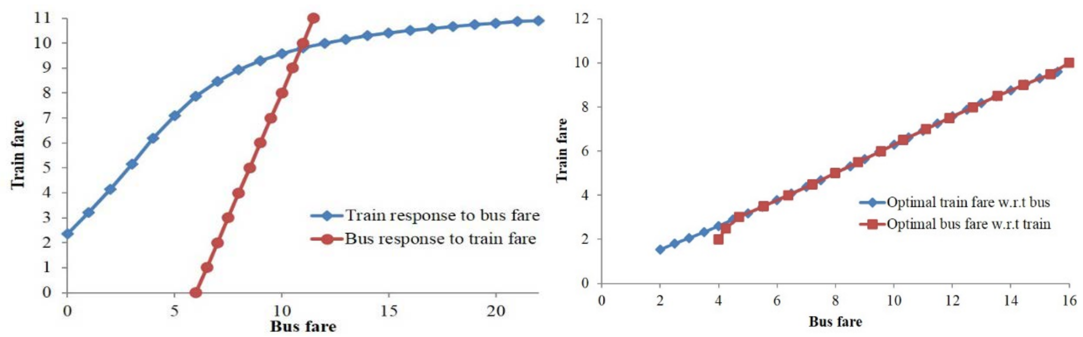
Figure 2a shows that with increasing bus fare, the profit and service region of train increase while the bus service region decreases. The bus profit shows a parabolic trend with the maximum at \$11, showing that a further increase in bus fare cannot compensate for the reduction of patronage due to the expansion of the service region by train. Furthermore, Figure 3a shows that the train has slight fare increases when the bus fare increases per dollar, whereas this slight increase is enough to shift demand from bus to train and maximizes train profit. When the total service regions of bus and train reduce with the increased fare, the auto flow increases and consequently the travel time on the roadway and the generalized travel cost of bus increase.

Figure 2b shows the pricing strategies of bus operator under increasing train fare. Though the bus profit increases and the service region of train reduces, the bus service region expands slightly when train fare is below \$3 and shrinks afterward. This is similar to Figure 2a that the bus company does not require a large service region to maximize its profit. The train operator can be benefited when the bus operator maximizes his profit despite the reduction of his service region. For the fare determination, Figure 3a shows the optimal fare in this duopoly market is when bus fare is \$11 (with a profit of \$25,890) and train fare is \$10 (with a profit of \$21,510).



(a) Different bus fare levels (b) Different train fare levels

Figure 2 Changes in profits and service regions with fare levels without B&R

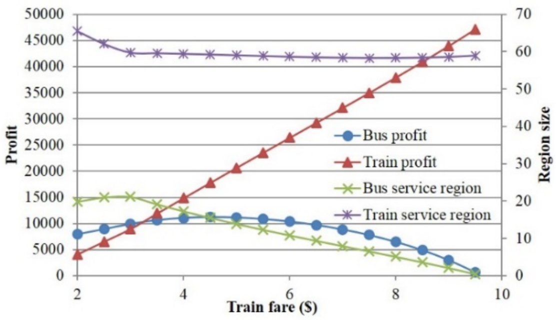


(a) Without B&R (b) With B&R

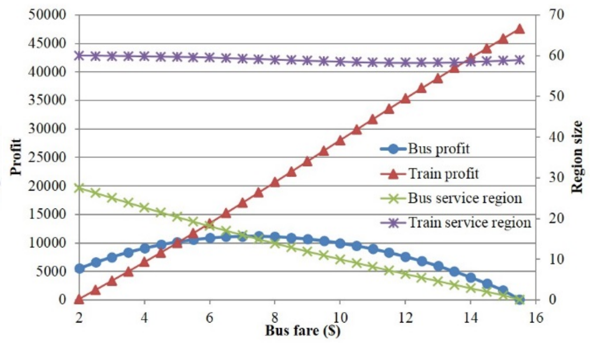
Figure 3 Optimal fares for bus and train corresponding to opponent's fare level

Figures 4a and b show the profits and service regions for bus and train when B&R mode is available. It is assumed that the B&R mode can reduce 6-minute access time and the setup cost of the bike station is $150/(km-hr). The profits of the train in both figures are increasing with the fare level, and the train service region remains constant with the increasing fare because B&R reduces the generalized travel cost of the train. Comparing figures 2 and 4, for the train operator, providing B&R service can improve his profit and expand his service region with the increased fare. The profit curve of the bus operator has a similar shape, bus the profit has reduced from $25,000 to $12,000. Figure 3b shows that there are multiple combinations of optimal fares in this duopoly market as the best response fare functions U_T and U_B overlaps when bus fare is greater than $5 and train fare is greater than $3. This overlap implies that the optimal fare of both operators can be determined when the fare level of one of the operators is known. For instance, when the train operator sets a higher fare, the bus operator needs to follow to maximize his profit.

The effect of reduced access time by the B&R mode is shown in Table 1 and Figure 5. The bus fare is set to be $11 and uses the optimal solution for the duopoly market without B&R for a simpler comparison. The profit and the service region of the train increase with the reduced access time because the travel time cost of train is reduced. Interestingly, the optimal train fare is reduced with increasing reduced access time. When the fare is reduced, the generalized cost for train can be further reduced and the service region can be maximized. The service region and profit of bus decrease with the increased reduced access time since train becomes more competitive. Figure 5 demonstrates that the market share of the sum of train and B&R mode increases when B&R mode can reduce more access time whereas train mode vanishes with high reduced access time.



(a) Different train fare levels



(b) Different bus fare levels

Figure 4 Changes of profits and service regions with different fare levels with B&R mode

Table 1 Changes of profit and service region of train and bus with reduced access time

Reduced time (min)	0	2	4	6
Train fare ($)	9.8	8.6	8.2	6.9
Train profit ($)	21509	23092	26847	31716
Bus profit ($)	25870	23616	19506	8966
Train service region (km)	21.9	30.6	39.8	58.4
Bus service region (km)	23.5	21.5	17.7	8.2

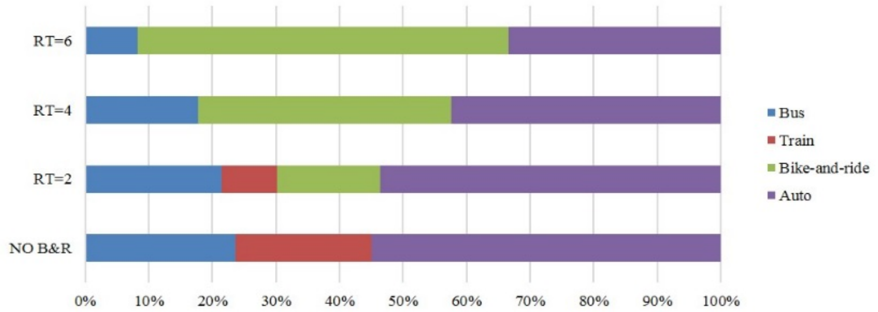


Figure 5 Market shares of three modes with different access time reductions by B&R mode

4. CONCLUSIONS

This paper proposes a bi-level intermodal equilibrium model for a deregulated transit market. In the model, the optimal fares for both transit operators and the regions for B&R mode are determined. The results show that B&R mode triggers the overlapping of the best response fare functions for train and bus. It is effective in dragging travelers from the highway to train, and its impact increases with the saved access time. It can be concluded that B&R mode is beneficial to the transit operator that manages the BSS as it can always improve the profit and market share. For the policy implication, the above analysis shows that B&R mode is beneficial to the private transit operator in the long run and able to shift auto travelers to use public transport. Nevertheless, the bike-train integration can be a fail attempt if the rail operator becomes the monopoly transit operator and sets a high fare. To avoid this, the local authority can consider fare capping or profit margin for the transit operators.

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