Recovering platform wait time patterns from revealed preference data for urban rail transit systems

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1 Introduction

The time spent waiting at a station platform prior to boarding is considered by passengers to be one of the most onerous phases of a transit journey. In terms of the generalised cost of travel, the value of platform wait time is typically weighted at least twice as much as the value of uncrowded in-vehicle time (Wardman et al., 2016). Transport for London applies a higher weighting of 2.5 for wait time, compared to a base weighting of 1 allocated for in-vehicle time in uncrowded conditions (Transport for London, 2013). Wait time therefore incurs at least double the amount of disutility to passengers as when they are travelling on empty trains. Therefore, to deliver a high quality of service, quantifying the underlying drivers of fluctuations in wait times is essential.

Train frequency is a primary determinant of passenger wait time (Newell, 1982; Osana and Newell, 1972). In the literature, the consensus is that passenger arrival patterns are influenced by train frequency levels. When train frequencies are high in peak times, passengers arrive in a random manner, resulting in an average wait time equivalent to half of the headway between trains. As service frequency decreases in off-peak periods, passengers tend to arrive in a non-random manner in order to minimise their wait times. In the most recent empirical work on rail transit covering both high frequency metro and low-frequency suburban rail modes, the transition from random to non-random passenger arrivals is reported to occur in the range from 5 to 11 minute headways (Berggren et al., 2019; Ingvardson et al., 2018; Fan and Machemehl, 2009; Luethi et al., 2007). With increasing access to advanced train arrival information available through open source data online and via mobile phone applications, the transition to timed passenger arrivals may occur at shorter headways.

In this paper, the prevailing hypothesis that passenger wait times tend to decrease at longer train headways as passenger arrivals transition from random to non-random is tested, using the London Underground metro system as a case study. Automated fare collection (AFC) data from the Oyster card system and automated vehicle location (AVL) data on train movements are used. Three parts of analysis are undertaken to establish the effect of headways on passenger wait times at the origin station as follows:

1. Total passenger journey times are decomposed via a probabilistic passenger to train

assignment algorithm to calculate passenger access time, the time taken from passenger tap-in to train boarding at the origin station.

2. Semiparametric regression modelling is undertaken to determine the partial effect of headways on access times while conditioning on other service supply and passenger demand characteristics.

3. The effect of headways on marginal passenger wait times is then isolated and quantified.

Marginal passenger wait times with respect to train headways are calculated in two-step process. First, expected values of access time with respect to train headway are calculated from the outputs of the semiparametric regression model of access times. This generates a relationship which can be considered analogous to a exposure-response function, a model framework commonly applied in the medical toxicology literature (Haschek et al., 2013; Wang, 2015). In this case, access time is the response and headway is the exposure. Second, the derivative of access time with respect to headway is calculated via finite differencing. This quantity is demonstrated to be equivalent to marginal wait time with respect to headway. Through this process, the passenger arrival process is able to be characterised and the point at which arrivals transition from random to non-random across three high frequency lines in central London is established.

The derivation of the exposure-response function of marginal wait times with respect to headway is a new methodological contribution to the literature on transit wait times. In the literature, models exploring the relationship between wait times and headway rely on direct measurement of wait times through observation of samples of boarding passengers at station platforms or from passenger stated preference survey data. Parametric functional forms are assigned to the wait time data, and inferences are then made regarding passenger arrival and wait time patterns (O'Flaherty and Mangan, 1970; Seddon and Day, 1974; Bowman and Turnquist, 1981; Luethi et al., 2007; Nygaard and Torset, 2016; Ingvardson et al., 2018). In this paper, revealed preference data on passenger trips and train movements from AFC and AVL data sources are used, and the relationship between wait times and headways is mathematically derived. The paper demonstrates that the mathematical derivation of marginal wait times generates valid results for the London Underground metro system, and so the method can be readily applied to other systems to infer wait time patterns from routinely collected automated data.

2 Data

Selected sections of the Central, Jubilee, and Victoria lines on the London Underground are analysed as follows: the entire length of the Victoria line (16 stations); West Acton to Oxford Circus on the Central line (12 stations); and Bond Street to North Greenwich on the Jubilee line (10 stations). The routes on the line represent single-line trips only excluding transfers, and the line sections are chosen such that route choice decisions do not need to be made under regular operating conditions. Since the line sections cover some of the busiest routes in central London, the passenger data capture a mix of regular work commuters, school children, tourists, and others.

Passenger trip data and train movement data over a period of 7 weeks from October to December in 2013 are used, focusing on weekday travel (Monday to Friday) only. The passenger trip data are recorded via the Oyster smart card system, and include the timestamps and locations of each trip between the origin and destination stations. The train movement data are recorded by the internal TfL NetMIS system and include departure timestamps and locations of each train movement at each station platform.

3 Methods

The journey times from the AFC data set report total journey times from tap-in at the origin station to tap-out at the destination station, and so to enable the journey times to be split into parts, passengers must first be allocated to trains. This is achieved by merging the AFC trip data with the AVL train movement data and applying a probabilistic train assignment algorithm based on the egress times associated with each feasible train itinerary. Full details of the assignment algorithm are not presented here but are available in Singh et al. (2018).

Through assignment of all trips to unique train itineraries, the total journey times of each trip are decomposed to obtain the access time component. The access time y_{ij}^{ac} associated with each trip i assigned to train itinerary j is the time taken from passenger tap-in at the origin station t_i^{entry} to the point where the train departs from the origin station platform DT_{oj}, i.e. $y_{ij}^{ac} = DT_{oj} - t_i^{entry}$.

A semiparametric regression model with access times set as the response is then developed as detailed in section 3.1, and marginal platform wait times are estimated from the outputs of the access time model as presented in section 3.2.

3.1 Access time regression model

To exploit the large volume of data, a semiparametric regression model framework is adopted. Semiparametric regression enables non-linear relationships between the independent and dependent variables to be modelled via basis functions in the form of penalised thin plate regression splines. The basis functions are fitted including a penalty to impose a trade-off between the degree to which the spline functions match the data and the degree of smoothness. Further details of the underlying theory are given in Wood et al. (2015) and Wood (2006). The modelling is undertaken using the R statistical software package 'mgcv'. The models are fitted using penalised iteratively reweighted least squares (PIRLS) and the restricted maximum likelihood (REML) technique is used to estimate the parameters of the models.

For each passenger trip i assigned to train itinerary j, an access time regression model can be formulated as a function of headway h_{ij} and other service supply and demand covariates V_{ij}, and estimated model parameters $\widehat{\theta}$:

$$y_{ij}^{ac} = m(h_{ij}, V_{ij}, \widehat{\theta}) + \epsilon_{ij} \tag{1}$$

where y_{ij}^{ac} is the access time component, and ϵ_{ij} is the random error term, assumed to be independently and identically distributed with mean 0 and given variance σ_ϵ^2, such that $\epsilon_i \sim \mathcal{N}(0, \sigma_\epsilon^2)$.

The headway and other service supply and demand covariates are modelled by applying the following forms: continuous variables are modelled with either a parametric or non-parametric form, and categorical factors are modelled with group-specific fixed effects. The general form of the access time regression model is therefore expressed as:

$$y_{ij}^{ac} = \alpha + X_{ij}^T \beta + \sum_{k=1}^{K} f_k(X_{ij}^*) + Z_{ij}^T u + \epsilon_{ij}, \tag{2}$$

where

α is the model constant,

β are the parameter coefficient estimates for the continuous covariates X_{ij} modelled parametrically,

$f_k, k = 1..K$ are the smooth basis functions based on penalised thin plate regression splines of the continuous covariates X_{ij}^* modelled non-parametrically,

u is a vector of estimated group-specific fixed effects for the categorical factors Z_{ij}, and

y_{ij}^{ac} and ϵ_{ij} are as previously defined.

The data set for the regression model consists of all trips within the defined study area, totalling approximately 5 million trips. The continuous covariates and fixed effects included in the model are listed in Table 1.

Table 1: List of covariates and fixed effects in access time regression model

Service supply covariates		Demand covariates	
Dynamic	Static	Passenger volumes	Passenger characteristics
Headway COV headway Normalised headway Time of day	Fixed effects for stations, and line/direction	Platform loading Line loading	Passenger age Card type Discount Travel frequency Egress time

3.2 Derivation of marginal platform wait times

There are two distinct phases of passenger movements within the access time component: 1) the time taken to walk from the ticket gates to the platform, t_{oi}^{walk} and 2) the wait time at the platform before boarding, w_{ij}. Based on this, the access time y_{ij}^{ac}, can therefore be expressed as per equation 3. In the access time model, walk times are quantified through fixed effects that capture the station- and line/direction-specific characteristics, and the passenger demand covariates.

$$y_{ij}^{ac} = w_{ij} + t_{oi}^{walk} \tag{3}$$

For calculation purposes, equation 3 can be expressed in terms of the expectations of the variables:

$$E[y_{ij}^{ac}] = E[w_{ij}] + E[t_{oi}^{walk}] \tag{4}$$

After having taken into account all other variables in the access time model, if the sole influence of headways h_{ij} on access times is analysed, the derivative of access time with respect to headway is:

$$\begin{aligned} \frac{\partial E[y_{ij}^{ac}]}{\partial h_{ij}} &= \frac{\partial E[w_{ij}]}{\partial h_{ij}} + \frac{\partial E[t_{oi}^{walk}]}{\partial h_{ij}} \\ &= \frac{\partial E[w_{ij}]}{\partial h_{ij}} \end{aligned} \tag{5}$$

Passenger walk times are independent of the headway between trains, and so in equation 5, the rate of change in passenger walk times with respect to headways $\frac{\partial E[t_{oi}^{walk}]}{\partial h_{ij}}$ is omitted. The derivative of access time with respect to headways therefore primarily represents the marginal wait time at the platform $\frac{\partial E[w_{ij}]}{\partial h_{ij}}$.

Two steps are involved to compute the derivative of access times with respect to headways. First, predicted values of access times with respect to headway are required, and these can be extracted from the regression model of access times from equation 1, i.e.:

$$\widehat{y}_{ij}^{ac} = m(h_{ij}, V_{ij}, \widehat{\theta}) \tag{6}$$

where $\widehat{y}_{ij}^{ac}$ is the predicted value of the access time regression function conditional on headway, the remaining supply and demand covariates denoted collectively by V_{ij}, and the model parameter estimates denoted collectively by $\widehat{\theta}$.

Second, finite differences between each pair of sequential data points are then required to be taken in order to compute the derivative of access times with respect to headways. Due to the large number of observations and high levels of dispersion in the predicted values generated from the access time model, the finite differences between a set of two consecutive individual points contain extreme outlying values which tend to artificially skew the results. In order to smooth out the outliers, the mean of the predicted values is calculated for a fixed value of headway at 1 second intervals. The 1 second interval corresponds to the equivalent level of accuracy in the train movement data timestamps. Repeating this calculation for the range of headways observed allows the generation of an exposure-response function, where headway is the exposure variable and the expected value of access time is the response. The calculation of the exposure-response function of expected access times with respect to headway is given in equation 7.

$$E[y^{ac}(h)] = \frac{1}{n} \sum_{h_c=h,\ ij=1}^{h_c=h+(1/60),\ ij=n} \widehat{y}_{ij}^{ac}(h_c) \tag{7}$$

where $E[y^{ac}(h)]$ is the expected value of access times with respect to headway h in minutes. The term $\widehat{y}_{ij}^{ac}(h_c)$ represents predicted values of access times for passenger trip i assigned to train itinerary j at a given point value of headway h_c. The summation refers to the summation of access times across all n number of passenger and train itinerary combinations ij over a 1 second interval of headways from $h_c = h$ to $h_c = h + (1/60)$ minutes. The bounds for headway are defined from $h = 1$ to $h = 9$ minutes 59 seconds, which represents approximately 99% of observations in the raw data set after excluding upper and lower outlying values of headway.

The second step is the estimation of the derivative of access times with respect to headways. The derivative is estimated through a discrete approximation by computing finite differences between the average predicted values calculated in equation 7. This process is undertaken for the total data set, and separately for peak and off-peak periods. Four models are estimated, segmented by time period: 1) all time periods, 2) peak of the AM peak, 3) inter-peak, and 4) peak of the PM peak. The equation for calculating the derivative from the average predicted values is given in equation 8.

$$\frac{\partial E[y^{ac}(h)]}{\partial h} = \frac{E[y^{ac}(h+a)] - E[y^{ac}(h)]}{(h+a)-h} \tag{8}$$

where $E[y^{ac}(h)]$ is the expected value of access time with respect to headway h as calculated per equation 7, and a is a small finite increment.

4 Results

The marginal wait time functions with respect to headway are plotted in Figure 1. In the literature, the prevailing consensus is that for high frequency services, passengers arrive randomly with a uniform random distribution. Under perfectly random conditions, the average passenger wait time (w) is equivalent to half of the headway between trains, i.e. $w = 0.5h = 0.5f^{-1}$, where h is train headway and f is the frequency of services. Under such conditions the marginal impact of headway on waiting time should remain 0.5, no matter what the initial frequency is. As shown in Figure 1, when headways are less than approximately 2-3 minutes, the impact of a change in headways is much greater than half of

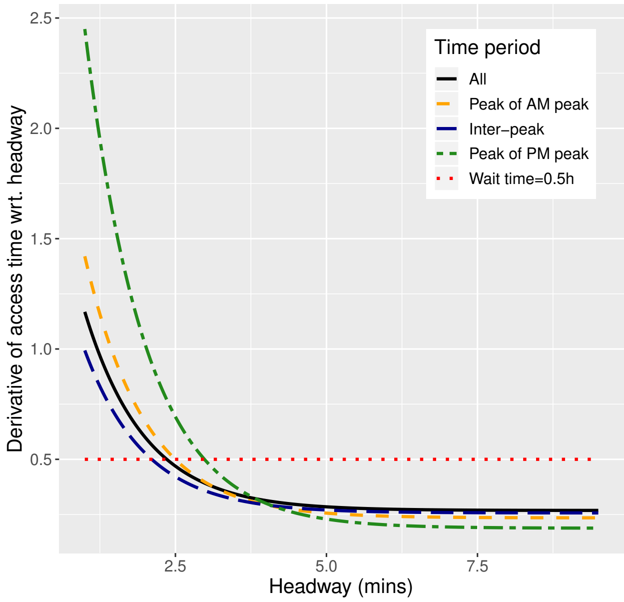

Figure 1: Marginal wait time with respect to headway

the headway. This outcome could be a consequence of station congestion and the possibility of denied boarding.

In low frequency periods, literature suggests that passenger wait times are less than half of the headway between trains i.e. $w < 0.5h$. In Figure 1, when headways are longer than approximately 2-3 minutes, the models predict that the average passenger saves less time than half of the headway. This finding is in line with the literature for wait times at long headways, albeit at lower values of headway compared to the conventional rule of thumb of a 10 minute headway transition value. Considering all trips together, passengers arrivals tend to become non-random at a headway of 2.35 minutes, and approximately 55% of passengers experience marginal wait times less than half of the headway. For those passengers that experience marginal wait times less than 0.5, the average marginal wait time is approximately 0.30, or almost one third of the operating headway. Considering the different time periods, the inter-peak function is equivalent to 0.5 at a headway of 2.10 minutes, and 78% of trips have marginal wait times less than half of the headway. In the AM and PM peak times, the functions cross 0.5 at longer headways of 2.50 and 2.98 minutes, respectively. This likely reflects a higher degree of congestion during the peaks compared to the inter-peak. A much lower percentage of trips, approximately 31% and 21%, of trips made during the AM and PM peaks respectively have marginal wait times less than half of the headway.

Wait times below half of the headway are likely to be caused by the fact that, on average, passengers do not arrive at the station randomly. A potential explanation for this could be that when the system is operating at longer headways, the system is also less congested, and this enables passengers to move more freely through stations. The reduced levels of congestion may better enable passengers to speed up when they are aware of an approaching train. Awareness may arise from real-time train arrival information on digital information display boards at the station prior to entry, from mobile phone applications, and/or from hearing and seeing the train approach en route to the platform.

5 Conclusions

In this paper, the impact of headways on passenger wait times on urban metro systems is derived. The analysis is undertaken for single-line trips excluding route choice and transfer options on three lines of the London Underground metro system. It is found that wait times are more impacted by changes to service frequency during high frequency operating periods, compared to low frequency operating periods. The results indicate that passenger arrival patterns transition from random to non-random as headways increase, and that marginal wait times for non-random arrivals are equivalent to approximately one third of the headway between trains. This observed behaviour could be a result of passengers consulting train arrival information prior to boarding, and if so, this would demonstrate that generalised travel time savings can be achieved through providing improved passenger information.

Extending the analyses presented here, a number of future research directions are possible. The observation of missed boardings at short headways could be further investigated to better understand potential generalised time benefits of capacity improvements. In terms of spatial scale, the data could be segmented at route, station, and line levels or by passenger demographics to obtain wait time patterns at a more disaggregate level. At a wider spatial scale, all methods presented here could be aggregated at a network level to enable comparisons of passenger arrival patterns and wait times across different systems.

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