STRATEGIC DESIGN OF A BIMODAL PUBLIC TRANSPORT NETWORK: AN EXPLORATORY ANALYSIS

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ABSTRACT

An approach for the strategic design of a bimodal public transport system (bus-subway) is presented and applied, using the case of a stylized urban corridor where all possible combinations of lines structures can be characterized and solved. Demand is completely described with few parameters such that the best system can be found for all cases and presented graphically. We show that bus lines spacing pays a key role in the search for the best lines structure. Lessons for the extension to a city are obtained.

Keywords: bimodal public transport, strategic design, lines structures

1. INTRODUCTION

The design of urban public transport systems is quite complex. Strategically, the challenge is to find the most appropriate set of lines, their frequencies and vehicle sizes considering all resources consumed by both operators and users (their time). In real cases the search for strategic designs is usually done by means of heuristics over complex networks. Analytical models have been found to be helpful under two gross approaches: searching for best structures over regular networks representing a city, such as a grid (e.g. Daganzo, 2010) or a circular model (e.g. Badia et al., 2014); and using simple networks with few nodes and links, enough to represent a specific transport problem, such as an extended cross-shaped network (Jara-Díaz et al., 2018). Recently Fielbaum et al., (2017) proposed a generic center-based city model that links the analytical approach with that of heuristics.

Most analytical optimization models consider single-mode networks, such as buses (Fielbaum et al., 2016) or rail (e.g. Saidi et al., 2016). The objective of this paper is to explore the main challenges that are faced when conceiving the strategic design of a transit network considering the potential use of two technologies represented by their corresponding operators' costs (e.g. bus-subway). With this purpose we will develop an analytical model over a corridor that can be considered a zone of a city including a periphery, a subcenter and a CBD, where all possible combinations of lines structures (single and bi-modal) can be described and solved. Demand distribution is completely represented with few parameters such that the best system cabe found for all cases. Such representation could be looked at as an extension of Jara-Díaz et al. (2012) who analyzed the effects of unbalanced demand on the optimal lines structure on a simplified urban corridor considering a single-mode. The corridor has three nodes with only one destination which can be served with either a single line, exclusive lines, lines with transfers or shared lines; the best lines structure is the one that minimizes social costs (operators' plus users') that results from the optimal design variables – frequencies and vehicle sizes – for each line in a given structure. The zonal corridor considering two technologies developed here will be shown quite useful for two reasons: it permits the analysis of the conditions under which different (potentially bimodal) lines structures dominate for all possible demand structures, and sets the basis to move forward towards the analysis of a multizonal city.

In the remainder of this Section a brief summary of the literature involving two technologies in the design of transit networks is offered. In Section 2 we present the stylized corridor representation with its demand pattern parametrically described, on which the design problem is formulated and lines structures are identified. Section 3 presents the solutions of the design problem for each lines structure and finds those of minimum cost for every demand pattern. Synthesis and conclusions are offered in the final section.

The combination of two technologies involves the design problem of a bimodal public transport network. The literature on bimodal structures shows some emphasis on the feeder-trunk structure. In corridors, Chien and Schonfeld (1998) develop a model that jointly optimizes the design variables of a trunk rail line (length, headway and stop spacing) and its feeder bus lines (headway, line and stop spacing), serving a demand pattern with many origins and many destinations. Sun et al. (2017) find the optimal rail length when a bus single line is replaced by a bus-train feeder-trunk structure, and analyze the effects of its transition in a dynamic model. The demand is assumed with origins distributed stretching from the CBD to the city boundary with a single destination at the CBD. Also, in a feeder-trunk structure over a corridor, Sivakumaran et al. (2012) assume a demand with a single destination, exploring how the coordination of arrivals of feeders and trunk vehicles - and the joint determination of their frequencies - reduce total costs (operators' plus users'). In a rectangular city, Sivakumaran et al. (2014) model bus feeder lines that intersect rail (or BRT) trunk lines and determine which structure dominates over single-modes networks (bus, BRT or rail) for different demand densities and trip lengths. Fan et al. (2018) model a potentially bimodal system composed of local (bus) and express (BRT or rail) lines that intersect perpendicularly in a grid form. The model determines the optimal design of each network (bus plus the other mode) including the headway, the local and express lines spacing, and the local stop spacing. The numerical results show that bimodal networks generate a lower total cost than a single-mode network for intermediate and high demand values, and that the joint search for the optimal design of the whole system is superior to the separated design of local and express services.

2. PROBLEM FORMULATION AND LINES STRUCTURES

The model is set topologically as a corridor considering three nodes: a periphery (P), a subcenter (SC) and a CBD. The distance between the CBD and the subcenter is L; the distance between the subcenter and the periphery is gL (with $0 < g < 1$ normally). The demand pattern represents morning peak, with trips generated at nodes P and SC, attracted by SC and CBD. Only three parameters are needed to have all the elements of the OD matrix: total patronage Y, the proportion of trips generated at the periphery (a) and the proportion of trips that go to the CBD (α). The spatial structure and the demand pattern are shown in Figure 1.

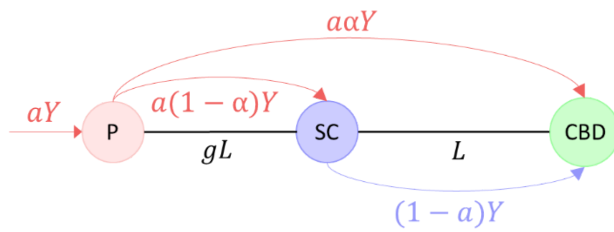
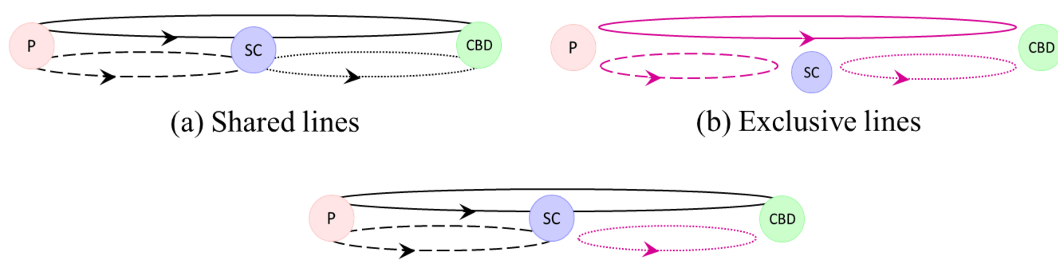


Figure 1. Corridor representation and its demand pattern

This setting expands on Jara-Díaz et al. (2012) and represents one zone of the parametric city model (Fielbaum et al. 2017)[1]. It has a nice property, namely that for a given total patronage Y all cases can be represented in the space (a, α), where a monocentric corridor can be defined as those cases in which most of the trips go to the CBD, i.e. if $(1-a)Y + a\alpha Y > a(1-\alpha)Y$, equivalent to those combinations that fulfill $\alpha > 1 - \frac{1}{2a}$.

Under this setting, we identify the following general lines structures: shared (C, one line can serve more than one OD pair), exclusive (E, each line serves only one specific OD pair) and mixed lines, as shown in Figure 2.



(a) Shared lines

(b) Exclusive lines

(c) Example of mixed lines: shared lines in the arc P-SC and one exclusive line for OD pair SC-CBD

Figure 2. Lines structures in a corridor

Each line is defined by its route, stops, frequency, vehicle size, and transport technology. Considering one transport technology, shared lines, as shown in Figure 2a, present four particular cases when some lines result with null frequency: single line (S), lines with transfers (T), and shared lines in either the P-SC arc (C1) or in the SC-CBD arc (C2). Mixed lines structures, as the one presented in Figure 2c, present six additional cases, resulting from the combination of one exclusive line with: a single line (S+E_{ij}, three cases), lines with transfers (T+E_{ij}, two cases), shared lines in the P-SC arc (C1+$E_{SC\text{-}CBD}$) and shared lines in the SC-CBD arc (C2+$E_{P\text{-}SC}$). With one technology (mode), there are 13 possible lines structures. If we consider two transport technologies, bus and subway, this set expands to 74 lines structures, resulting from all possible combinations of each technology in all lines.

[1] In the parametric city model n zones are considered, each one with a periphery and a subcenter; besides the trips shown in Figure 1, there are trips from each periphery and each subcenter to the other subcenters. In Jara-Diaz et al (2012) the intermediate node does not attract trips.

The objective is to find the lines structures that minimize the value of the total resources consumed (VRC) that results from the addition of both operators' costs (C_O), and users' costs (C_U). Operators' costs include those associated with running the system (drivers, rolling stock use, and maintenance) and fixed costs (infrastructure development and maintenance, and general costs). Running (or variable) costs are obtained multiplying the fleet of line i, B_i, by an hourly unit cost associated to the technology j (c^j) which will be assumed linear in the vehicle size K_i of the line i (Jansson, 1980), with c_0^j a fixed hourly cost per vehicle and c_1^j an hourly cost per vehicle-passenger of technology j:

$$c^j = c^j(K_i) = c_0^j + c_1^j K_i \tag{1}$$

B_i results from the frequency f_i times the cycle time t_{c_i} of line i (equation 2). Cycle time is given by the time in motion plus standing time at stops, which is the product of the number of passengers that board and alight a vehicle (given by line flow divided by its frequency) and the boarding and alighting time per passenger, t^j. We assume that time in motion depends only on the length of the links traveled (independent of flow), meaning that there is no congestion between vehicles (either of the same or different technology).

$$B_i = f_i\, t_{c_i} \tag{2}$$

Fixed costs are assumed to be proportional to the length X^j of network of technology j with c_2^j as cost per unit length and time. If $\delta_{ij} = 1$ when the line i is of technology j (0 if not), the operator cost over all I lines is equal to:

$$C_O = \sum_{i \in I} \sum_j \delta_{ij} \left(c_0^j + c_1^j K_i \right) f_i\, t_{c_i} + \sum_j c_2^j X^j \tag{3}$$

User's cost includes those costs associated with waiting time (t_w), in-vehicle time (t_v), access time (t_a), and transfers (R). Waiting time is determined by a proportion θ_w^j of the headway (inverse of frequency) between vehicles of technology j. In-vehicle time has three components: in-motion time, time spent in bus waiting for other passengers to board and alight, and own alighting time. Access time refers to walking time to the line of mode j (t_a^j); it is mode specific as buses usually require walking at surface level while subway includes access to platforms in a different level. With p_h the value of time of activity $h = \{w, v, a\}$ – the same for all technologies – and p_R the pure transfer penalty[2], the user cost is expressed in average values over all OD pairs of the network[3]:

$$C_U = Y\, (p_w \bar{t}_w + p_v \bar{t}_v + p_a \bar{t}_a + p_R \bar{R}) \tag{4}$$

[2] The pure transfer penalty is expressed in equivalent in-vehicle minutes (EIVM) and represents the value of interrupting the trip beyond additional waiting and walking. See for example García-Martínez et al (2018).

[3] We assume that pv is independent of travel conditions, e.g. no discomfort due to the possible high rate of occupancy of vehicles. This can be handled by letting K represent seats rather than space.

Problem (5) below solves the design problem for each lines structure, finding the optimal frequencies (fleets) and vehicle sizes for all lines involved. Constraints (5a) impose that vehicle capacities have to be large enough to carry the maximum vehicle load k_i given by the ratio between the maximum flow of the line and its frequency. These constraints are always active as cost increases with K_i. Vehicle size has an upper limit given by technology j. Constraints (5b) impose that the sum of frequencies in each link e cannot be larger than a maximum value given by capacity of the infrastructure (ways and stops) and safety considerations of each technology ($\delta_{ie}^{j} = 1$ if link e of corridor belongs to route of line i, 0 if not).

$$min\ VRC = \sum_{i\in I}\sum_{j}\delta_{ij}\left(c_0^j + c_1^j K_i\right) f_i\, t_{c_i} + \sum_{j} c_2^j X^j + Y\,(p_w \bar{t}_w + p_v \bar{t}_v + p_a \bar{t}_a + p_R \bar{R}) \tag{5}$$

$$s.t \qquad k_i \le K_i \le K_{max}^j \qquad \forall i \in I \tag{5a}$$

$$\sum_{i\in I}\delta_{ie}^j f_i \le f_{max}^j \qquad e \in \{P-SC, SC-CBD\},\ \forall j \tag{5b}$$

Once the optimal frequencies and capacities have been found, replacing them in VRC yields the cost function of the corresponding lines structure, depending on a, α and Y. The best lines structure for a given triad is the one that exhibits the minimum cost across candidate structures.

3. APPLICATION

To solve each of the 74 cases previously identified, we have to characterize both transport technologies considered: conventional buses running on exclusive lanes and subway. Conditions and parameters are based on information from Santiago, Chile. Table 1 shows the specific modal parameters; operational costs were estimated with data from cost studies and annual reports (DTPM, 2013; SECTRA, 2015). Subway runs under centrally operated control, such that vehicles operate with high speed and regular arrivals. The boarding and alighting process is sequential for both buses and subway. Values of time and geometrical parameters are shown in Table 2. We use the value of 16 EIVM (equivalent in-vehicle minutes) as pure transfer penalty, a value that is within the range reported for multimodal networks (Garcia-Martinez et al., 2018).

Table 1. Modal parameters

Mode	c_0 (\$us/h-veh)	c_1 (\$us/h-pax)	c_2 (\$us/h-km)	v (km/h)	θ_w	t (s/pax)	f_{max} (veh/h)	K_{max} (pax/veh)	t_{a_t} (min)
Bus	8.61	0.30	0.00	20	0.7	2.50	150	160	0.00
Subway	80.91	0.15	933.15	40	0.5	0.33	40	1440	1.00

Table 2. Values of time and spatial parameters

p_v (\$us/h)	p_w (\$us/h)	p_a (\$us/h)	p_T (\$us)	L (km)	g
2.74	5.48	8.22	0.73	10.00	0.85

Numerical simulations were made for each combination of a and α, considering four values of patronage Y associated with trips in corridors running from the center in all eight different directions in Santiago: 7000, 12500, 20000, and 27000 passengers/h. These values are shown in Figure 3 in the space (a, α), where the dotted line represents the hyperbola that generates the subspace where the corridor is monocentric (shadowed in gray).

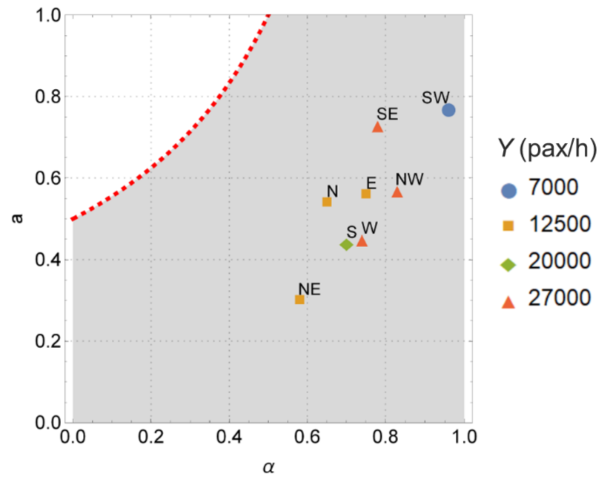


Figure 3. Representation of corridors in Santiago in space (a, α)

Figure 4 shows 19 lines structures that dominate - i.e. that minimize VRC^* - for different combinations (a, α) and different levels of Y, simulated without considering lines spacing so access time at street level plays no role. Our results show that bus-only lines structures dominate over bimodal or subway-only for low and intermediate demand values. Only when the capacity of the bus system is reached, the advantages of metro and bimodal lines structures emerge.

Dominant structures are sensitive to both demand pattern parameters, a and α. The single line structure dominate only for high values of a and α (most trips are made on the OD pair P-CBD) for all demand levels, because fewer trips on the remaining OD pairs (P-SC and SC-CBD) generate low boarding and alighting times in the subcenter, and the vehicles have low idle capacity in all links. For relatively low demand levels the bus technology dominates in that zone but, as demand increases to high levels, buses reach capacity and the single subway line structure becomes the best solution.

On the other extreme - very low values for a and α - lines with transfers dominate (most trips are made on the OD pair SC-CBD and few on the OD pair P-CBD). This happens because relatively few users make transfers, generating low transfers costs (including additional waiting and access time, and pure transfer penalty), and no idle capacity in vehicles of each line. For all demand values, the line on the P-SC arc operates with buses, but, when patronage gets large the line on the SC-CBD arc changes technology from bus to subway. Surprisingly, this feeder-trunk-like bimodal structure dominates in a very limited space.

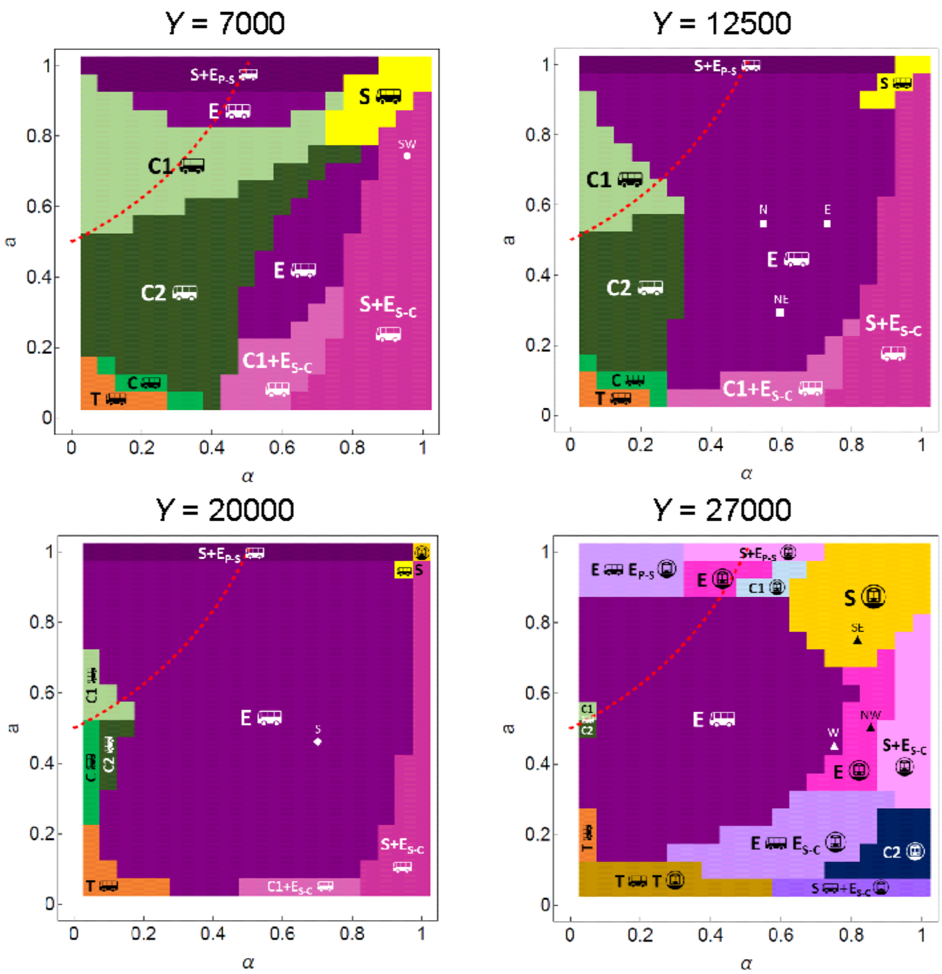


Figure 4. Minimum cost lines structures for all demand patterns

Shared lines structures C1 and C2 have a large dominant space for low demand – when it is concentrated in P-SC and SC-CBD link, respectively – and they practically disappear for high demand. These structures can be seen as short-turning operation strategies in the most loaded link. Exclusive lines of buses dominate with low demand when generation is highly concentrated either in the periphery or in the subcenter, and with intermediate demand in much of the (a, α) space until the capacity of the bus system is reached. In both cases, the demand in each OD pair is high enough to have reasonably high frequencies; also, in the exclusive lines, the travel time is not affected by intermediate boarding and alighting times and there is no idle capacity.

Mixed lines structures of buses dominate for low demand values and mainly for high values of α . In general, these structures dominate in sub-spaces where some exclusive lines structure would be best if the mixed did not exist. This happens because mixed structures combine the advantages of shared and exclusive lines, adapting frequencies and vehicle sizes according to the flow of each OD pair: an exclusive line for the OD pair with high demand and a shared lines structure for the remaining OD pairs (with fewer demand).

Six of the eight corridors in Santiago generate the minimum social cost with buses lines structures (S+$E_{S\text{-}C}$ and E) with an average frequency of 61 veh/h and vehicle sizes within the range observed in the city: medium (44 pax/veh) and articulated buses (160 pax/veh). The resulting frequencies in some lines are very high, but this is due to the aggregation of demand on a single street, without considering lines spacing. The two remaining corridors operate in subway lines structures (U and E) with average frequency of 24 veh/h and average vehicle size of 493 pax/veh.

The next scenario considers the potential operation of parallel lines of buses (lines spacing) with each former line i representing $D^b = 4$ lines per corridor[4], keeping one subway line per corridor (i.e. $D^m = 1$). Now, frequency f_i is treated as $F_i = f_i D^j$ in problem (5) and walking time to the line plays a role. Figure 5 shows the minimum cost lines structures considering a 2 km corridor width and a 4 km/h walking speed. The design considering buses lines spacing causes this mode to dominate over all the space (a, α) in 10 lines structures for the same Y values considered before, due to the amplification of its capacity in the corridor, reaching levels comparable to that of the subway. It is very interesting to note that the pattern of dominance of lines structures is relatively stable with respect of the level of demand, and converges to the case of low demand ($Y = 7000$) without buses lines spacing, which is visualized when comparing Figure 5 with Figure 4. This result is explained because the demand of the axis is distributed over four equally spaced lines, amplifying the dominance of each structure according to the demand acting on each line. This suggests that the optimal lines structures would depend more on a and α than on Y .

Due to the dispersion of the demand in four equally spaced lines, the exclusive lines reduce the frequencies and increase the waiting times, so they lose their competitiveness considerably in the first three simulated levels of demand. The shared lines dominate over a large portion of the space (a, α), with the structure C2 in the monocentric sub-space, C1 mainly in the complementary sub-space, and C with very low participation between the division of C2 and T. Single line and lines with transfers maintain the dominance observed in the previous case – with high and low demand parameter values, respectively – but increasing it in low demand scenarios.

The resulting frequencies are lower than in the previous case. In the eight corridors of Santiago, the average frequency in the lines structures of minimum cost is 33 veh/h. In some corridors, the frequencies reduce such that the dominance of exclusive lines is lost to the shared lines in the SC-CBD link. Vehicle sizes are smaller on average, within a limited range, between 21 and 85 pax/veh.

[4] Db=4 is a reasonable value for Santiago that is in the range of optimum density values found by Fielbaum et al., (2020) with numerical analysis with similar parameters but considering buses only.

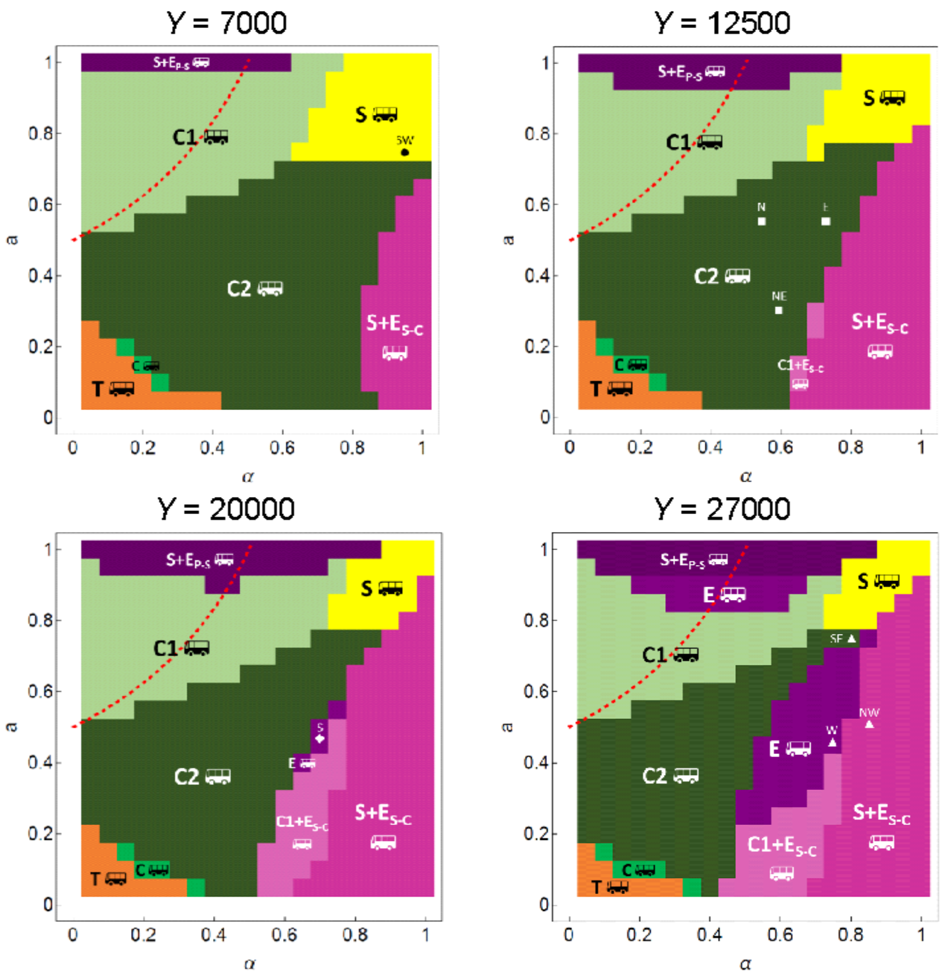


Figure 5. Minimum cost lines structures for all demand patterns including bus lines spacing

Searching for dominants lines structures in the corridor – composed by two links and three nodes – reveals some key points that could difficult the extension to the analysis on a general parametric description of a city. In synthesis, these elements are:

1. The number of possible lines structures considering two technologies grows substantially: in the corridor (zone) case, the enumeration of all structures is treatable; in the case with *n* zones, solving the design problem for all structures is practically infeasible.
2. The number of possible users' routes grows notably in most OD pairs.
3. When considering buses lines spacing, the access time (walking to the line time) in the origin and (possible) transfer nodes is particularly complex to incorporate in a simple way.

4. SYNTHESIS AND CONCLUSIONS

We have explored the design of a transit network operating with up to two technologies over a stylized corridor involving a periphery, a subcenter and a CBD. In the simplified corridor model, it is possible to a) solve the design problem covering all demand patterns represented by different parameters combinations for given levels of patronage, and b) to enumerate all possible lines structures. Considering one transport technology, there are 13 lines structures, which are classified into three groups: shared (five structures), exclusive and mixed (seven). When two technologies are considered, this set grows substantially to 74 structures – about six times higher – resulting from all the possible combinations of structures and lines.

The numerical analysis with conditions and parameters based on information from Santiago, Chile, shows that the subway-only and bimodal lines structures become superior only when the capacity of the bus system is reached. The inclusion of bus lines spacing generates structures of minimum cost based only on this technology. In this case, the dominance pattern is relatively stable for all levels of demand, suggesting that it depends more on the demand distribution in space than on the total patronage.

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