Demand-driven optimization method for shared mobility services

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ABSTRACT

Shared mobility services are announced as a game-changer in transportation and a promising solution to reduce congestion and improve the performance of urban mobility. They could prefigure the arrival of autonomous vehicles. Modeling of these new services is a real challenge, especially because existing approaches are mainly an adaptation of methods devoted to classic transportation services. Consequently, this paper introduces a new data-driven optimization method fully devoted to shared mobility service. First, the proposed approach decomposes the recurrent demand based on its spatio-temporal features to overcome the drawbacks of the existing methods. Notably, it makes it possible to consider larger instances and to build robust solutions. Thus, recurrent demand patterns are identified to capture the potential demand of shared mobility services using a tailored clustering process. Second, a variant of Dial-a-Ride Problem is implemented to design robust lines to serve this demand. Such a hybrid method makes it possible to define relatively massive transport lines while maintaining spatial and temporal proximity to users real demand. The method is then tested with an open-source dataset released by the New York City Taxi and Limousine Commission.

Keywords: Clustering, Shared mobility, Dial-a-Ride, Similarity, Ride-sharing.
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1. (a), (b), (c), (d) clusters with different characteristics, the pick-up are depicted in green and drop off in red. \( n_k \) denotes the number of trips in the cluster \( k \), \( \bar{L}_k \) denotes the average length of trips in \( k \) and \( \bar{\tau}_k \) denotes the average duration of trips in \( k \). (e) Ratio of clustered trips per day in Midtown and Upper East Side from 8h to 11h. (f) Example of demand graph for a randomly selected meta-cluster.

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INTRODUCTION

Shared mobility services are announced as a game-changer in transportation and a promising solution to reduce congestion and improve the performance of urban mobility. Moreover, it appears that recent studies on shared mobility, particularly regarding the real-time satisfaction of the demand, could prefigure the arrival of autonomous vehicles. Shared mobility consists in the shared use of a vehicle (car, motorcycle, scooter, bicycle, or other travel modes). Modeling of these new services is a real challenge, especially because existing approaches are mainly adaptation of methods devoted to classic transportation services. These methods can be classified into two main categories: the conventional methods and the dynamic methods.

The conventional methods are used to design rapid transportation lines such as subway, streetcar, or bus lines (1, 2, 3). These approaches can be qualified as long term methods because the transportation supply is defined according to both urban planning purpose and transportation demand. Consequently, the goal is motivated by serving an existing demand but also to modify the travel behavior at a long time scale. These approaches are well known in the literature for many years. However such methods involve long term demand changes. Indeed, the deployment of such lines affects the socio-economic development around the lines stops, and so affects the demand of mobility. Thus, unlike dynamic methods, conventional methods are not adapted to an instantaneous mobility demand. These methods aim to respond to a demand for global mobility; the lines designed follow mobility corridors with a high concentration of departure and arrival points. Generally, the demand is estimated using online questionnaires, surveys, or historical moving data. However, these methods have some limits; the calculated flows do not take into account each trip’s specificities but only a rough estimation of the movements of a large number of users. This spatial and temporal aggregation has led to the design of significant lines for which the stops are located relatively far from the real desired departure and arrival points for users. It has been shown in numerous studies that this problem of the last mile is one of the major brakes which prevents users of personal vehicles from deporting to shared modes of transport. It is to overcome this problem of the last mile that for the past ten years, taking advantage of the emergence of smartphones and underlying geolocation technologies, research has turned towards a new approach so-called dynamic.

Contrary to conventional approaches, dynamic methods aim at adapting the service supply to the real-time demand characteristics. One of the main benefits is that such approaches provide users with short-term access to a travel mode on an as-needed basis. These transport services may take different forms: station-based roundtrip services, station-based one-way services, free-floating services, etc. Similarly, many economic models exist to meet diverse user needs: public or private, membership-based, peer-to-peer (P2P), for-hire, or public transit system. Moreover, sharing can include either sequential sharing (i.e., different users sharing the same vehicle one after the other), or simultaneous sharing (i.e., sharing the same vehicle with multiple users for the same trip). Simultaneous sharing is a particular challenge that many services try to tackle: transportation network companies (TNCs) now offer ride-sourcing services (including shared taxi, shuttle, etc.); ridesharing (including carpooling, vanpooling, etc.) is becoming more and more popular. The main objective of real-time methods (or highly dynamic) is to match a maximum number of requests while minimizing objective functions, such as the users’ waiting time or the total travel time. (4, 5) provide a list of objective functions and matching policies well known in the literature. The dynamic method is well adapted to large fleets of vehicles. The dynamic matching between users and vehicles is efficient when the number of vehicles is significant. It allows reducing the
waiting time and detours to pick up or drop off a user. The dynamic approach is considered as a bridge technology that will be replaced by autonomous vehicles when the technology will be mature \((6, 7, 8)\). Despite their many advantages, the dynamic methods also have certain limitations. The real-time matching between travelers and vehicles is complex, and it can not be performed on large instances. The number of passengers served by a vehicle is often low. That is why the dynamic approach is not adapted to design massive customized lines of transport.

This paper aims to propose a new hybrid approach that allows the design of massive and robust lines of transport adapted to the daily demand. The main interest that is driven by the method, is to detect a large number of similar and recurrent trips to estimate the potential demand of shared mobility, then to design transport lines allowing pickups and deposits as close as possible to the real demand of the users. The approach is defined as hybrid because it takes into account the regularity of trips over time (as in conventional methods); however, the distances to be covered and the delays observed for users remain relatively short. For this, groups of similar trips are firstly searched using a function to assess the similarity between 2 trips, and a clustering method to build groups of similar trips. Secondly, we analyze whether these clusters are episodic or regular. The method has been developed for detecting clusters of similar and regular trips in time (meta-clusters). Finally, an optimization method (DARP) is applied to design transport lines adapted to this demand for mobility.

The rest of the paper is organized as follows. Section 2 presents the data set and introduces the methodology used to estimate demand patterns. Then, Section 3 is dedicated to the design of customized transport lines to satisfy the demand. Section 4 focuses on the analysis of the results of the demand estimation and the planning of new lines. Finally, Section 5 is devoted to a final discussion.

**ESTIMATION OF THE DEMAND**

This section’s main objective is to show how the demand can be decomposed into spatio-temporal areas containing a significant number of similar trips. The study focus on the demand of shared mobility in Midtown and Upper East Side. The objective is to present the method used to obtain clusters of similar and recurrent trips over time (meta-clusters). These clusters will be used to define the instances of the optimization problem presented in Section 3. The methodology is based on three steps: (i) definition of a similarity function to estimate the likeness between two trips; (ii) implementation of a clustering method to create clusters of similar trips; (iii) development of a method to detect recurrent clusters over time. However, it is essential to underline the fact that the demand is analyzed from a transportation point of view even if many other aspects could be taken into account: economic, social or behavioral. Our method determines an upper bound of the potential of shared mobility.

We use an open-source dataset released by the New York City Taxi and Limousine Commission\(^1\). Although these data are not fully representative of human mobility since they only correspond to taxi trips, such a dataset provides an attractive proxy for studying the individuals’ routes within a city. The study focuses on morning peak hours from 8h to 11h of June 2011. The area studied is a well known high-density area in terms of mobility in New-York City: Midtown and Upper East Side \((9, 10)\). For each trip \(i\), the dataset gives access to the following information: departure time \(t_i^{PU}\) and location \(p_i^{PU} = (x_i^{PU}, y_i^{PU})\) of the pick-up of the passenger(s); arrival time \(t_i^{DO}\) and location...\(^1\)data source: https://www1.nyc.gov/site/tlc/index.page.
Modeling similarity between individual trips

Firstly it is essential to define a similarity function to estimate the likeness between trips. The goal of such a function is to quantify the similarity between two trips. We use the similarity function presented in (11) because it was shown that it provides excellent results to estimate the similarity for the trips defined by a pair origin-destination. The similarity is calculated according to the spatio-temporal commonalities between the trips. Let \( S(i, j) \) the similarity function between trips \( i \) and \( j \). From a physical point of view, the intuition is that two (or more) travelers may have an interest to share their trip if they start in the same neighborhood and at the same moment, and want to go to the same destination. The function \( S \) must encompass these different spatio-temporal attributes of the trips. We proposed the following function:

\[
S(i, j) = \sum_{l \in [PU, DO]} \alpha_l e^{f_l(i,j)}
\]

(1)

where \( f_l(i,j) \) is the feasibility function and \( \alpha_l \) is a coefficient. Function \( f \) describes the service’s potential to operate the shared trips, i.e. the ability to pick up (or drop off) the two travelers before both of their desired departure times:

\[
f_l(i,j) = |t_l^i - t_l^j| - \gamma d(p_l^i, p_l^j)
\]

(2)

where \( \gamma \) is the average pace to connect travelers that want to share a trip. This parameter is a general and synthetic formula to describe the operation of the service and the way in which this service gathers two demand requests into the same vehicle: defining a meeting point, successive pick-ups, etc. For example, if the first traveler must walk to the second traveler’s pick-up point, then \( \gamma \) is equal to the inverse of the walking speed. If this distance is traveled by car, meaning that the service offers door-to-door service, then \( \gamma \) is equal to the inverse of the vehicle speed. Consequently, \( f \) is positive if the match can be realized before the two desired departure times \( t_l^i \) and \( t_l^j \), whereas \( f \) is negative if travelers have to experience delay to make the match possible. Moreover, \( \alpha_l \) is equal to \( \frac{1}{2} \) if \( f_l(i,j) > 0 \) and to \( \frac{3}{2} \) otherwise because it is more disadvantageous to be delayed. In addition to this first index of similarity \( S(i,j) \), excessive distances/durations for rendezvous are penalized. Thus, penalties \( \theta_x \) and \( \theta_t \) are added when, respectively, the distances between origin (or destination) locations and departure (or arrival) times of trips \( i \) and \( j \) exceed, respectively, specific thresholds, \( \delta_x^l \) and \( \delta_t^l \):

\[
\theta_x^l = e^{d(p_l^i, p_l^j) - \delta_x^l} \quad \forall t / d(p_l^i, p_l^j) > \delta_x^l
\]

(3)

\[
\theta_t^l = e^{|t_l^i - t_l^j|/\delta_t^l - \delta_t^l} \quad \forall t / |t_l^i - t_l^j| > \delta_t^l
\]

(4)

Otherwise, these penalties are null. In this manner, \( S(i,j) = S(i,j) + \theta_x^l + \theta_t^l \) defines a sharp function that enhances the differences between trips and facilitates identification of similar travelers in the dataset. Notice that \( S \) is minimal (and equal to 1) when the two trips are exactly identical.
Detection of similar trips for individual days

The method presented here allows us to detect spatio-temporal areas where there are a significant number of similar trips. The function of similarity exposed above only estimates the likeness between two trips but does not detect clusters of similar trips. In order to detect such groups, a clustering algorithm is used. The function of similarity allows us to compute the similarity matrix requested by a clustering algorithm.

A variant of a well-known clustering density-based method (12) is applied for each day to detect groups of similar trips. This method only requires two parameters: a threshold $\varepsilon$ and a minimum number of points $MinPts$, which have to be in a radius $\varepsilon$ so that the studied point is considered as an element of the cluster, see (13) for more details. The parameter $\varepsilon$ is the maximal distance between trips, i.e., the maximal value of $S$, allowed to consider them as similar and group them into the same cluster. However, this method must be slightly adapted to detect groups of different density. Thus, a successive DB-SCAN clustering is performed, i.e. itdbscan ((14) for more information), using the similarity function $S$ as the distance, while updating iteratively the values of the parameters.

Starting with a large value of $MinPts = M$ and a drastic $\varepsilon$, it makes it possible to identify large groups of travelers in the initial data set of trip $T$. In other terms, we first detect large and high-density clusters. Then, the DB-SCAN method is applied on the remaining non-clustered trips to detect groups of size $M - 1$. This process is repeated until $MinPts = 2$.

Clusters detected have different sizes, from 2 trips to 74 trips gathered into the same group. It brings to light that the shared mobility demand may take many aspects requiring different forms of transportation services to be optimally satisfied. Figure 1 depicts four clusters with different sizes and characteristics. The average travel length $l_k$ is directly the arithmetic average of the length of $n_k$ trips within the cluster $k$, whereas the average travel time $t_k$ is the arithmetic average of the duration of the $n_k$ trips. Figure 1.e shows the number of clustered trips and the total number of trips per day. The developed method detects almost 85% of similar trips per day on average in the studied zone.
(a) $n_k = 4$  $\bar{l}_k = 0.93\text{km}$  $\bar{\tau}_k = 6.5\text{min}$

(b) $n_k = 19$  $\bar{l}_k = 2.58\text{km}$  $\bar{\tau}_k = 13.3\text{min}$

(c) $n_k = 30$  $\bar{l}_k = 1.95\text{km}$  $\bar{\tau}_k = 9.2\text{min}$

(d) $n_k = 74$  $\bar{l}_k = 1.56\text{km}$  $\bar{\tau}_k = 9.8\text{min}$

(e) (f) FIGURE 1 (a),(b),(c),(d) clusters with different characteristics, the pick-up are depicted in green and drop off in red. $n_k$ denotes the number of trips in the cluster $k$, $\bar{l}_k$ denotes the average length of trips in $k$ and $\bar{\tau}_k$ denotes the average duration of trips in $k$. (e) Ratio of clustered trips per day in Midtown and Upper East Side from 8h to 11h. (f) Example of demand graph for a randomly selected meta-cluster.
Identification of regular demand pattern for multiple days

Once this daily analysis is done, we investigate if commonalities in the clusters can be identified. Many approaches exist to derive the most representative partition from a group of partitions, such as meta-clustering or consensus learning (15). Here, we use the same clustering method to maintain consistency when scaling-up. In the following, to reduce the computational time, we focus the study on the 14 days of the dataset for which the ratio of clustered trips is the highest: June 6 to 19, 2011. The objective is now to find out if there are similar trips (relatively close departure and arrival locations and times) made several times during the studied period. These recurrent spatio-temporal areas are called meta-clusters. For that purpose, each cluster previously found is considered as a new trip, formed by the centroid of its pick-up and the centroid of its drop off. Centroïds correspond to the mean origin/destination locations and mean departure/arrival times of the clustered trips. This information can be useful to design the transportation supply because centroïds can be the locations of common meeting points of the standby areas of shared vehicles. A second clustering is then performed, it returns clusters with similar characteristics (without taking into account the initial day when the trips were made). In other words, two clusters are in the same meta-cluster, if their centroïds have close departure and arrival locations and times. Interestingly, we observe that more than 94% of the daily clusters are recurrent from one day to another.

The representation of a meta-cluster on a 2D map is difficult to analyze because the time dimension is not accounted for. Consequently, we prefer to focus on the evolution of the daily clusters’ size and the localization of the related origin/destination whereabouts. Each meta-cluster can be depicted as a graph of the demand. Figure 1.f shows the graph of the demand for a randomly selected meta-cluster. This figure shows that in the same spatio-temporal area, similar trips can be seen every day, except on weekends. Each meta-cluster provides precise information about its location, its estimated departure and arrival times and the total number of trips performed per day. It is important to note that different individuals perform these trips from one day to another. However, global human mobility is remarkably regular; this is a valuable insight to tune transportation services and favor shared mobility efficiently.

CUSTOMIZED SUPPLY DESIGN

As mentioned in the previous section, the spatio-temporal areas containing similar and regular trips (meta-clusters) are detected. The study’s next objective is to find a way to serve the pick-up and drop off points in each of these meta-cluster while respecting a set of constraints: time windows, vehicle capacity, size of the fleet, etc. A minimalist example of the developed method is depicted in Figure 2. Figure 2.a shows a set of 3 meta-clusters; each of them contains several clusters of similar trips. A green marker and a red marker linked by a blue line depict a cluster containing several similar trips. The green and red markers designate respectively the points of pick-up and drop off of a cluster. A meta-cluster is depicted by an aggregation of clusters in the same spatial area. On average, each cluster contains 6.22 similar trips. Moreover, a meta-cluster contains, on average 8.64 clusters. In other words, each meta-clusters contains on average 53 trips with very similar characteristics (see Section 4). The method consists to design a line of transport serving a set of meta-clusters with characteristics compatible (size, time-windows, etc.). Depending on the meta-clusters chosen, the number and size of vehicles required will be different. To quantify the potential demand of a tour, we plot the total number of trips per day served by a tour going through these meta-clusters. Figure 2.c depicts the total number of trips served per day for the set
of meta-clusters depicted in Figure 2.a

(a) Green markers depict the pick-up and red markers the drop off (a) 3 meta-clusters randomly chosen between 08h15 and 08h35. (b) example of line designed, serving the centroids of pick-up and drop off of each meta-cluster. (c) Total number of trips per day served on the set of 3 meta-clusters. (d) Shows the number of meta-clusters for each time slot in function of the minimal median value of trips per day required

FIGURE 2  Green markers depict the pick-up and red markers the drop off (a) 3 meta-clusters randomly chosen between 08h15 and 08h35. (b) example of line designed, serving the centroids of pick-up and drop off of each meta-cluster. (c) Total number of trips per day served on the set of 3 meta-clusters. (d) Shows the number of meta-clusters for each time slot in function of the minimal median value of trips per day required

Then, the selection of a set of meta-clusters to find a potential tour of vehicles is performed. We use the median number of trips per day in a meta-cluster as an indicator of its size. For each studied period, a minimal median value required is defined; we filter the meta-clusters with a
median inferior to this value. This method allows us to obtain a reasonable number of meta-clusters to solve a relatively small instance of the optimization problem. However, it should be noted that the solution is optimal for each period on which we solve the problem, but an optimal result is not guaranteed for a set of periods. The meta clusters found are used to define a Dial-a-Ride Problem (DARP) instance. Given the large number of meta-clusters, it is impossible to directly model vehicle tours from the whole set of meta-clusters. That is why we focus this study on high capacity vehicle tours. This method allows us to serve a large number of passengers by solving smaller problem instances. The main advantage of this method is that the calculation time depends on the number of meta-clusters served and not directly of the number of passengers. That is why it is crucial to find the largest possible meta-clusters. If we solve the problem for a set of large meta-clusters, the number of passengers effectively served will be significantly higher.

There are many variants for the DARP problem, \((16, 17)\) give a list of them based on different objective functions. In our case, we use a variant presented in \((18)\). This model is depicted below Eq.5 to Eq.17. The model is based on a three index formulation.

Let \(G = (V, A)\) a directed graph. The set of vertices \(V\) is partitioned as follow: the first and the last element are two copies of the depot, elements from index 1 to \(n\) are pick-up and elements from index \(n + 1\) to 2\(n\) are drop off. \(P\) denotes the set of pick-up and \(D\) the set of drop off. A request is a couple \((i, n + i)\), where \(i \in P\) and \(n + i \in D\). The load of each vertex is defined as \(q_i\), with \(q_0 = q_{2n+1} = 0\), \(q_i \geq 0\) for \(i \in \{1, \ldots, n\}\) and \(q_i = -q_{i-n}\) for \(i \in \{n+1, \ldots, 2n\}\). A service duration \(d_i \geq 0\) with \(d_0 = d_{2n+1} = 0\). \(K\) denotes the set of vehicles. The capacity of a vehicle \(k \in K\) is \(Q_k\), and \(T_k\) denotes the maximal duration of a route for a vehicle \(k\). The arc set is defined as: \(A = \{(i, j) \mid i = 0, j \in P \text{ or } i, j \in P \cup D, i \neq j \text{ and } i \neq n + j, \text{ or } i \in D, j = 2n + 1\}\) the cost of traversing an arc \((i, j)\) with a vehicle \(k\) is \(c_{ij}^k\), and the travel time between two nodes \(i\) and \(j\) is \(t_{ij}\). \(L\) denotes the maximal ride time and the time window of a vertex \(i\) is \([e_i, l_i]\). \(x_{ij}^k\) is a binary variable equal to 1 if and only if \((i, j)\) is traversed by a vehicle \(k \in K\). Let \(u_i^k\) the time at which a vehicle \(k\) starts servicing a vertex \(i\) the load of vehicle \(k\) leaving vertex \(i\), and \(t_i^k\) the ride time of user \(i\).

\[
\text{(DARP)}
\]

\[
\text{Minimize} \sum_k \sum_i \sum_j c_{ij}^k x_{ij}^k \tag{5}
\]

subject to

\[
\sum_{k \in K} \sum_{j \in V} x_{ij}^k = 1 \quad (i \in P), \tag{6}
\]

\[
\sum_{i \in V} x_{0i}^k = \sum_{i \in V} x_{i,2n+1}^k = 1 \quad (k \in K), \tag{7}
\]

\[
\sum_{j \in V} x_{ij}^k - \sum_{j \in V} x_{n+i,j}^k = 0 \quad (i \in P, k \in K), \tag{8}
\]

\[
\sum_{j \in V} x_{ji}^k - \sum_{j \in V} x_{ij}^k = 0 \quad (i \in P \cup D, k \in K), \tag{9}
\]

\[
u_i^k \geq (u_i^k + d_i + t_{ij}) x_{ij}^k \quad (i, j \in V, k \in K), \tag{10}
\]

\[
w_j^k \geq (w_j^k + q_j) x_{ij}^k \quad (i, j \in V, k \in K), \tag{11}
\]
\[
\begin{align*}
  r^k_i &\geq u^k_{n+i} - (u^k_i + d_i) & (i \in P, k \in K), \\
  u^k_{2n+1} - u^k_0 &\leq T_k & (k \in K), \\
  e_i &\leq u^k_i \leq l_i & (i \in V, k \in K), \\
  r^k_i &\leq r^k_i & (i \in P, k \in K), \\
  \max (0, q_i) &\leq w^k_i \leq \max (Q_k, Q_k + q_i) & (i \in V, k \in K), \\
  x^k_{ij} & = 0 \text{ or } 1 & (i, j \in V, k \in K),
\end{align*}
\]

This model presents several interesting aspects: multiple vehicles, time-windows for Pick-up, or Drop Off. The main objective of this method is minimizing the total route length. However, several other constraints can be added, such as vehicle capacity, maximum route duration, or maximum ride time for users. Nevertheless, it is essential to note that the meta-clusters previously found are independent of the method chosen to serve them and vice versa. Indeed depending on the objective searched, an approach may be more interesting that another. For example, it could be interesting to use a method to minimize the total route length for a Transportation Network Company. From a user point of view, it could be more interesting to use a technique allowing to reduce the waiting time to be served. Several methods aim to satisfy an objective function depicted as a combination of constraints such as transportation time, ride time, excess of maximum ride time, waiting time, time windows violations, etc. (19). A comparison with these sophisticated methods will be studied in a future study.

**RESULTS**

This Section is devoted to the results of the proposed method for the case of NYC. First, the meta-clusters are presented and analyzed. Secondly, based on this demand decomposition, the optimization method is tested and evaluated.

**Selection of the spatio-temporal areas**

First of all, it is interesting to analyze the characteristics of the meta-clusters found. As mentioned in Section Method - Estimation of the demand, in the studied area between 08h00 and 11h00, almost 85% of trips can be considered similar. Moreover, more than 94% of trips are recurrent, i.e., these trips can be observed almost every day. 2136 spatio-temporal areas are detected as zones where there is a recurrent potential demand of shared mobility. On average, each meta-cluster contains 53 trips. Once again, it is important to notice that different users surely perform these trips. In the following, it is considered that the users’ meeting point is defined as the centroids of pick-up (respectively drop off) of a meta cluster. Thereby, it is interesting to know the spatial and temporal difference between the centroids and the points of pick up and drop off. Table 4.1 shows that the spatial distances are close to 200m. The average temporal shifts are nearly 6 minutes.

---

1. \[i^k_i \geq u^k_{n+i} - (u^k_i + d_i) \quad (i \in P, k \in K),\]
2. \[u^k_{2n+1} - u^k_0 \leq T_k \quad (k \in K),\]
3. \[e_i \leq u^k_i \leq l_i \quad (i \in V, k \in K),\]
4. \[r^k_i \leq r^k_i \quad (i \in P, k \in K),\]
5. \[\max (0, q_i) \leq w^k_i \leq \max (Q_k, Q_k + q_i) \quad (i \in V, k \in K),\]
6. \[x^k_{ij} = 0 \text{ or } 1 \quad (i, j \in V, k \in K),\]
which is entirely acceptable. It shows that the meta-clusters found are relatively close to the initial
clusters estimated from the real rides of users. The average travel distance and time in the meta-
clusters are respectively 1.71 km and 11.1 min. Although these data are not fully representative of
human mobility since they only correspond to taxi trips, such a dataset provides an attractive proxy
for studying the individuals’ routes within a city.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average spatial distance Pick-up / centroid of Pick-up</td>
<td>0.21 km</td>
</tr>
<tr>
<td>Average spatial distance Drop off / centroid of Drop off</td>
<td>0.21 km</td>
</tr>
<tr>
<td>Average shift between departure times / centroid of departure times</td>
<td>6.16 min</td>
</tr>
<tr>
<td>Average shift between arrival times / centroid of arrival times</td>
<td>6.27 min</td>
</tr>
</tbody>
</table>

**TABLE 1** Average spatial and temporal distances between pick-up, drop off and the centroïds of the meta-clusters.

In a first time, it is necessary to select a reasonable number of meta-clusters to solve a
relatively low instance of the optimization problem. As said in Section Customized Supply Design,
the median number of trips per day in a meta-cluster is used as an indicator of its size. Figure 2.d
shows us for each one hour period the number of meta-clusters depending on the chosen minimal
median value. In other words, a meta-clusters is counted if and only if its median value of trips
per day is greater or equal to the chosen value. It is important to note that the number of points
effectively treated in the DARP will be for each period $2 \times \text{number of clusters}$, because a vehicle
serve a pick-up and a drop-off for each meta-cluster. According to Figure 2.d, the minimal median
value 24 has been selected to find potential routes with a large number of users with very short
execution times.

**Demand-driven route optimization**

According to the results showed in Section - Results - Selection of the spatio-temporal areas, three
periods of one hour for which the meta-clusters contain at least a median of 24 trips per day are
selected. Figure 3.a shows the potential number of trips per day that can be served on the period
08h00-11h00. This result illustrates one of the method’s interests: the 18 meta-clusters selected
represent actually 614 trips per day on average. The median number of trips served per day for this
set of meta-clusters is 756. Also, we note that the demand is extremely regular every day of the
week (except weekends) for the two weeks of analysis. In the case of an effective implementation
of optimized lines, it would be interesting for the service to be operated from Monday to Friday.
The parameters used for DARP are adjusted for each period of one hour. Each centroïd of pick-up
and drop-off of the meta-clusters are inserted in the sets $P$ and $D$. The number of vehicles $V$ for
each period is depicted in Table 2. For each vehicle, its capacity $Q_k = 80$, which corresponds to the
average capacity of a bus. For each arc $(i, j)$, the cost $c_{ij}$ is defined as the spatial distance between
$i$ and $j$. For each node $i$, we set the service duration $d_i = 2$ minutes. The load of each pick-up $q_i$
is defined as the number of users to serve. The load of each drop off is defined as $-q_i$. A time
window of 20 minutes is defined to serve the different points. This value is not representative of the
real difference between the desired service times and the effective times of service, but it provides
an upper and lower limit that should not be exceeded. If this value is not enough, the constraint
will often be violated then; no solution will be found. The travel time between two nodes $i$ and $j$
is estimated according to the results presented is (10). We set the average speed for a vehicle to 9.65 km/h.

Figure 3.a, b and c show for each period the a map of the designed routes. Each color designates a specific transport line. Table 2 indicates the result of the DARP. For the three periods, routes allowing to serve all the selected meta-clusters in less than 9 seconds are found. This result proves that the method is a good way to design lines serving many users (more than 600 trips per day on average). Besides, for each period, we calculate the average delays and time advances for each point served by the optimized tour. This value is estimated by the difference between the wished times of departure and arrival (given by the centroids of the meta-clusters) and the hour of service given by the solving of DARP. These values show that the developed method relatively little impact on demand. Indeed, on average, the delay is 12 minutes and the advance is 10.6 minutes, which is acceptable since the number of users served is high.

To the best of our knowledge, there are no classical optimization methods to find round serving such a quantity of similar and recurrent trips in such a tight timeframe. The theoretical studies on DARP (17) show that the exact method used in this paper can solve instance until 36 points. It would not be possible to solve instances with so many passengers without using an aggregation method in meta-clusters. The existing dynamic methods such as (20, 21, 22) obtain trips delay between 2 and 6 minutes; however, these services work with large fleets of vehicles with limited capacities (between 2 and 10). Moreover, these methods work only on networks with a limited number of nodes.

<table>
<thead>
<tr>
<th></th>
<th>Period 1</th>
<th>Period 2</th>
<th>Period 3</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>08h00 - 09h00</td>
<td>09h00 - 10h00</td>
<td>10h00 - 11h00</td>
<td>08h00 - 11h00</td>
</tr>
<tr>
<td>Number of meta-clusters served</td>
<td>8</td>
<td>6</td>
<td>4</td>
<td>18</td>
</tr>
<tr>
<td>Number of vehicles required</td>
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<td>2</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>Average delay (min)</td>
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<td>11</td>
<td>9</td>
<td>12</td>
</tr>
<tr>
<td>Average advance (min)</td>
<td>9</td>
<td>12</td>
<td>11</td>
<td>10.6</td>
</tr>
<tr>
<td>Total travel distance</td>
<td>36.15 km</td>
<td>19.2 km</td>
<td>12.43 km</td>
<td>67.78 km</td>
</tr>
<tr>
<td>Computation Time</td>
<td>6.85 s</td>
<td>1.37 s</td>
<td>0.05 s</td>
<td>8.27 s</td>
</tr>
</tbody>
</table>

**TABLE 2 Result of the search of rounds for the 3 time periods from 08h00 to 11h00**
FIGURE 3  (a) depicts the total number of users effectively served in the set of meta-clusters selected in Section - Selection of the spatio-temporal areas. (b),(c),(d) shows for each period presented in Table 2, the customized lines found.
CONCLUSION

This article presents an optimization method based on a decomposition of the demand and a resolution of DARP on a reduced instance. This data-driven method allows in a first time to identify clusters of similar and regular trips over time (meta-clusters). Then these meta-clusters are considered as points to serve in an instance of DARP. This method’s main interest is to design tours of vehicles to serve a large number of potential users. As shown in Section 4, the main advantage of this method is that the execution time of the optimization problem does not depend on the number of users served, but only on the number of meta-clusters served.

As part of this study, the analysis of pattern recurrence was carried out over two weeks. However, the proposed method makes it possible to detect regular patterns over much more extended periods. This can be particularly useful in the case of effective implementation of transport lines. Besides, it is possible to integrate many other parameters into the objective function of DARP to design lines as close as possible to actual user demand.

The results obtained show us that rounds of large-capacity vehicles (80 peoples) can be identified. Making it possible to serve on average more than 600 trips per day with calculation times lower than 9 seconds. This hybrid method between classical and dynamic approaches allows to design high capacity lines based on the real demand of mobility. Moreover, it allows to obtain lines with restricted spatio-temporal deviations from the demand described by the centroids of the meta-clusters. Setting up massive lines close to the initial demand of users provides a partial response to the last mile problem, which is one of the main obstacle to shifting users of private vehicles to shared modes of transport. As we mentioned in the introduction, this study is part of the adaptation of current methods to new requirements for the deployment of autonomous vehicles. The method presented makes it possible to overcome the incompatibilities between current methods and new approaches adapted to the needs of services based on the use of autonomous vehicles.

Several ways are studied in order to complete the current method. The first is to take real-time aspects into account in the method. This can be done in several ways, either with instant classification or by taking into account the results obtained to anticipate future demand. Another interesting aspect is the design of more or less dynamic lines according to the demand in a studied area. For example, depending on the number of users and the required responsiveness of the service, different solutions can be implemented: classic or dynamic bus lines, taxi, etc. Finally, taking into account the dynamic aspects of the network to choose routes according to the network’s particular events: congestion, roadworks, etc. seems to be an excellent way to improve the current method.

Finally, the method’s scalability will be widely studied to maximize the number of data processed and thus the veracity of the results obtained.
AUTHORS CONTRIBUTIONS
The authors confirm contribution to the paper as follows: study conception and design: NC; data collection: CV; analysis and interpretation of results: NC, CV; draft manuscript preparation: NC, CV. All authors reviewed the results and approved the final version of the manuscript.
References


