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Optimal freight loading zones: a graph-theoretic approach

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Abstract The design of Freight Loading Zones (FLZ) aims to identify the appropriate number and locations of FLZ. We propose a new methodology based on solving the well-known optimization problem of minimum vertex cover on a predefined hypergraph for the design of FLZ. The model is applied on Paris city urban network and have been proven to produce an enhanced saving in terms of average delivery walking distances by around 46% compared to the actual distribution of FLZ.

Keywords Urban freight · Freight Loading Zone · Truck parking design · Graph theory

1 Introduction

Parking spots and especially the search for on-street available spots are key determinants of the urban mobility and are now strongly related to the transportation policies that cities want to deploy. As the mobility is becoming more and more multi-modal, parking spots can be dedicated to various uses: private vehicles, car-sharing stations, bike-sharing stations, terraces, etc. The literature is already large on the studies analyses of these different approaches, *e.g.* Ortuzar and Willumsen (2001) [5], Marcucci *et al.* (2015) [10], Nourinejad and

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Roorda (2017) [12], Muñuzuri *et al.* (2017) [11], Alho *et al.* (2018) [3], Letnik *et al.* (2018) [9]. One of the most common usage is to dedicate on-street parking spots to freight in order to facilitate the urban good transportation by saving times in the spot search and in the pickup/delivery operations.

Even if this so-called Freight Loading Zones (FLZ) exist in almost all the cities over the world, they clearly still deserve attention of the research community. Especially, we can identify, in the both academic and practical literature, only a few studies that aims at determining optimal location for FLZ deployment. This problem reveals two main questions: how many parking spots must be dedicated to freight and where should they be located? Some existing approaches are of particular interest to answer to these questions. Muñuzuri *et al.* (2017) [11] estimated the number and the location of FLZ on a given street. More particularly, the authors proposed an two-step methodology: estimate (i) the number of FLZ and (ii) its locations. Their case study was four streets in Seville, Spanish. However, Muñuzuri *et al.* considered the number of FLZ as a static parameter of their model. More precisely, they calibrated the number of FLZ according to a CERTU procedure [4]. In the same vein, Tamayo *et al.* (2017) [17] considered the calibration of the number of FLZ by the policy maker knowledge. Nevertheless, the policy maker may not be aware of the proper value or the optimal number of FLZ to calibrate. Letnik *et al.* (2018) [9] proposed a dynamic management of FLZ based on the fuzzy c-means clustering of commercial establishments. Their case study was the Lucca city center in Italy. One inherent limitation of their method is the high sensitivity of the c-means algorithm to the input parameters, such as the initial cluster centers, [19] similarly as the k-means algorithm. Pinto *et al.* (2019) [14] identified the number, the location and the size of FLZ. The authors use the concept of *radius*, which represents the longest distance that delivery driver is willing to walk from the FLZ to the given commercial establishment. This later was used also by [9,17]. They addressed the problem as a long-term facility location problem. Their objective is to minimize the number of FLZ and to determine the sizing of FLZ as well. Their case study was a central district of the city of Bergamo, Italy.

To determine the optimal number and location of FLZ for any given network, we propose a novel approach formulated as an optimization problem. Notice that the number of parking spots to dedicate to freight is a variable of our modeling. Some papers considered the number of FLZ as a parameter of the model, *e.g.* Muñuzuri *et al.* [11] and Tamayo *et al.* [17]. To our best knowledge, no studies have considered the number of FLZ endogenously. More particularly, we address an optimization of two objective functions in only one step. Muñuzuri *et al.* [11] solved the location-allocation problem in the second step of their method. In this paper, the number and the location of FLZ are determined simultaneously with a unique optimization problem. To this end, we model the situation with an hypergraph, linking potential FLZ and commercial establishment, that allow to solve the problem as a classical minimum vertex cover problem. A very simple algorithm is proposed and tested in the network of Paris, France. Based on various values of the radius, it makes it possible to calculate and compare the optimal number and locations of FLZ but also to determine the optimal deployment according a given number dedicated parking spots, for example the actual number of FLZ in Paris.

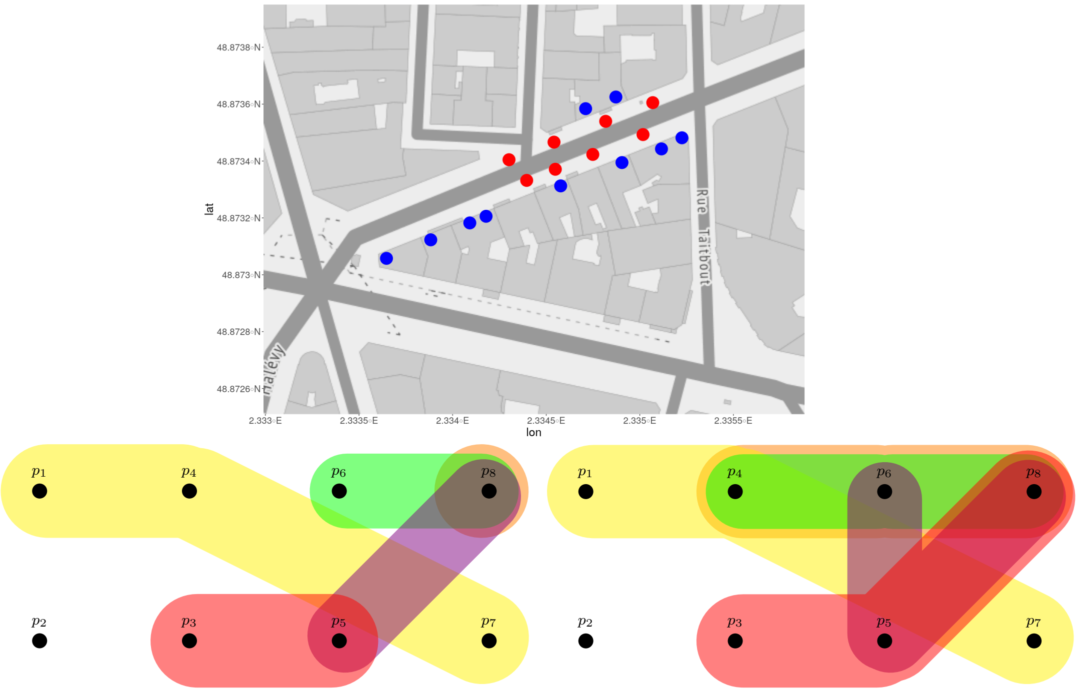
The paper is organized as follows. Section 2 presents the methodology, while section 3 describes data used for our case study. Section 4 shows some experimental results for the case study of Paris city. Finally, section 5 concludes the study and suggests furthers research.

2 Methodology

The proposed algorithmic framework aims to simultaneously find the optimal FLZ number and their locations. We address the problem by first constructing a weighted undirected hypergraph. Then, we model and solve an optimization based on the minimum vertex cover problem on this hypergraph, as said before. We denote the set of existing parking spots and commercial establishments in the studied area respectively by $\mathcal{P}$ and $\mathcal{S}$. The distance between each couple $(p, s) \in \mathcal{P} \times \mathcal{S}$ is computed. This distance is supposed to be symmetric, since it represents the walking distance the delivery operator is willing to cross between p and s. We denote by $D = (d_{ps})_{(p,s)\in\mathcal{P}\times\mathcal{S}}$ the walking distance matrix. The hypergraph is characterised by vertices corresponding to the set $\mathcal{P}$, and by undirected hyperedges given by $\mathcal{S}$ in the following way. For each commercial establishment $s \in \mathcal{S}$, the set of parking spots p that are of interest, denoted by $\mathcal{P}_s$, are those selected from $\mathcal{P}$ within a pre-specified distance parameter, called radius R, *i.e.* $\mathcal{P}_s = \{p \in \mathcal{P} : d_{sp} \leq R\} \subseteq \mathcal{P}$. To provide a starting cover for their optimizations, Pinto *et al.* [14] and Tamayo *et al.* [17] also make use of a radius parameter R. Therefore, for each commercial establishment s, we define an hyperedge[1] equal to the subset $\mathcal{P}_s$. The proposed hypergraph is then $\mathcal{H} = (\mathcal{P}, \mathcal{E})$, with $\mathcal{E} = \{\mathcal{P}_s : s \in \mathcal{S}\}$ is the set of edges. Fig. 1 shows an example of hypergraph modelization applied on the first section of Lafayette Street in Paris. In this example, 8 parking spots and 10 commercial establishments are considered. Hyperedges induced by the commercial establishments are determined conditional on the value of R. For instance, if $R = 0$ meter, no hyperedge subsists, while in case of $R = 1000$ meters, 10 similar hyperedges are generated, which is the maximum number of edges. Each one of these hyperedges actually includes all the 8 parking spots. The example of Fig. 1 is given for $R = 20$ and 30 meters.

Each commercial establishment receives an average delivery number per week that lasts for an average time. This average demand highly influences the usage of the FLZ. For instance, pharmacies and drugstores usually receive a

[1] In hypergraphs, edges are called hyperedges, and they correspond to edges with any number of endpoints, thus they are a subset of the vertex set.



R = 20 meters R = 30 meters

Fig. 1: An example of constructing the hypergraph $\mathcal{H} = (\mathcal{P}, \mathcal{E})$ on the first section of Lafayette Street in Paris. We notice 8 parking spots (red dots) and 10 commercial establishments (blue dots). The set of vertices is given by the parking spots: $\mathcal{P} = \{p_1, p_2, ..., p_8\}$. The set of hyperedges on the other hand depends on the value of the radius R. If $R = 20$ meters, five commercial establishments can be selected with the corresponding set of hyperedges: $\mathcal{E} = \big\{\{p_8\}, \{p_1, p_4, p_7\}, \{p_6, p_8\}, \{p_5, p_8\}, \{p_3, p_5\}\big\}$. If $R = 30$ meters, the same five commercial establishments are selected but with different underlying hyperedges: $\mathcal{E} = \big\{\{p_8, p_4, p_6\}, \{p_1, p_4, p_7\}, \{p_6, p_8, p_4\}, \{p_5, p_8, p_6\}, \{p_3, p_5, p_4\}\big\}$. Notice that the parking spot p_2, which actually corresponds to the extreme left red dot, is not used at all due to the long crossing distance.

high number of deliveries per week that are generally short[2], at the opposite of clothing retail stores which have low number of deliveries that may last long. There are several ways to quantify the significance of a selection of parking spots in function of commercial establishments in their surroundings. We chose one criterion to start with. It relates the significance of a parking spots p to the weighted distance to commercial establishments s in its vicinity, *i.e.* $d_{ps} \leq R$, where weights are given by the average weekly number of deliveries w_s. If we denote by $W = (w_s)_{s \in \mathcal{S}}$ the vector of demands, the criterion formulation will

[2] For France, national averages given by CERTU are equal to 31.8 deliveries per week for 2 minutes in general [15].

be,

$$c(p, D, W) = \sum_{s \in \mathcal{S},\, d_{sp} \leq R} w_s \, d_{sp} \Big/ \sum_{s \in \mathcal{S},\, d_{sp} \leq R} w_s, \tag{1}$$

which represents the expected value of walking distances for the parking spots p according to the distribution of commercial establishments' demands in its vicinity. In this paper, we disregarded the average delivery times, although it would be interesting to add them later. The formulation of the optimization problem we solve is as follows:

$$\min_{x_p} \quad \sum_{p \in \mathcal{P}} c(p, D, W) \, x_p \tag{2}$$

$$\text{s.t.} \quad \sum_{p \in \mathcal{P}_s} x_p \geq 1, \quad \forall s \in \mathcal{S} \tag{3}$$

$$x_p \in \{0, 1\}, \quad \forall p \in \mathcal{P} \tag{4}$$

This problem actually coincides with the basic formulation of the minimum vertex cover problem for the hypergraph $\mathcal{H} = (\mathcal{P}, \{\mathcal{P}_s : s \in \mathcal{S}\})$, where the weight of vertices is provided by the function $c(p, D, W)$. The minimum vertex cover problem is a classical problem in graph theory and computer science in general. Its goal is to find the smallest subset of the graph vertex set, such that each edge has at least one of its endpoints in this subset. The weighted version targets the subset of vertices with the smallest sum of weights that cover all edges. This problem has been proven to be $\mathcal{NP}$-complete in 1972 by Karp [8], and has several practical applications, like in the area of computational biology [2]. The objective function of our formulation (2) corresponds to the weighted version. As for (3) and (4) constraints, they respectively impose the coverage of all edges of $\mathcal{H}$, which as said before are derived from the set of commercial establishments $\mathcal{S}$, and states the binary variables, which indicate which vertex is belonging to the minimal cover. For comparison matters, we also solve the following unweighted version:

$$\min_{x_p} \quad \sum_{p \in \mathcal{P}} x_p \quad \text{s.t.} \quad (3) \quad \text{and} \quad (4).$$

As said before, we did not consider times of deliveries by the operators, our model is thus macroscopic in nature. The underlying assumptions adopted in the modeling and in the study case application are as follows: (i) deliveries at the same parking spot do not happen at the same time, (ii) FLZ are supposed to be solely dedicated to deliveries, and (iii) the size of FLZ is large enough to park all used delivery vehicles. These assumptions are not realistic in practice, but they allow us to focus on the optimization task of determining the FLZ number and locations.

For the solving approach, we rely on the approximation algorithm provided by Ramadan *et al.* (2004) [16], who applied it to the protein complex network modeled as an hypergraph in order to find near-optimal covers. This algorithm is greedy in nature, and conceptually similar to the algorithm proposed by Johnson, Chvatal and Lovasz [18] to solve the set cover problem. It is described in the following pseudocode. The idea is to choose at each iteration the vertex with the minimum cost, defined by its weight distributed over the set of the not yet covered hyperedges, to include it in the cover. This algorithm yields good results [16].

Algorithm 1 The approximate algorithm for the minimum weight vertex cover of hypergraphs, given by Ramadan *et al.* (2004) [16]

$i := 1$ (iteration number)
$C := \emptyset$ (optimal cover)
$F_i := F$ (set of uncovered hyperedges)
while $F_i \neq \emptyset$ **do**
    for $v \in V \setminus C$ **do**
        $v_i = \arg\min \frac{w(v)}{|adj(v) \cap F_i|}$ ($w(v)$ and $adj(v)$ are resp. the weight and the set of adjacency hyperedges of vertex v)
        $C := C \cup \{v_i\}$
        $F_{i+1} := F_i \setminus adj(v_i)$
        $i := i + 1$
    end for
end while

3 Data description

The proposed method is tested on Paris city, more specifically in the area covering the 1st to the 6th boroughs and a part of the 7th one, as shown in the perimeter of Fig. 2. Our dataset is composed of parking spots for cars and actual FLZs. Real-world data of overall parking spots were provided by *Open Data Paris* [1]. Parking spots for motorcycles and bike-sharing have been removed from the dataset as their length can be too short for handling of goods. On-street and off-street parking spots are considered as well. The second element of the dataset concerns commercial establishments. Real-world data of overall commercial establishments were provided by *Open Street Map* [13]. Fig. 2 shows the studied Paris network, wherein blue (resp. red) dots represent commercial establishments (resp. parking spots), which account for a total number of 7649 (resp. 8261) objects. The third element of the dataset is the actual delivery demand in terms of delivery frequencies for the considered commercial establishments. We have used averages given by the French service CERTU[3] for these values, which are expressed in function of the type of the activity of the commercial establishment, and can be found in Plantier and Bonnet (2013) [15].

[3] *i.e.* "Centre d'Études sur les Réseaux, les Transports, l'Urbanisme et les constructions publiques", which is a service of the French institute"Centre d'Études et d'expertise sur les Risques, l'Environnement, la Mobilité et l'Aménagement (CEREMA)".

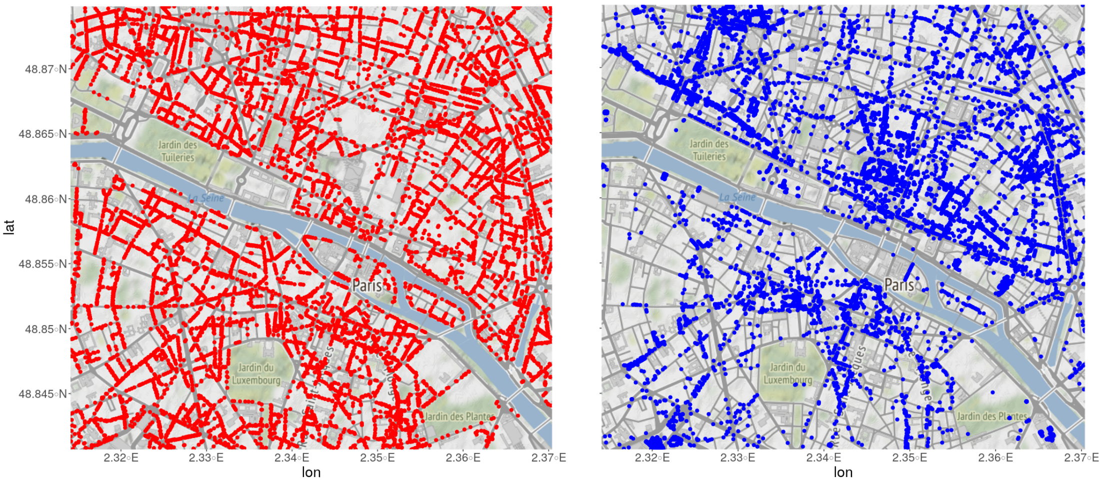


Fig. 2: Paris' network - covering the 1st to the 6th boroughs, and a part of the 7th one - where commercial establishments (right plot) and parking spots (left plot) are displayed.

To construct the distance matrix $D = (d_{ps})_{(p,s)\in\mathcal{P}\times\mathcal{S}}$, we initially considered three formulations of the distance: (i) the Euclidean distance, (ii) the traveled distance and (iii) travel times. In our study, results are reported the second case (ii) through the computation of walking distances. Thus, we considered an undirected graph of the transportation network of Paris in order to calculate those traveled distances. We used the R package OSRM [7] for this purpose. The R package 'hypergraph' [6], which provides the data structure and algorithms for hypergraphs is also used.

4 Study case application: Initial results

This section presents the application of the methodology on the real study case of Paris, presented in the previous section. We compare our optimal design with the existing FLZ system. For the reported metrics, in addition to the objective function given by expression (2) and the parking cover size, we also used statistical values, such as the mean, the median and the standard deviation, taken on the distribution of traveled distance between each commercial establishment and its first nearest FLZ. In other terms, given a selection of parking spots $\mathbb{P} \subseteq \mathcal{P}$ generated by a given method, we affected each commercial establishment $s \in \mathcal{S}$ of the dataset to its closest parking spot in $\mathbb{P}$, and then studied the distribution of distances $(d_{s\mathbb{P}})_{s\in\mathcal{S}}$, where $d_{s\mathbb{P}} = \min_{p\in\mathbb{P}} d_{sp}$.

Fig. 3 displays the evolution of the value of the objective function, the size and the ratio of commercial establishments covered by the the optimal cover

	Mean (m)	Median (m)	Maximum (m)	Standard deviation (m)
Existing FLZ	130.1	89.0	1461.0	136.2
Optimal cover ($R = 50$m)	70.19	31.0	1305	105.02

Table 1: Statistics related to the distributions shown in Fig. 4.

and the existing FLZ in function of the input radius R. Concerning cover size, as expected, it is inversely correlated to R. Lower R values require a large cover, which varies from ≈ 7950 for $R = 10$ meters, to ≈ 1507 parking spots for $R = 220$ meters. The actual FLZ system is implemented for 2017 parking spots, which is corresponding to the optimal cover size for a radius distance of 185 meters. Although this number of elements is small, the repartition however does not cover all commercial establishments in the area, as it is shown in the lower plot. In this lower plot, percentage of commercial establishments covered varies in function of R used in the evaluation, given always an advantage to optimal covers over the existing FLZ system. For instance, for $R = 10$ (resp. 50 and 220 meters), shops coverage for the optimal cover is 29.58 (resp. 66.81 and 90.93%), which is better that the corresponding coverage for the FLZ system, 9.48 (resp. 32.71 and 81.22%). Optimal solutions do not always cover 100% commercial establishments. This is due to the fact that for a number of commercial establishments, no parking spot is at R walking distance. However, they cover the maximum number of shops given the input R. On the other hand, evaluating the objective function shows an increasing trend in function of R for the optimal covers. It is sticking to notice that results of the weighted and unweighted hypergraphs are almost matching. Thus, the impact of the frequency demands seems to be negligible compared to the physical position of parking spots and commercial establishments.

Fig. 4 represents the distribution of the walking distance $(d_{s\mathbb{P}})_{s\in\mathcal{S}}$ between all commercial establishments and their first nearest FLZ for the optimal weighted cover when the radius is fixed to $R = 50$ meters, and for the existing FLZ system as well. Table 4 shows the mean, the median and the standard deviation for both distributions. The mean distance (*commercial establishment-nearest parking spot*) for the existing FLZ is equal to 130.1 meters in our studied area, which is almost twice the corresponding value of the optimal cover, *i.e.* 70.19 meters. The gap between both values is huge, and account for $46\% = |130.1 - 70.19| * 100/130.1$ of additional walking distance the delivery operator is ought to cross. Fig. 5 shows visual examples of the optimal repartition of parking spots when the radius is fixed to $R = 10$, 50 and 100 meters, in addition to showing the existing FLZ system. This figure confirms the previous observation that a lower R generates a higher number of parking spots. When $R = 10$ meters, almost all streets have to be used for delivery parkings.

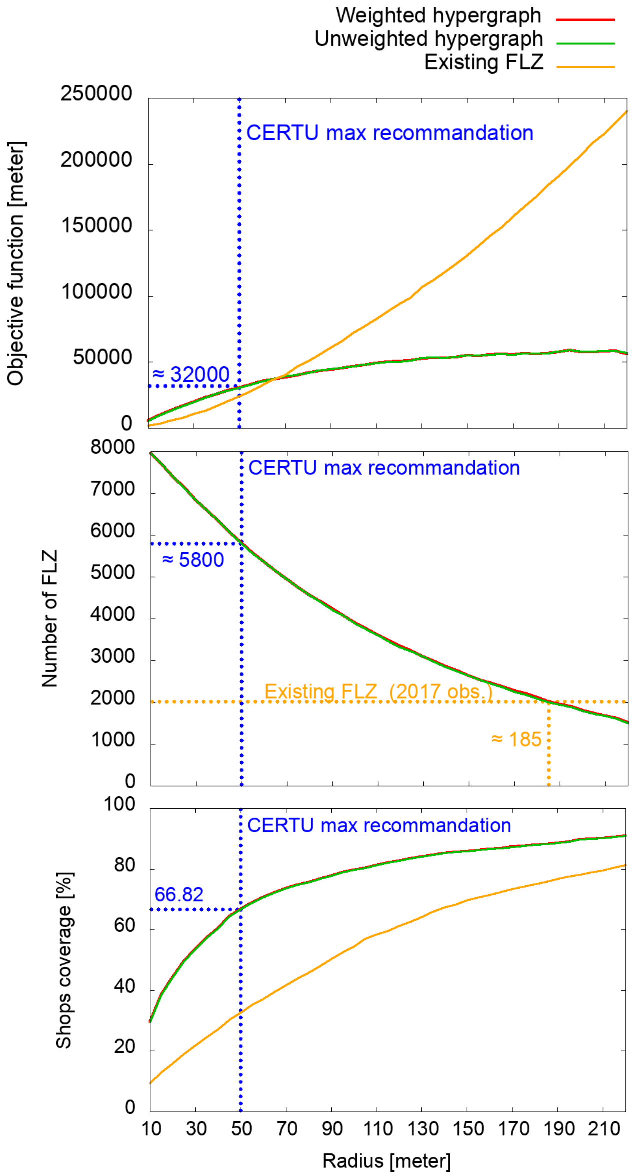


Fig. 3: Evolution of the objective function (upper plot) and the size (middle plot), the commercial establishments coverage ratio (lower plot) for the optimal covers and the existing FLZs in function of the radius R. Note that the evaluation of the objective function, given by expression (2), and the percentage of commercial establishments coverage both depend on the value of R. This is the reason why evaluations of the existing FLZ cover (in orange color) for the first and the third plots vary in function of R. The CERTU recommendation to not exceed 50 meters for the walking distance is added as well [15].

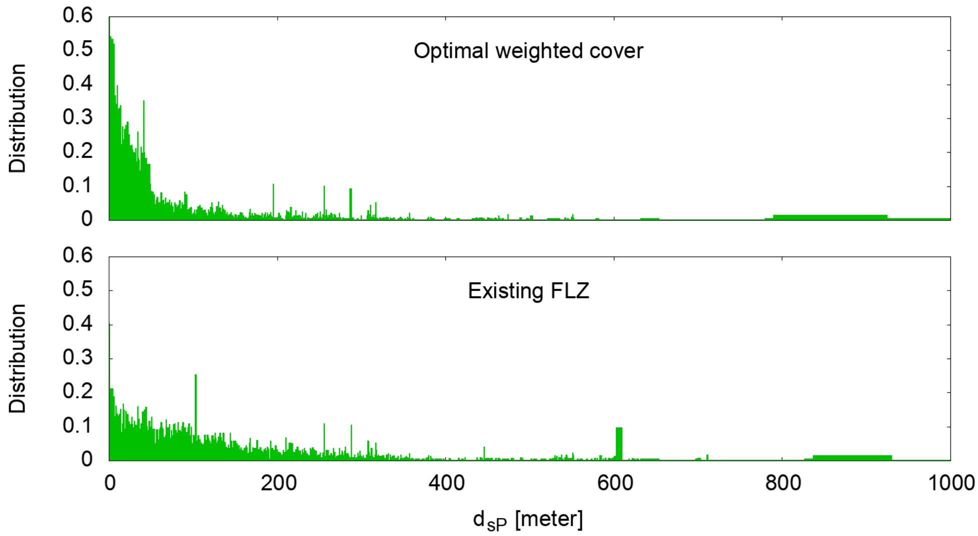


Fig. 4: Distribution of the distances $(d_{s\mathbb{P}})_{s\in\mathcal{S}}$ when the parking cover $\mathbb{P}$ is equal to the optimal cover for a weighted hypergraph with a radius $R = 50$ meters (upper plot), or when $\mathbb{P}$ is equal to the existing FLZ system (lower plot).

5 Conclusion

This paper aims to design the optimal FLZ system. A novel methodology based on hypergraph modeling is introduced, and is applied to the real case study of Paris, France. The traveled (walking) distances between commercial establishments and parking spots are calculated based on the transportation network.

Numerous studies have previously used optimization to provide a design of parking spots: locations and number. Nevertheless, to our best knowledge, existing bibliography mainly proposed macroscopic approaches, where both objectives, locations and number, are separately determined, plus optimization is solely used for obtaining the locations. We proposed a graph oriented approach that simultaneously accounts for both objectives.

We have captured a difference of performance of 46% (60 additional meters to walk in average for delivery operators) between the existing FLZ system and our optimal cover for an input radius of $R = 50$ meters. Our novel approach has several promising applications. We could cite for instance, transit planning and car-sharing. For further works, we are currently comparing our optimal cover

Fig. 5: Optimal parking cover solution for the weighted hypergraph given $R = 10$ (upper-left), $R = 50$ (upper-right) and $R = 100$ meters (lower-left), in addition to the FLZ parking spots implemented by local authorities (lower-left).

to outputs of other optimization models in the literature, namely to Muñuzuri *et al.* (2017) [11], Tamayo *et al.* (2017) [17] and Pinto *et al.* (2019) [14] [4].

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[4] As said, this work is currently ongoing. In case the paper is accepted for presentation, this section will be presented at the conference as well.

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