

A Rideshare-oriented Mobility as a Service (MaaS): An Activity-based Approach

In this paper, we mathematically formulate a novel MaaS-based activity travel pattern (ATP) generator to facilitate ridesharing in a system in which drivers and passengers interact by sharing their full activity diary. The proposed formulation extends the definition of MaaS beyond an intermodal trip planner, by incorporating an inclusive set of travel attributes including the choices of activity, activity sequence, departure time, and mode, and the transitions among the modes in ATPs of all participants. Furthermore, this paper introduces a dynamic rideshare-oriented MaaS model in which the conventional rideshare modelling structure is synchronised with the proposed MaaS-based ATP generator in a unified structure. These models explicitly bridge the gaps in the missed-out connection between ATPs planning and rideshare models in the current literature.

1. Introduction

MaaS, in its common definition, combines services on different transport modes to provide customised mobility services via a single interface (MaaS Alliance, 2017). However, MaaS is in its introduction and growth stages and its implementation is still limited. Also, there exists a lack of understanding on the entire premise of MaaS and its divergence about what exactly constitutes MaaS (Wong et al., 2019). So, it would be beneficial to improve its coverage and concept beyond an intermodal journey planner.

The current MaaS platforms usually work as an intermodal journey planner which provide combinations of different transport modes such that the system participants can buy mobility services (Kamargianni et al., 2016) not a mode of transport, a membership or a subscription only. Therefore, the assumption is that the tour (a consequence of activities to be visited) is already determined and the objective is to provide the means of mobility while the mobility provision is conditional on travellers' needs. For example, activity location is a dominant factor in determining the mode of transport. If the activity is shopping, there can be different candidate locations that can be selected for this purpose. Advising the participants of a MaaS system can enhance the performance of the MaaS systems. Scheduling tour and activity visits could be a new feature to be included in MaaS systems to grow the coverage which is called a MaaS-based ATP generator in this paper. While there are some tools for integrated trip planning (e.g. TriMet in Portland) to provide multimodal trip information to participants, there is no integrated model for tour, activity visit and intermodal journey planning.

Regardless of the MaaS concept, the tour and activity schedulers have been extensively discussed in the literature. Generating trips, and choosing destinations, departure times, and modes are the core of such models where they are usually determined through some disjoint but interrelated sub-models (Najmi et al., 2018). Nonetheless, modal interactions are not explicitly considered in most traditional activity planning models mainly due to lack of spatiotemporal constraints among activity locations. Supernetwork-based models can take advantage of expanded networks to formulate multimodal TPMSs considering their interactions and spatiotemporal constraints (Najmi et al., 2019). Still, the multimodal formulations are usually simplified in the literature so

that the formulations may consider only walking, cycling, private car, and public transport at their simplest forms (Friedrich et al., 2018). Thus, despite of the prosperous future of rideshare modes in our society, its influence on travellers' ATPs is often ignored.

On the other hand, the state of the art in rideshare modelling is to formulate some matching problems with maximisation of different objective functions such as distance savings (Agatz et al., 2011), number of matches (Masoud and Jayakrishnan, 2017), and distance proximity (Najmi et al., 2017). While the performances of the ridesharing models are highly correlated with the participation rates, travel request attributes such as origins, destinations, and time windows of the requested trips, the travel attributes are usually considered fixed and sometimes stochastic but predictable in their mathematical formulations. Interestingly, the travel attributes should be the outputs of participants' ATPs but, unfortunately, the incorporation of the mode in the tour and activity schedulers and on persons' mobility has not been received enough attention neither in the literature.

In fact, the reciprocal interaction between ridesharing and MaaS-based ATP planning concepts has been neglected despite their significant mutual impacts. The main reason for this separation can be attributed to the fact that ridesharing is interpersonal and dynamic in its nature while tour planning is usually static and solely depends on individuals' choice or behavior restrictions and preferences. Therefore, this paper seeks a new formulation to effectively synchronise the new MaaS-based ATP generator and rideshare concepts in a unified structure.

2. Rideshare oriented MaaS optimisation

In this paper, two different models are introduced for generating MaaS-based ATP: 1) static model and 2) dynamic model. In the models, rideshare mode plays a key role. We assume that the participants with MaaS-based requests enter the system hoping to fulfil their requirements with minimum cost. The static and dynamic models are briefly discussed in this section and next section, respectively.

Constraint (1) is the flow conservation constraint and indicates that, in an ATP, if a node is visited, it must also be left.

$$\sum_{i:(i,j) \in E(k)} x_{ij}^{p,k} - \sum_{i:(j,i) \in E(k)} x_{ji}^{p,k} = 0 \quad \forall p \in P^M, \forall j \in V \quad (1)$$

There are some nodes in the system that are visited several times. Enumerating the nodes is implemented only for shared car nodes (as in Constraint 3) in this paper which can be extended to other nodes.

$$\sum_{j:(i,j) \in E} x_{ij^+}^{p,k} - \sum_{j:(i,j) \in E} x_{ij^-}^{p,k} = 0 \quad \forall p \in P^M, \quad \forall k \in K^*, \forall j \in V(CS) \exists(j^+, j^-) \quad (2)$$

Constraint (3) is a time-window constraint and ensures that a feasible route (sequence of nodes) in the space-time prism will be selected. We denote the travel time on the link (i,j) by τ_{ij} .

$$t_i^p + \tau_{ij} + d_j^p - t_j^p - M(1 - x_{ij}^{p,k}) \leq 0 \quad \forall p \in P^M, \forall k \in K, \forall (i,j) \in E(k) \quad (3)$$

Constraint (4) is the connectivity constraint (sub-tour elimination).

$$\sum_{\substack{k \in K, (i,j) \in E(k) \\ i \in B, j \notin B}} x_{ij}^{p,k} \geq 1 \quad \forall p \in P^M, \forall B \subset V \quad (4)$$

where B is a sub-tour formed in the ATP solution.

Constraints (5) assures that no more than one of the candidate locations for each activity can be visited.

$$\sum_{\substack{i \in \psi, k \in K \\ j: (i,j) \in E(k)}} x_{ij}^{p,k} = 1 \quad \forall p \in P^M, \forall \psi \in C^p \quad (5)$$

Constraints (6) and (7) ensure that departure times and activity durations are properly handled.

$$\underline{t}_i^p \leq t_i^p \leq \bar{t}_i^p \quad \forall p \in P^M, \forall i \in V \quad (6)$$

$$\underline{d}_i^p \leq d_i^p \leq \bar{d}_i^p \quad \forall p \in P^M, \forall i \in V \quad (7)$$

Constraints (8) and (9) specify the range and values of the decision variables.

$$x_{ij}^{p,k} \in \{0,1\} \quad \forall p \in P^M, \forall k \in K, \forall (i,j) \in E(k) \quad (8)$$

$$t_i^p, d_i^p \in \mathbb{R}_{\geq 0} \quad \forall p \in P^M, \forall i \in V \quad (9)$$

Having a properly pre-processed transport network, the modes of walking, private/shared car and bicycle, and public transport (with some simplifying assumptions) can be addressed by the equations discussed above. However, addressing the rideshare mode in the multimodal structure is complicated because of its interpersonal nature which considers the interaction of participants in the system. The rideshare mode is formulated in the next section.

Rideshare modelling

Having the customised networks and tagging the rideshare links (RS), the ATP formulation can be extended to incorporate the rideshare mode. Constraint (10) ensures that if participants $p \in D$ and $p' \in R$ are matched on the link $(i,j) \in E(R)$, both participants must traverse the link. In the formulation, $y_{ij}^{p,p'}$ represents the matching variable which is 1 if driver $p \in P$ and passenger $p' \in P$ are matched on link $(i,j) \in E(RS)$ and 0 otherwise.

$$x_{ij}^{p,A} x_{ij}^{p',RS} - y_{ij}^{p,p'} = 0 \quad \forall p \in D^M, \forall p' \in R^M, \forall (i,j) \in E(RS) \quad (10)$$

Constraint (11) enforces that a participant $p' \in R$ can traverse a rideshare link only if it is matched with one driver.

$$x_{ij}^{p',RS} - \sum_{p \in D} y_{ij}^{p,p'} = 0 \quad \forall p' \in R^M, \forall (i,j) \in E(RS) \quad (11)$$

Constraint (12) specifies the departure time of the driver and passenger to be the same if they are matched.

$$\sum_{l \in L} y_{ij}^{p,p'} (t_i^{p'} - t_i^p) = 0 \quad \forall p \in D^M, \forall p' \in R^M, \forall (i,j) \in E(RS) \quad (12)$$

Constraints (13)-(14) are matching constraints and guarantee that, in each of the links $(i,j) \in E(RS)$, each passenger is matched by at most one driver and each driver is matched by at most its available seats, Cap^p . In other words, it is a car-pooling formulation which is incorporated in the model. In the cases where $Cap^p = 1$, the formulation reduces to the peer-to-peer ridesharing formulation.

$$\sum_{p' \in P} y_{ij}^{p,p'} \leq Cap^p \quad \forall p \in D^M, \forall (i,j) \in E(RS) \quad (13)$$

$$\sum_{p \in P} y_{ij}^{p,p'} \leq 1 \quad \forall p' \in R^M, \forall (i,j) \in E(RS) \quad (14)$$

Objective function

A weighted objective function is defined in Eq. (15).

$$\begin{aligned} \min \quad & \sum_{\substack{k \in K, (i,j) \in E(k) \\ p \in P^M}} \tau_{ij} x_{ij}^{p,k} + \sum_{\substack{k \in K, (i,j) \in E(k) \\ p \in P^M}} (t_j^p - t_i^p - d_j^p - \tau_{ij}) x_{ij}^{p,k} \\ & - \sum_{\substack{k \in K, (i,j) \in E(k) \\ p \in R^M}} \bar{\tau}_{ij} x_{ij}^{p,RS} \end{aligned} \quad (15)$$

3. Dynamic rideshare oriented MaaS optimisation

The proposed ridesharing constraints of the rideshare oriented MaaS model (eq. (10)-(14)) significantly increase the complexity of the model. To overcome this difficulty, we introduce the dynamic version of the model in which the system periodically solves a number of smaller problems and re-optimise the ATPs of participants in a stagewise manner, based on the active rideshare opportunities/offers in the solution space. To generalise the algorithm to rideshare models, we also incorporate participants with single-trip announcements in the system. Participants with single-trip announcements are participants who either request a single ride or offer a single drive while their origin and destinations are predetermined and fixed. We denote $i(p)$ and $j(p)$ the origin and destination of a single-trip participant p .

3.1. Individual route optimisation

A MaaS-based participant uses the rideshare mode if his/her participation results in lower cost. Therefore, we call a rideshare participation *feasible* if it results in cost reduction. To find feasible matches, detour lengths before and after detour should be calculated and compared. To calculate the cost before detour, the model in Section 2 (without rideshare constraints) should be

optimised for each of the MaaS-based participants. The route includes their category of participant (driver or passenger), and the modes that they will use to get their activity locations.

3.2. Matching feasibility

Having the driver/passenger and single-trip/MaaS-based categories of announcements, the participants in the system are categorised into four groups of 1) single-tip drivers, 2) single-trip passengers, 3) MaaS-based drivers, and 4) MaaS-based passengers. Therefore, each participant p , whether his/her request is for a single-trip or MaaS ATP, is either a driver or a passenger looking for their best matches using the platform. We, henceforth, refer to d and r participants as the driver role and the passenger role, respectively.

The detour by driver d to give a ride to passenger r , referred to as the pair (d, r) , may be *feasible* if 1) the time windows of driver d and passenger r overlap, and 2) the detour cost of the driver is justifiable. Depending on the type of participants' requests, the matching pairs (d, r) can be categorised into: 1) single trip drive announcement and single trip ride announcement (S&S, see Fig. 1), 2) MaaS-based driver and single-trip passenger (M&S, see Fig. 2), 3) single-trip driver and MaaS-based passenger (S&M, see Fig. 3), and 4) MaaS-based driver and MaaS-based passenger (M&M, see Fig. 4). Checking the feasibility of the pairs which include MaaS-based participants is not straightforward and is excluded from this short paper due to word limit.

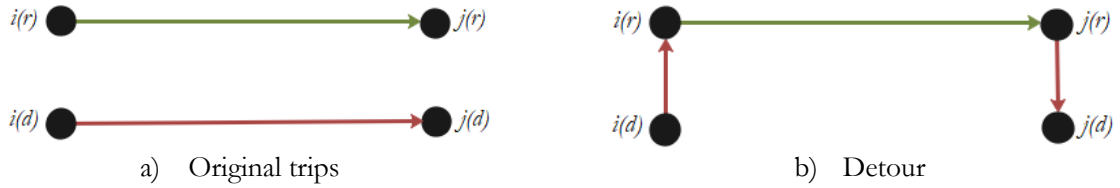


Fig. 1 Detour feasibility of pair (d, r) in S&S

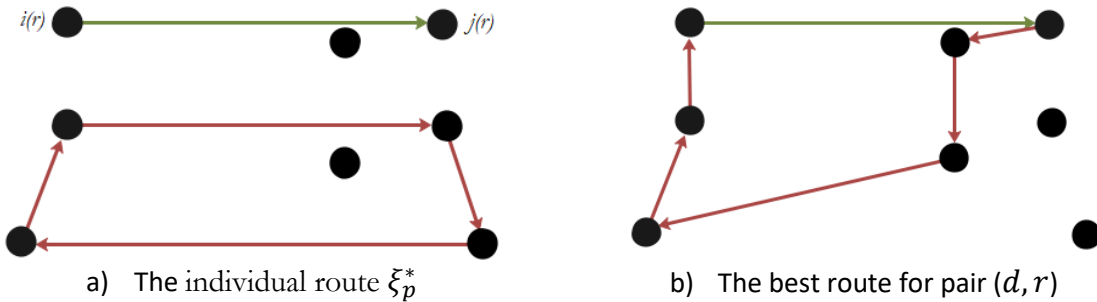


Figure 2 Detour feasibility of pair (d, r) in M&S

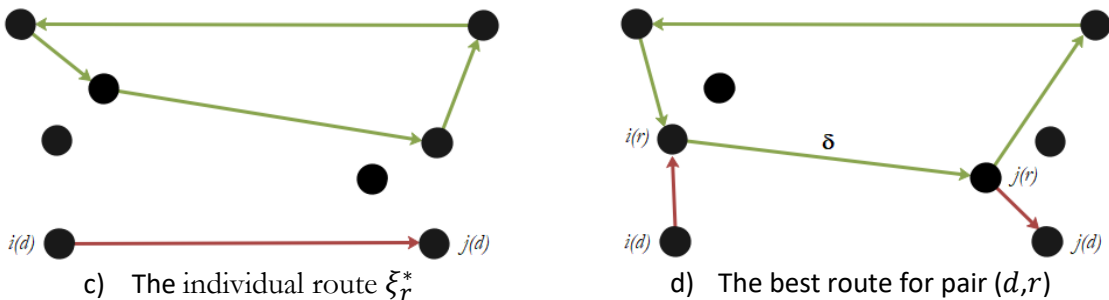


Figure 3 Detour feasibility of pair (d, r) in S&M

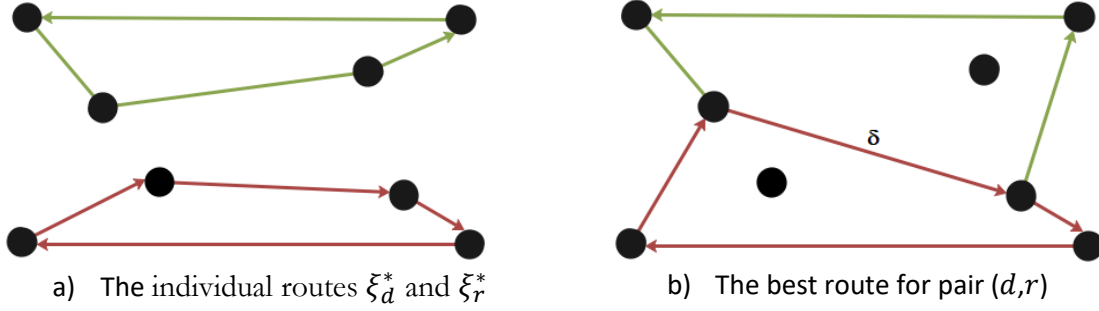


Figure 4 Detour feasibility of pair (d, r, δ) in M&M

3.3. Matching drivers and passengers

This section presents a matching algorithm to optimally determine which drivers and passengers and how they should be matched to each other. The objective of the ridesharing matching problem is to maximise the weighted sum $\sum_{(d,r,\delta) \in \bar{P}} w_{d,r,\delta} z_{d,r,\delta}$ and the complete formulation of the problem is summarised in Eq. (16)-(19).

$$\max \sum_{(d,r,\delta) \in \bar{P}} w_{d,r,\delta} z_{d,r,\delta} \quad (16)$$

Subject to:

$$\sum_{\substack{r \in R, \delta \in E(d): \\ (d,r,\delta) \in \bar{P}}} z_{d,r,\delta} \leq 1 \quad \forall d \in D^S \cup D^M \quad (17)$$

$$\sum_{\substack{d \in D, \delta \in E(r): \\ (d,r,\delta) \in \bar{P}}} z_{d,r,\delta} \leq 1 \quad \forall r \in R^S \cup R^M \quad (18)$$

$$z_{d,r,\delta} \in \{0,1\} \quad \forall (d,r,\delta) \in \bar{P} \quad (19)$$

The weights $w_{d,r,\delta}$ of each edge play a critical role in forming the best solution. We use the index of distance saving DS and NM in the objective function for numerical experiments in Section 4 (see Najmi et al., 2017 for illustration purposes).

3.4. “STATIC” solution algorithm

The pre-processing and matching formulations are summarised in Algorithm 1. This algorithm will be used in each iteration of the dynamic MaaS-based ATP generator algorithms to periodically find the best ATPs for MaaS participants based on the rideshare opportunities that appear in the system.

Algorithm 1: STATIC

```
1 Input: Set of driver announcements  $D^S \cup D^M$ , set of passenger announcements  $R^S \cup R^M$ ,  $w_{d,r,\delta}$ 
2 Output: A weighted bipartite graph  $G$ , a matching vector  $\mathbf{x}$  for all pairs of driver-passenger in  $\bar{P}$ 
3  $P \leftarrow \{(d, r, \delta) : d \in D^S \cup D^M, r \in R^S \cup R^M, \delta \in E(d) \cup E(r)\}$ 
4  $\bar{P} \leftarrow \emptyset$ 
5 for  $(d, r, \delta)$  in  $P$ :
6   if  $TF(d, r, \delta) = \text{True}$ , and  $\Delta S(d, r) \geq 0$  then:
7      $\bar{P} \leftarrow \bar{P} \cup \{(d, r, \delta)\}$ 
8   end if
9 end for
10  $\mathbf{w} \leftarrow$  Determine weight vector based on the objective function and  $\bar{P}$ 
11  $G \leftarrow (D^S \cup D^M \cup R^S \cup R^M, \bar{P}, \mathbf{w})$ 
12  $\mathbf{z} \leftarrow$  Execute the generalized maximum-weight matching formulation proposed in Section 4.4 on  $G$ 
```

3.5. Rolling Horizon Framework

Single-trip and MaaS-based announcements may enter the rideshare oriented MaaS system continuously at any time, thus making the problem dynamic. Algorithm 2 shows the *rolling horizon* we have used in this paper.

Algorithm 2: ROLLING HORIZON

```
1 Input: Announcements sets  $D^S$ ,  $D^M$ ,  $R^S$  and  $R^M$ , objective function type
2 Output: A finalised vector  $\bar{\mathbf{z}} = [\bar{z}_{dr\delta}]$ 
3 for  $t \in \{0, h, 2h, 3h, \dots\}$ :
4    $D_t^S \leftarrow \{p \in D^S : q(p) \geq t, a(p) \leq t\}$ 
5    $R_t^S \leftarrow \{p \in R^S : q(p) \geq t, a(p) \leq t\}$ 
6    $D_t^M \leftarrow \{p \in D^M : |V(p)| > 0\}$ 
7    $R_t^M \leftarrow \{p \in R^M : |V(p)| > 0\}$ 
8    $G \leftarrow \text{STATIC}(D_t^S, R_t^S, D_t^M, R_t^M, \text{objective function})$ 
9    $\mathbf{z} \leftarrow$  Execute the maximum-weight bipartite matching algorithm on  $G$ 
10  for  $(d, r, \delta) \in \bar{P}_t : x_{dr\delta} = 1$ :
11    if MatchingType = S&S then:
12      if  $\min[q(d), q(r)] < t + h$ :
13         $\bar{z}_{dr\delta} \leftarrow 1$ 
14         $D_t^S \leftarrow D_t^S \setminus \{d\}$ 
15         $R_t^S \leftarrow R_t^S \setminus \{r\}$ 
16      end if
17    else:
18       $\bar{z}_{dr\delta} \leftarrow 1$ 
19      if  $d \in D_t^S$  then:
20         $D_t^S \leftarrow D_t^S \setminus \{d\}$ 
21      else:
22         $D_t^M \leftarrow V(p) \setminus \{j(\delta)\}$ 
23      end if
24      if  $r \in R_t^S$  then:
25         $R_t^S \leftarrow R_t^S \setminus \{r\}$ 
26      else:
27         $R_t^M \leftarrow R_t^M \setminus \{j(\delta)\}$ 
28      end if
29    end if
30  end for
31  for  $p \in (D_t^S \cup R_t^S)$ :
32    if  $q(p) < t + h$  then:
33      remove the announcement from either  $D_t^S$  or  $R_t^S$ 
34    end if
35  end for
36  for  $p \in (D_t^M \cup R_t^M)$ :
37    if the departure time of the first trip in the ATP is due and the first trip in the ATP is the same as the
      first trip in the ATP for previous iteration then:
38      remove the activity node of the first trip in the ATP from the activity list of  $p$ 
39    end if
```

Algorithm 2: ROLLING HORIZON

```
40   if there is no feasible ATP for  $p$  then:  
41       remove the announcement from either  $D_t^M$  or  $R_t^M$   
42   end if  
43   end for  
44   end for
```

4. Numerical Experiments

4.1. Data and Simulation

To test the performance of the proposed rideshare system, we use tempo-spatial data from the inner and eastern suburbs of Sydney, Australia. Sydney is the most populated city of Australia and capital of the state of New South Wales. As it is depicted in Fig. 5, we consider pools of 20 shopping centres, 20 service centres, and 10 educational centres to be chosen in the experiments.

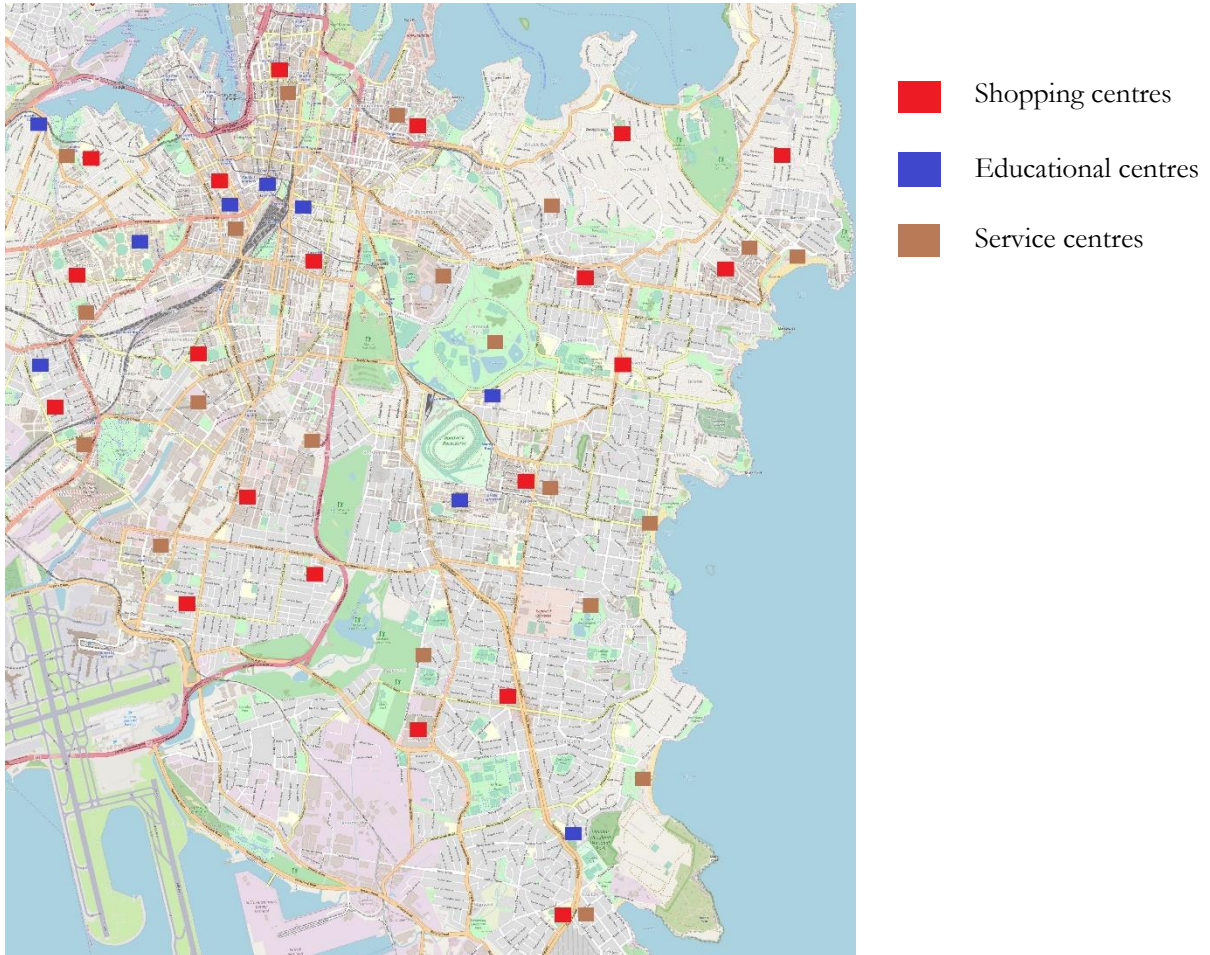


Figure 5 Simulation area

To evaluate the performance of the proposed MaaS model, three variants of the model are used which are abbreviated as follows: DSM (Dynamic Single-trip based Matching), SSM (Static Single-trip based Matching), and DMM (Dynamic MaaS-based Matching). DSM is the same as the models in Agatz et al. (2011) and Najmi et al. (2017) and is discussed in this section for comparison purposes (as the benchmark). In DSM, the classic dynamic matching formulation is

used to find the matching rate. In SSM, all the settings are as the same as for DSM except that all announcements are known prior to the start of the day. In DMM, which is the core in this research, the activity lists are pre-defined but the ATPs are not fixed and should be dynamically rescheduled.

4.2. Computational results

4.2.1 Performance evaluation

The following measures are used in this paper for comparison purposes:

1. Matching rate (MR): the total number of matched driver and passenger announcements divided by the total number of trip announcements;
2. Average total vehicle-kilometres savings (AKS): total kilometres saved as a result of matching algorithms versus the scenario in which all individual trips are performed; and

4.2.2 Dynamic Problem Benchmark

Table 1 summarises the results of running different variants with both objective functions for matching. Comparing to DSM, DMM significantly improves the quality of the objective functions in terms of both MR and AKS. While the improvement in MR for DS is %7.27, the value is %17.22 for NM. Furthermore, the improvement in AKS for DS is %22.22 while the corresponding value for NM is more than %60. Similar to the results in Stiglic et al. (2016) and Najmi et al. (2017), the MR and AKS for the scenario with higher spatial density of participants is significantly higher which shows the important of participation density in ridesharing systems.

Table 1. Performance measures for the dynamic problem benchmark

Scenario	Variant	Model	Matching objective function			
			NM		DS	
			MR	AKS	MR	AKS
Low participation rate	V1	Greedy	-	-	12.23%	1.60%
	V2	DSM	22.18%	2.10%	18.44%	5.22%
	V3	SSM	25.18%	3.45%	21.16%	6.49%
	V4	DMM	26.00%	3.43%	19.78%	6.38%
		Improvement of DMM over DSM	17.22%	63.33%	7.27%	22.22%
high participation rate	V1	Greedy	-	-	12.28%	1.66%
	V2	DSM	24.69%	2.17%	21.55%	6.61%
	V3	SSM	27.31%	3.54%	24.32%	8.10%
	V4	DMM	28.94%	3.60%	23.53%	8.26%
		Improvement of DMM over DSM	17.21%	65.90%	9.19%	24.96%

Table 2 summarises the detailed statistics and performance of DMM with higher participation rate scenario. In the table, the MaaS-based requests incorporates both the requests for drive and ride in the system. Comparing the MaaS-based and single-trip statistics highlights the following issues: First, having the opportunity to choose among multiple destinations plays a key role in changing the utility of participants. While the total number of trips for MaaS-based requests accounts for %44.9 (1956 over 4356) of the total number of trips, they incorporate %60.48 of the total saving in the system. Second, the comparison of success rate for MaaS-based and single trip requests reveals that %39.83 of MaaS-based participants are matched at least once which is

much higher than the corresponding value (about %13) for single-trip based. Furthermore, the MaaS-based participants have obtained higher average distance saving compared to single-trip based participants. The higher chance to find a match and the higher saving rate are incentives for single-based participants to shift to MaaS-based stream.

Table 2. Detailed performance of the rideshare model

Model	Requests	Total No. of requests	Total No. of trips	Total original distances (km)	Distance saving contribution (km)	Saving contribution (%)	Success rate (%)	Saving contribution per trip (km)
DSM	ATP-based trips	600	1956	7279.52	875.53	32.38%	17.58%	0.45
	Single trips - Riders	1200	1200	6634.2	901.25	33.33%	24.41%	0.75
	Single trips - Drivers	1200	1200	6537.23	926.84	34.28%	25.13%	0.77
	Sum	3000	4356	20450.96	2703.62	100		
DMM	MaaS-based requests	600	1956	7279.52	2043.35	60.48%	39.83%	1.04
	Single trips - Riders	1200	1200	6634.2	687.06	20.34%	13.25%	0.57
	Single trips - Drivers	1200	1200	6537.23	648.1	19.18%	13.92%	0.54
	Sum	3000	4356	20450.96	3378.5	100		

Fig. 6 shows the matching rate patterns of MaaS-based requests versus their total number of matched trips which are categorised based on their original distances from home to fixed destinations (work or education purposes) (henceforth referred to as the home-based tour gyration). The tour gyration is important as it can significantly limit the choice of routes for the participants. As it can be perceived from the figure, the MaaS-based requests with the tour gyration of $\leq 2\text{km}$ and $2\text{km} < x \leq 5\text{km}$ are with 51% and 42% probability of failure to find even one match in their tour. The success rates for the tour gyration of $5\text{km} < x \leq 8\text{km}$ and $\geq 8\text{km}$ are about 13% and 9.5% respectively which shows the significant role of tour gyration in finding a match. The MaaS-based requests with the shorter tour gyration (less than 5 km) have on average 42% chance to find exactly one match; while the value drops to around 10% and 0 on average for finding 2 and 3 matches, respectively. Interestingly, the MaaS-based requests with longer tour gyration (greater than 5 km) have more than 50% chance to find at least 2 matches. For the tour gyration of greater than 8 km, there are the chances of 25% and 8%, respectively, to find at least 3 and 4 matches in the tour. These shows the critical role of tour gyration in the MaaS-based ATP generator which can encourage people to participate in MaaS system.

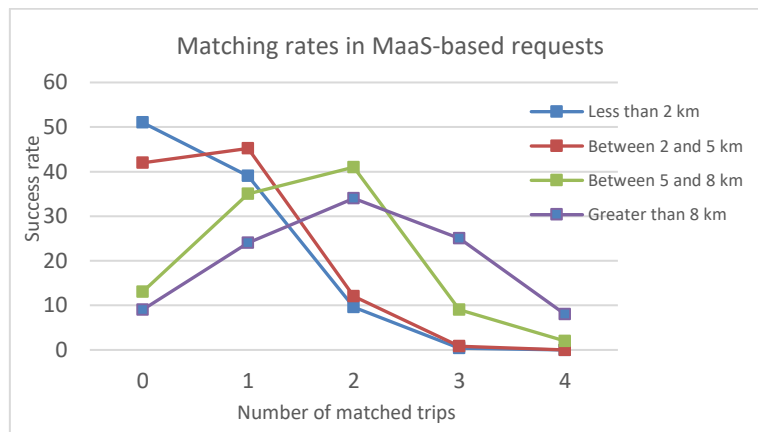


Figure 6 Potential of MaaS-based planning in ridesharing context

5. Conclusion

The presence of MaaS-based announcements in the system has a remarkable impact on the performance of the rideshare systems as not only they increase the success rates of finding a match in the system but also do increase the attractiveness of MaaS mode for the participants. Also, the results showed that the MaaS-based participants in the system, despite their fewer number of participants, incorporate a major portion of the successful participants in finding a match. Furthermore, the participants incorporate the biggest share of total saving contribution in the system. Other than these, we found that the MaaS-based participants have obtained higher average distance saving compared to single-trip based participants. Furthermore, we showed the key role of tour gyration on the performance of MaaS system.

References

- Agatz, N., Erera, A.L., Savelsbergh, M.W.P., Wang, X., 2011. Dynamic ride-sharing: A simulation study in metro Atlanta. *Transp. Res. Part B Methodol.* 45, 1450–1464. <https://doi.org/10.1016/J.TRB.2011.05.017>
- Friedrich, M., Hartl, M., Magg, C., 2018. A modeling approach for matching ridesharing trips within macroscopic travel demand models. *Transportation (Amst)*. 45, 1639–1653. <https://doi.org/10.1007/s11116-018-9957-5>
- Kamargianni, M., Li, W., Matyas, M., Schäfer, A., 2016. A Critical Review of New Mobility Services for Urban Transport. *Transp. Res. Procedia* 14, 3294–3303. <https://doi.org/10.1016/j.trpro.2016.05.277>
- Masoud, N., Jayakrishnan, R., 2017. A decomposition algorithm to solve the multi-hop Peer-to-Peer ride-matching problem. *Transp. Res. Part B Methodol.* 99, 1–29. <https://doi.org/10.1016/J.TRB.2017.01.004>
- Najmi, A., Duell, M., Ghasri, M., Rashidi, T.H., Waller, S.T., 2018. How Should Travel Demand and Supply Models Be Jointly Calibrated? *Transp. Res. Rec. J. Transp. Res. Board.* <https://doi.org/10.1177/0361198118772954>
- Najmi, A., Rey, D., Rashidi, T.H., 2017. Novel dynamic formulations for real-time ride-sharing systems. *Transp. Res. Part E Logist. Transp. Rev.* 108, 122–140. <https://doi.org/10.1016/J.TRE.2017.10.009>
- Najmi, A., Rey, D., Rashidi, T.H., Waller, S.T., 2019. Integrating Travel Demand and Network Modelling: a Myth or Future of Transport Modelling.
- Stiglic, M., Agatz, N., Savelsbergh, M., Gradisar, M., 2016. Making dynamic ride-sharing work: The impact of driver and rider flexibility. *Transp. Res. Part E Logist. Transp. Rev.* 91, 190–207. <https://doi.org/10.1016/J.TRE.2016.04.010>
- Wong, Y.Z., Hensher, D.A., Mulley, C., 2019. Mobility as a service (MaaS): Charting a future context. *Transp. Res. Part A Policy Pract.* <https://doi.org/10.1016/j.tra.2019.09.030>