Learning link marginals from dynamic simulation to calculate sustainable system optimum

Ruiwei CHEN, Cécile BECARIE and Ludovic LECLERCQ

University Gustave-Eiffel, University of Lyon, ENTPE, Lyon, France
{ruiwei.chen, cecile.becarie, ludovic.leclercq}@entpe.fr

Extended abstract submitted for presentation at the hEART 2020 9th Symposium
Sept. 1–3, 2020, Lyon, France

Word count: 2702 words (excluding the references)
February 24, 2020

Abstract

This work focuses on dynamic sustainable system optimum (SSO). When carrying out dynamic SSO assignment, instead of minimizing their own travel cost of each user, we aim to define an SSO status to minimize the on-road traffic emissions of the whole system. The method is based on an system-optimization program. This work proposes a simple, intuitive, simulation-based methodological framework for solving the SSO problem. The main contribution of this paper is to propose a statistically reliable learning process of link marginals that permits to derive link performance function from any DTA simulator. Then the obtained functions can be used to compute network equilibrium including SSO. A test case is carried out on a toy network. Numerical results show that with the help of the proposed method, we can reduce 5.79 % of carbon monoxide (CO) emission on the whole system under SSO equilibrium, when compared with UE.

1 Introduction

Exhaust emissions from road transport are one of the most important sources of air pollution at urban areas. With given traffic demand and infrastructure, various researches are focused on reducing on-road traffic emission via (i) emission pricing, (ii) traffic signal control and (iii) eco-cost-based traffic assignment models, etc. (Chen and Yang, 2012, Ma et al., 2015, Osorio and Nanduri, 2015, Aziz and Ukkusuri, 2012, Wang et al., 2018). This paper focuses on the third topic. We propose a methodological framework to determine the dynamic traffic assignment (DTA) aiming at minimizing the total traffic emissions of the whole system. The obtained equilibrium status is called sustainable system optimum (SSO), in reference to the system optimum (SO) equilibrium where travel time is minimized (Ghali and Smith, 1995, Peeta and Mahmassani, 1995b, Shen et al., 2007, Qian et al., 2012, Lu et al., 2013). While calculation methods for SO have received lots of attention in the literature, very few studies focus on the environmental counter part. In recent years, some pioneering researches propose eco-system optimal DTA models that extend cell-based DTA model (Aziz and Ukkusuri, 2012) and agent-based DTA model(Lu et al., 2016), with emission considerations. Aziz and Ukkusuri (2012) compute CO emission in each cell during a time interval based on a nonlinear regression model (Lin and Ge, 2006, Zhang et al., 2010), combined with computed average space-mean speed in each cell. In their mathematical program, the SO objective function takes (i) CO emission and (ii) travel time into account. In classical SO problem, the path marginal cost (PMC) represents the increase of travel cost to the whole system due to an additional vehicle on that path. At SO state, users are on routes with equal and minimum PMC so that no user can shift to any other paths that have less marginal travel cost (Sheffi, 1985). In SSO problem, we consider path marginal emission (PME) and try to minimize it. Based on this concept, Lu et al. (2016) propose an integer linear program based on a bottleneck model to compute link travel speed. They extended path marginal delay formulation proposed by Shen et al. (2007), Qian et al. (2012), Lu et al. (2013), in order to model path marginal emission. Their numerical results show that when users choose paths under eco-system SO criteria, we can reduce significantly emissions and total travel time of the whole system. However, as mentioned in Lu et al. (2016), there are approximation errors when the bottleneck capacities are highly dynamic, especially when the capacies changes due to the traffic signal and/or the queue spills back from the downstream link. Basically, this approach is only functional for network with slow varying traffic conditions and homogeneous vehicle distribution. In dense and large urban network, many signalised intersections and local heterogeneity prevent from using this approach to determine PME.

The main contribution of this work is that we extend the method of Peeta and Mahmassani (1995b) in order to compute time-dependent link marginal emission (LME) and path marginal emission (PME), based on link performance functions learned from DTA simulation results. The core idea is to learn link performance functions (speed v.s. density) from different network loading and then to derive SSO. Briefly, the methodological framework of this work is as follows:

(1) The dynamic SSO problem is formulated as a minimization program with total emission as objective function. COPERT model (Ntziachristos et al., 2009, EEA, 2016, Lejri et al., 2018) is used for computing on-road traffic emission.

(2) We learn the link performance function, i.e., speed v.s. density, from dynamic simulation with multiple demand scenarios. This process permits us to estimate the link response to most loading scenarios and defines the link cost function for equilibrium calculation.

(3) Based on the link-cost function, we can compute time-dependent LME and PME. We can then solve the optimization problem using the classical fixed-point problem formulation and the MSA algorithm (Peeta and Mahmassani, 1995a, Sbayti et al., 2007). In fact, for the same Origine-Destination (OD) pair and for each iteration of the MSA loop, users can switch to optimal routes with minimum PME in order to converge towards SSO. The solution approach consists of simulation-based heuristic

and iterative algorithm in order to obtain equal PME of all OD pairs of the whole system.

(4) Finally, a case study is carried out on a toy network during 30 minutes to evaluate the efficiency of the method.

The remainder of this paper is organized as follows. Section 2 overviews the general SSO problem, methodological framework, the calculation of link emission, link marginal emission, path emission and parth marginal emission. Section 3 present the case study and numerical results.

2 Method

This section presents mathematical formulation, methodological framework and solution algorithm for modeling and solving SSO problem. Table 1 (on the last page) shows the main notations used in this paper.

2.1 Mathematical program

The SSO refers to the status where no user of the network can reduce the whole system emission by changing its routes. The path emission is obtained by summing up link emission. The latter can be obtained as the product of (i) link traffic flow and (ii) emission factor computed by COPERT model. Similar to the classical SO problem (Sheffi, 1985, Peeta and Mahmassani, 1995b), the mathematical program of SSO problem reads:

$$\min E_{total} = \sum_{\tau}\sum_{o}\sum_{d}\sum_{p} E^{\tau}_{odp} = \sum_{\tau}\sum_{a} E^{\tau}_{a}, \quad \forall o,d,p,\tau,a \tag{1a}$$

subject to

$$\sum_{p} f^{\tau}_{o,d,p} = q^{\tau}_{o,d} \quad \forall o,d,p,\tau \tag{1b}$$

$$f^{\tau}_{o,d,p} \geq 0 \quad \forall o,d,p,\tau \tag{1c}$$

$$x^{\tau}_{a} = \sum_{\tau}\sum_{o}\sum_{d}\sum_{p}(f^{\tau}_{o,d,p} \times \delta^{\tau a}_{odp}) \quad \forall o,d,p,\tau,a \tag{1d}$$

In Equation (1a), E_{total} is used to evaluate the total emission of the whole system. E^{τ}_{odp} and E^{τ}_{a} represent path emission and link emission, respectively. Equation (1b) and Equation (1c) are contraints that the traffic flows of the DTA problem should respect. Equation (1d) explains the relation between the link traffic flow and path traffic flow, with the indicator function $\delta^{\tau a}_{odp}$ which: (i) equals to 1 if link a belongs to the path p departing at time τ from o to d, and (ii) equals to 0 otherwise.

The total duration of the simulation period $\mathcal{H}$ is divided to discret assignment periods $\mathcal{T}$ that is indexed by τ: $\tau \in [\tau_0, \tau_0 + \Delta T, \tau_0 + 2\Delta T, \ldots, \mathcal{H}]$. We assume that the demand is constant over an assignment period for all OD pairs. Furthermore, traffic condition including TT are averaged over the period to transform the DTA problem into a quasi-static one. During $\mathcal{T}_i$, the program (1) can be reformulated as Equations (2) based on marginal emissions in order to transform the optimization problem as a fixed-point problem (Sbayti et al., 2007, Lu et al., 2016).

$$\pi_{p,\tau}(PME_{p,\tau} - PME^*_{p,\tau}) = 0; \quad \forall o, d, p \in \mathcal{P}_{od,\tau}, \tau \in \mathcal{T} \tag{2a}$$

$$PME_{p,\tau} - PME^*_{p,\tau} \geq 0; \quad \forall o, d, p \in \mathcal{P}_{od,\tau}, \tau \in \mathcal{T} \tag{2b}$$

$$\pi_{p,\tau} \geq 0; \quad \forall o, d, p \in \mathcal{P}_{od,\tau}, \tau \in \mathcal{T} \tag{2c}$$

We firstly present in subsection 2.2 the optimization process embedded with MSA method for searching the least PME paths. Then we present in detail the calculation of LME and PME in subsection 2.3 and subsection 2.4.

2.2 Equilibrium calculation for one assignment period

The optimization process embeds two layers (Sbayti et al., 2007): (i) the outer layer for updating the candidate least PME paths set of all OD pairs and (ii) inner layer to update the traffic assignment on cadidate paths using classic MSA method (Peeta and Mahmassani, 1995b) based on the calculation of PME. Figure 1 illustrates the simulation-based two-layer optimization process. In this work, we use Symuvia as a trip-based DTA simulator for computing variables that are used to solve the program 1. Symuvia is developed by the LICIT laboratory at Gustave Eiffel University. It is a microscopic simulator based on a Lagrangian discretization of the LWR model (Leclercq et al., 2007). The convergence check of the outerloop (Step 4) is based on the total gap on PME during the corresponding time interval. Regarding the innerloop convergence check, we rely on the total PME gap of all paths of the path set. The convergence check of the outerloop (Step 4 in Figure 1) is based on the total gap on PME during the corresponding time interval. Regarding the innerloop convergence check, we rely on the total PME gap of all paths of the path set. We explain in detail the calculations of LME and PME in the following subsection 2.3.

2.3 Emission and marginal emission of links and paths

2.3.1 Link level emission

The key step is to calculate E_a in Equation (1a). It is calculated as $E_a = TTD^\tau_a \times e$, which represents the product of (i) total traveled distance of all vehicles on a and (ii) emission factor $e(v)$. On the one hand, based on the assumption that the traffic demand during $[\tau_0, \tau_0 + \Delta T]$ is constant, the traffic flow is obtained as $x^\tau_a = k^\tau_a \times v^t_a(n^t_a)$. The total traveled distance on link a reads: $TTD^\tau_a = L_a x^\tau_a \Delta T = L_a \Delta T k^\tau_a v^\tau_a = \Delta T n^\tau_a v^\tau_a$. On the other hand,

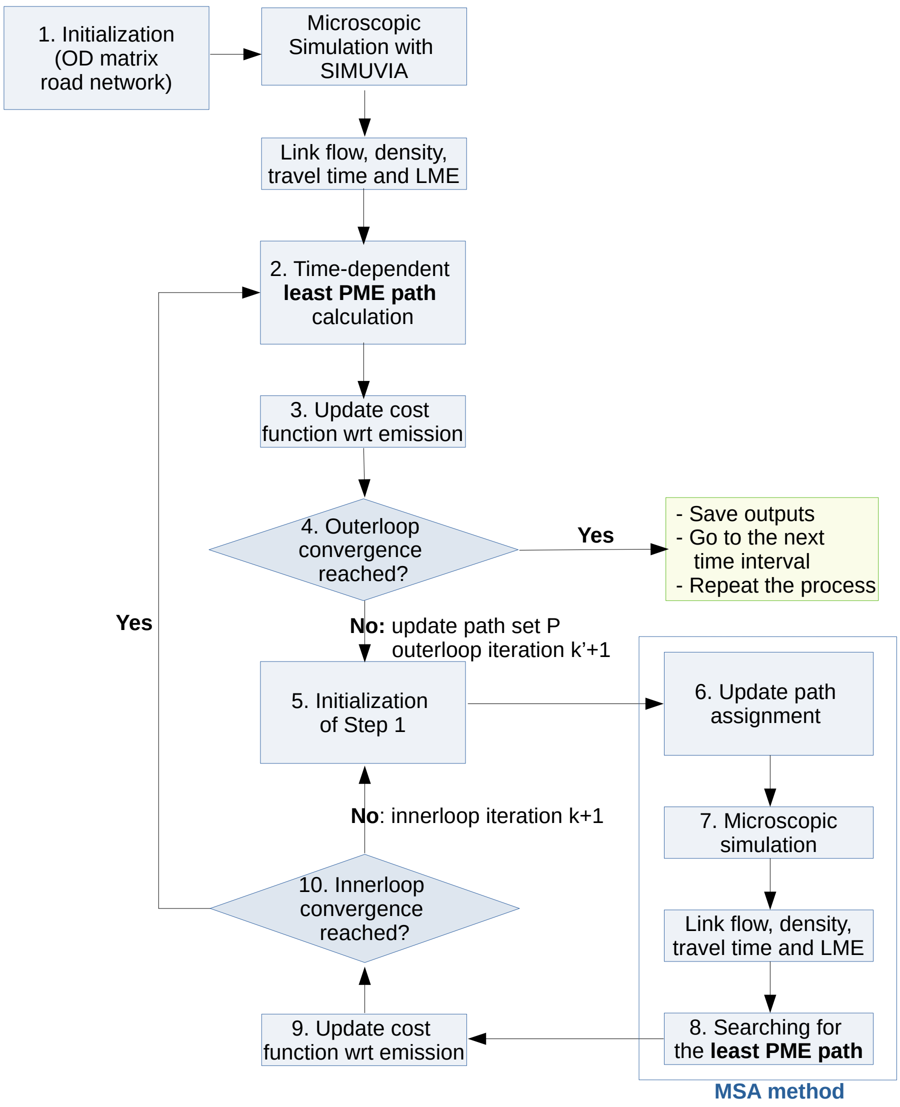

Figure 1: Flow chat of optimization process

according to COPERT model, the emission factor can be modeled as a function of average speed: $e_a(v_a^\tau)$. Therefore, the link emission E_a^τ is calculated as:

$$E_a^\tau = \Delta T n_a^\tau v_a^\tau e_a(v_a^\tau). \tag{3}$$

The link average speed can be obtained via a learning process, in order to get a map from n to v: $v_z^\tau(\cdot) : n_a^\tau \mapsto v_a^\tau$. We focus on the spatially-averaged number of vehicls because it represent the link density ($k_a = \frac{n_a}{L_a}$), yet it is easy to be obtained via DTA simulator: $n^\tau = \frac{TTT^\tau}{\sigma}$. Therefore, at link level during a fixed time interval $\mathcal{T}$, link emission can be considered as a function of n_a^t: $E_a^t = F_a(n_a^t)$. F_a^t depends only on (i) time-dependent n on link a and (ii) link characteristics of a (link length, speed limit, jam density, traffic light cycle, etc.). For simplicity, in the following parts of this subsection, we focus on link-level formulations on a during the time interval $[\tau, \tau + \Delta T]$ and ignore the subscript a and suscript τ. To ensure that the solution of the mathematical program exits, the first and second derivative of $F(n)$ should be non-negative, i.e., $\frac{\partial F(n)}{\partial n} \geq 0$, $\frac{\partial^2 F(n)}{\partial n^2} \geq 0$. Nonetheless, we cannot ensure the nonnegativity of $\frac{\partial F(n)}{\partial n}$ since the term $n\frac{dv}{dn}$ is negative as shwon in Equation (4).

$$\frac{\partial F(n)}{\partial n} = \frac{\partial nv(n)e(v(n))}{\partial n} = e(v)(v + n\frac{dv}{dn}) + nv\frac{\partial e(v)}{\partial n}. \tag{4}$$

Furthermore, computationally speaking, we cannot solve the problem over the full time horizon $\mathcal{H}$. Therefore, we calculate the optimal solution over $\mathcal{T}$ where path flow and traffic condition can be assumed constant (quasi-static approximation). In the latter case, the link marginal definition may drive the system to a solution where link speed are very small as it reduces the travel distance and thus emission. It appears that the term in Equation (3) is not the most adequate formulation for quasi-static simulations. We then assume that all vehicles have to travel the full link distance after some time. This assumption seems reasonable since we set the assignment period ΔT to 15 min. Then TTD_a is calculated as $L_a \times n_a$. We can then reformulate the link emission function depending on n, and at the same time, respects the definition of link traffic emission. The link emission is calculated by the product of (i) total traveled distance of all vehicles on link a and (ii) the emission factor, as shown in Equation (5a). The corresponding link marginal emission is then calculated by Equation (5b).

$$\hat{F}(n) = L \times n \times e(v). \tag{5a}$$

$$LME = \frac{\partial \hat{F}(n)}{\partial n} = L \times (e(v(n)) + n\frac{\partial e(v)}{\partial n}) = L \times (e(v(n)) + n\frac{de(v)}{dv}\frac{dv(n)}{dn}) \tag{5b}$$

There are two keys to ensure the nonnegativity of $\frac{\partial^2 \hat{F}(n)}{\partial n^2}$: (i) to choose a reasonable emission factor model $e(v)$ with $\frac{d^2 e(v)}{dv^2} \leq 0$, and (ii) to build link performance function $v(n)$ with non-positive second derivative. On the one hand, based on COPERT model, $e(v)$ is approximated as a polynomial regression of v. In urban areas, the link average speed is usually limited under 60 km h^{-1}, which ensures the condition of $\frac{d^2 e(v)}{dv^2} \leq 0$ (Ntziachristos et al., 2009, EEA, 2016, Lejri et al., 2018). On the other hand, $v(n)$ can be learned from data obtained from DTA simulators.

2.3.2 Path marginal emission

To compute PME, we take similar assumptions made in (Peeta and Mahmassani, 1995b), stating that the PME is the sum of time-dependent LME. Therefore, based on Equation (5b), the PME can be calculated as

$$PME^{\tau}_{o,d,p} = \sum_{\tau}\sum_{o}\sum_{d}\sum_{p} LME^{\tau}_{a}(n^{\tau}_{a})\delta^{\tau a}_{odp}, \tag{6}$$

where $\delta^{\tau a}_{odp} = 1$ if the link a is on the path p of the OD pair od with departure time τ. Otherwise, $\delta^{\tau a}_{odp} = 0$.

2.4 Learning of link performance function v-n

The main objective is to define the $v - n$ curve of all links based on data obtained from microscopic DTA simulator. We assume that $v(n)$ represents the intrinsic characteristic of link a. It represents the response of the speed to the level of loading (n) at link level. Here, we assume that we can estimate a single valuated decreasing function for each link of the network from previews simulations (learning process). There are three main concepts for building the $v - n$ curve, learned from DTA simulator results.

(1) We should make sure that the regression curve is monotone on n, and the second derivative should be non-positive: $\frac{d^2v}{dn^2} \leq 0$.

(2) We need enough observations of (n, v) on links to obtain good quality fits. This requires a thorough and comprehensive design of simulation experiments deriving the learning. To this end, we introduce a full OD matrix with various demand profiles, in order to reach a relatively congested status on the network, yet without gridlock. The simulations are carried out with sufficient iterations to ensure that a maximum number of links have enough sample points (n, v). Furthermore, we classify different values of n into K classes and update the statistical mean values and standard deviations of TTD and TTT of each class. This helps us to get statistically reliable link average speed computed by $\frac{TTD}{TTT}$. This also enables us to eliminate points with extreme values of TTD and TTT.

(3) We rely on quadratic regression to approximate the $v - n$ curve based on all the mean values of n and v in each class. Figure 2 illustrates a $v - n$ curve learned from simulation results.

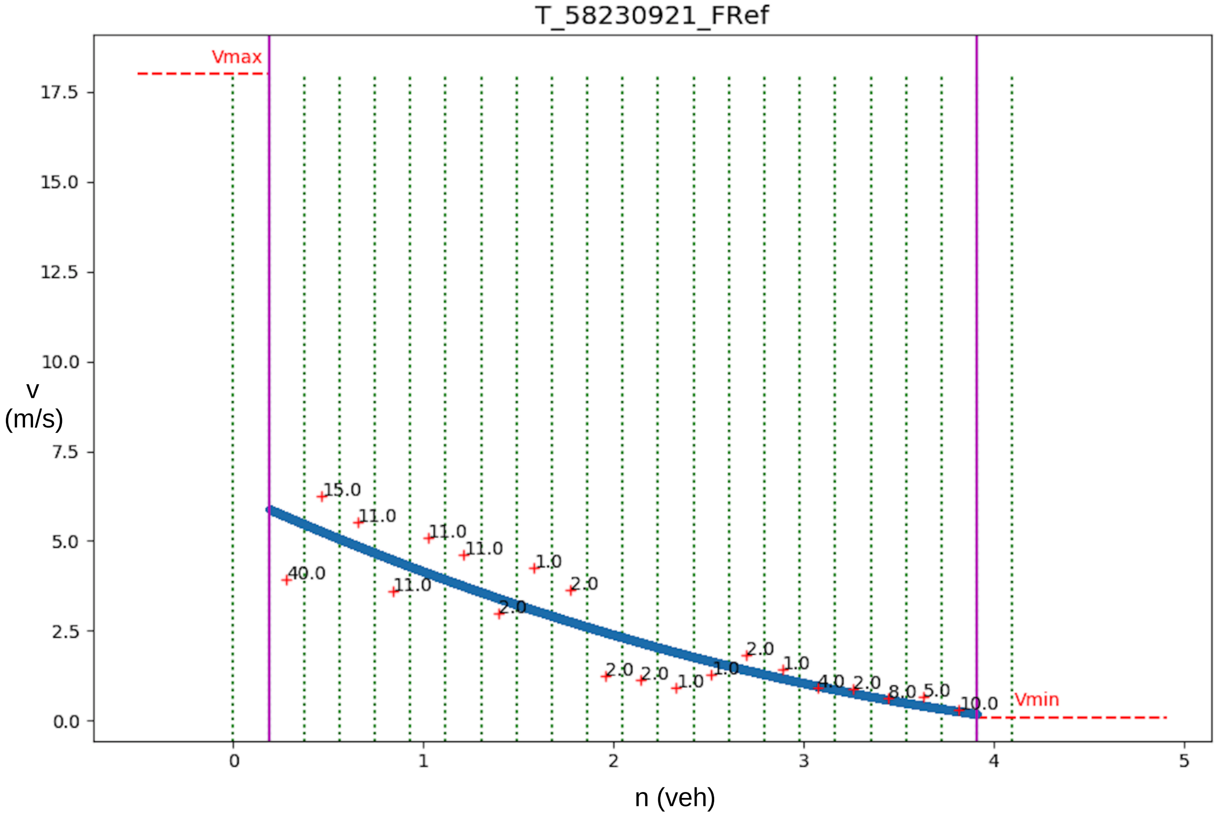

Figure 2: Link $v-n$ curve earned from simulation results. v_{min} and v_{max} are minimum and maximum link average speed. The red points are obtained from sample points (n_k, v_k). The numerical number beside each point denotes the total number of points in each class k.

2.5 Final expression for the LME

We devide the values of samples of n into K classes on each link. The outputs of the method from Section 2.4 give sample points (n_k, v_k) of each class k. We then carry out a quadratic regression between all sample points (n_k, v_k) and the final learned function $v(n)$ reads:

$$v(n) = a \times n^2 + b \times n + c, \tag{7}$$

where the values of a and b are bounded in order to respect $\frac{d^2v}{dn^2} \leq 0$. Therefore, at time t, the calculation of LME reads:

$$LME^t = \frac{\partial E(n)}{\partial n} = \frac{\partial Lne(v)}{\partial n} = L \times (e(v^t) + n^t \frac{de(v)}{dv}|_{v=v^t} \frac{dv(n)}{dn}|_{n=n^t}). \tag{8}$$

With the help of COPERT formula (Ntziachristos et al., 2009, EEA, 2016, Lejri et al., 2018), the generalized formulus for computing the emission factor is considered as a 3 order polynomial,

$$e(v) = p_3 \times v^3 + p_2 \times v^2 + p_1 \times v + p_0, \tag{9}$$

where p_0, p_1, p_2 and p_3 are scalar coefficients, obtained from empirical regression of COPERT model (Lejri et al., 2018). By substituting Equation (9) and Equation (7) into Equation (8). We then get Equation (10) for computing link level emission in function of n,

$$E(n) = L \times (a_6 \times n^6 + a_5 \times n^5 + \ldots + a_2 \times n^2 + a_1 \times n + a_0). \tag{10}$$

3 Simulation results

We carry out DTA simulation under SSO equilibrium condition on a toy network during 30 minutes. The reference case is DTA simulation under UE condition with the same networks and OD matrix. We then compare the total emissions of the whole network under UE and SSO condition.

The test network has 5 links with one OD pair and one traffic light, presented in Figure 3 (left). Figure 3 (right) presents the traffic demand rate (in $veh\ h^{-1}$). We carry out two DTA simulations under UE and SSO equilibrium condition, respectively. The total simulation horizon $\mathcal{H} = 30$ min. $\mathcal{H}$ is divided into 3 successive assignment periods of 10 min. The results are shown in table 2. Results show that when users choose their path based on SSO condition, the total emission of CO is 5.79 % less than the total emission when users are traveling based on UE condition.

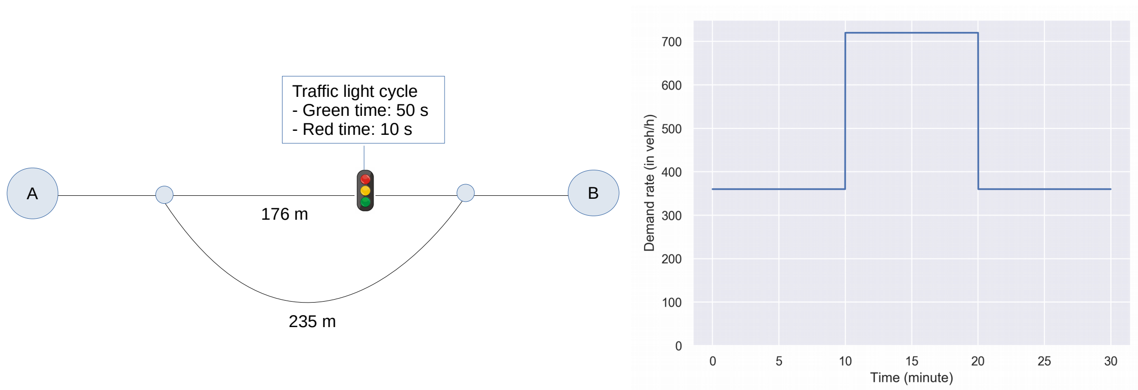

Figure 3: Input for case study on a simple network. Left: small network with traffic light, right: traffic demand rate from node A to node B.

Table 2: Total computed CO emissions on the network in Figure 3. The difference is calculated with respect to emissions obtained when users travel under UE condition.

Equilibrium type	Total CO emissions (in g)	Relative difference of $\Delta E = \frac{E_{\text{SSO}} - E_{\text{UE}}}{E_{\text{UE}}}$
UE	24672	–
SSO	23244	$-5.79\,\%$

4 Conclusions and perspectives

This work proposes a methodological framework to solve the dynamic sustainble system optimum (SSO) problem. The SSO is formulated as an optimization program in terms of minimizing the total emissions of the whole system. It is then transformed into a fixed-point problem using classical Beckman transformation and solved with a MSA algorithm. The key contribution of this paper are twofold. *First*, we provide new definitions for link marginals that now address the SSO equilibrium instead of the classical SO problem. *Second*, the calculation of link marginals with dynamic simulation has always been a challenge which is not well-documented in the literature. Here, we propose a new framework to learn the link performance function from multiple simulations on the same network with different loading scenarios. The implementation of the learning process is simple and can be coupled with other networks and/or DTA simulators.

To evaluate the method efficiency we carry out a case study on a simple network during 30 minutes. The proposed method solves the SSO problem and success to obtain a reduction of total emissions from UE status to SSO equilibrium. At SSO status, the total emissions of CO is 5.79 % less than the CO emissions when users choose their routes based on classic user equilibrium (UE). The authors are working on case studies on a real-world network.

Keywords: Sustainable system optimum (SSO), dynamic traffic assignment (DTA), link marginal emission, link performance function, learning process, COPERT

5 Acknowledgment

This project has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation program (grant agreement No 646592 – MAGnUM project).

References

HM Abdul Aziz and Satish V Ukkusuri. Integration of environmental objectives in a system optimal dynamic traffic assignment model. *Computer-Aided Civil and Infrastructure Engineering*, 27(7):494–511, 2012.

Linxi Chen and Hai Yang. Managing congestion and emissions in road networks with tolls and rebates. *Transportation Research Part B: Methodological*, 46(8):933–948, 2012.

EEA. *EMEP/EEA air pollutant emission inventory guidebook - 2016*. European Environment Agency, 2016. ISBN 978-92-9213-806-6. URL https://www.eea.europa.eu/publications/emep-eea-guidebook-2016.

MO Ghali and MJ Smith. A model for the dynamic system optimum traffic assignment problem. *Transportation Research Part B: Methodological*, 29(3):155–170, 1995.

Ludovic Leclercq, Jorge Andres Laval, and Estelle Chevallier. The lagrangian coordinates and what it means for first order traffic flow models. In *Transportation and Traffic Theory 2007. Papers Selected for Presentation at ISTTT17Engineering and Physical Sciences Research Council (Great Britain) Rees Jeffreys Road FundTransport Research FoundationTMS ConsultancyOve Arup and Partners, Hong KongTransportation Planning (International) PTV AG*, 2007.

Delphine Lejri, Arnaud Can, Nicole Schiper, and Ludovic Leclercq. Accounting for traffic speed dynamics when calculating copert and phem pollutant emissions at the urban scale. *Transportation Research Part D: Transport and Environment*, 63:588–603, 2018.

Jie Lin and YE Ge. Impacts of traffic heterogeneity on roadside air pollution concentration. *Transportation Research Part D: Transport and Environment*, 11(2):166–170, 2006.

Chung-Cheng Lu, Xuesong Zhou, and Kuilin Zhang. Dynamic origin–destination demand flow estimation under congested traffic conditions. *Transportation Research Part C: Emerging Technologies*, 34:16–37, 2013.

Chung-Cheng Lu, Jiangtao Liu, Yunchao Qu, Srinivas Peeta, Nagui M Rouphail, and Xuesong Zhou. Eco-system optimal time-dependent flow assignment in a congested network. *Transportation Research Part B: Methodological*, 94:217–239, 2016.

Rui Ma, Xuegang Jeff Ban, and WY Szeto. Emission modeling and pricing in dynamic traffic networks. *Transportation Research Procedia*, 9:106–129, 2015.

Leonidas Ntziachristos, Dimitrios Gkatzoflias, Chariton Kouridis, and Zissis Samaras. Copert: a european road transport emission inventory model. In *Information technologies in environmental engineering*, pages 491–504. Springer, 2009.

Carolina Osorio and Kanchana Nanduri. Urban transportation emissions mitigation: Coupling high-resolution vehicular emissions and traffic models for traffic signal optimization. *Transportation Research Part B: Methodological*, 81:520–538, 2015.

Srinivas Peeta and Hani S Mahmassani. Multiple user classes real-time traffic assignment for online operations: a rolling horizon solution framework. *Transportation Research Part C: Emerging Technologies*, 3(2):83–98, 1995a.

Srinivas Peeta and Hani S Mahmassani. System optimal and user equilibrium time-dependent traffic assignment in congested networks. *Annals of Operations Research*, 60(1):81–113, 1995b.

Zhen Sean Qian, Wei Shen, and HM Zhang. System-optimal dynamic traffic assignment with and without queue spillback: Its path-based formulation and solution via approximate path marginal cost. *Transportation research part B: methodological*, 46(7):874–893, 2012.

Hayssam Sbayti, Chung-Cheng Lu, and Hani S Mahmassani. Efficient implementation of method of successive averages in simulation-based dynamic traffic assignment models for large-scale network applications. *Transportation Research Record*, 2029(1):22–30, 2007.

Yosef Sheffi. Urban transportation networks. 1985.

Wei Shen, Yu Nie, and H Michael Zhang. On path marginal cost analysis and its relation to dynamic system-optimal traffic assignment. In *Transportation and Traffic Theory 2007. Papers Selected for Presentation at ISTTT17Engineering and Physical Sciences Research Council (Great Britain) Rees Jeffreys Road FundTransport Research FoundationTMS ConsultancyOve Arup and Partners, Hong KongTransportation Planning (International) PTV AG*, 2007.

Yi Wang, WY Szeto, Ke Han, and Terry L Friesz. Dynamic traffic assignment: A review of the methodological advances for environmentally sustainable road transportation applications. *Transportation Research Part B: Methodological*, 111:370–394, 2018.

Yunlong Zhang, Jinpeng Lv, and Qi Ying. Traffic assignment considering air quality. *Transportation Research Part D: Transport and Environment*, 15(8):497–502, 2010.

Table 1: Basic notations in this work

Notations	Explanations
o, d, p, τ	Index for OD pxair from node o to node d via path p with departure time τ
t	Current time
a	Index of a link in a road network
$\mathcal{P}_{od,\tau}$	Set of paths from node o to d departing at τ
$\mathcal{P}^*_{od,\tau}$	Set of optimal paths (with minimum path marginal emissions) from node o to d departing at τ
$\mathcal{H}$	Total simulation duration
$\mathcal{T}$	Discret assignment period
$\mathcal{T} + \Delta T$	Discret prediction horizon
ΔT	Length of discretized traffic assignment period
σ	Time step of microscopic traffic simulator to export outputs
q^{τ}_{od}	Traffic demand from origin node o to destination node d departing at time τ
E^{τ}_{odp}	Emission due to traffic departing from node o at time τ to node d via path p
f^{τ}_{odp}	Path traffic flow from origin node o to destination node d departing at time τ, using path p
x_a	Traffic flow on link a
e	Emission factor
$\delta^{\tau ta}_{odp}$	Time-dependent link-path incidence indicator
L_a	Length of link a
v_a	Link average speed
k_a	Traffic density on link a
κ_a	The congestion traffic density on link a
TTD	Total traveled distance
TT	Travel time
TTT	Total travele time
n_a	Spatially-averaged number of vehicle on link a. $n_a = \frac{TTT_a}{\Delta T}$ and $n_a = L_a \times k_a$
K	The total number of classes of values of n on link a
$\pi_{p,\tau}$	Number of users departing at τ using the path p
LME	Link marginal emission
PME	Path marginal emission