Analysis of the first-come-first-served mechanisms in car-sharing services

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ABSTRACT
Car-sharing services (CSS) have recently drawn increasing attention. As widely used in CSS, the first-come-first-served (FCFS) principle specifies that travelers arriving first should be served first for the sake of equity. This study formulates four different FCFS mechanisms in a multi-modal supernetwork. For each variant, we formulate the supply-demand dynamics of CSS and the generalized costs of using CSS. A boundedly rational dynamic traffic assignment model is formulated as a variational inequalities (VI) problem to study the effects of the FCFS mechanisms. Numerical examples on two FCFS mechanisms demonstrate different supply-demand dynamics.

Keywords: car-sharing services; first-come-first-served; bounded rationality; dynamic user equilibrium

1. Introduction

Car-sharing services (CSS) have been drawing increasing attention recently due to the mobility flexibility, comfort and low-costs. Without the necessity of car-ownership, CSS show potential solutions to tackle traffic congestion, mitigate CO2 emission, and save travel costs. Unlike the traditional car-renting services that have a full-day or multi-day time frame, CSS usually focus on short-term trips in the urban environments. Based on the way of disposing the shared cars after reaching the destinations, business-to-customer CSS usually have three categories: round-trip based, one-way station-based and free-floating.

Responding to the growing interest in CSS, a large number of studies have emerged to investigate the travel behaviors, demand, and supply management of CSS. Focusing on user preferences, researchers (Becker et al., 2017; Le Vine and Polak, 2019) have explored various factors, including personal attitudes and privacy-seeking, affecting the preference on CSS and analyzed the impact of CSS on car ownership. Travel demand analyses are essential for the efficient operations of CSS. The majority of demand analyses rely on network-based equilibriums analyses (Li et al., 2018) and micro-simulations (e.g., MATSim (Ciari et al., 2016) and mobiTopp (Heilig et al., 2018)). Besides, an increasing number of studies (Nair and Miller-Hooks, 2014; Ströhle et al., 2019) have been dedicated to manage the supplies and improve efficiency through deployment and operation strategies.

Comparatively, less attention has been paid to the queueing mechanism of CSS. With short-term dynamics of CSS (e.g., one-way CSS in urban areas), supply-demand imbalance may occur due to insufficient supply and unevenly distributed demand in time and space. It is common at certain periods of time that the demand cannot be satisfied by the supply and queue of CSS users cumulates. The service mechanisms of treating the queues when supply insufficiency arises have significant impacts on the efficiency of the CSS. Amongst, first-come-first-served (FCFS) restricting that travelers arriving first are served first, has been a widely used strategy. Despite an important modeling component in CSS, few studies have explicitly considered the FCFS principle in CSS. Specifically, Clemente et al. (2013) characterized the shared car renting process by six main phases, where the FCFS principle underlies the rental and use phases. However, their proposed discrete-event simulation approach did not provide any detailed descriptions. Levin et al. (2017) assumed that CSS users were served in a FCFS order if there were multiple travelers waiting at the same location. Although the congestion phenomenon was captured by the dynamic network loading model, the FCFS shared mobility was not formulated by mathematical expressions. The mathematical optimization proposed by Chang et al. (2017) could handle the FCFS principle of CSS by several constraints, but one limitation is that the underlying queuing mechanism is not captured in the event of supply insufficiency. Li et al. (2018) modeled the CSS supply-demand interactions in multi-modal transport networks by a FCFS principle; however, the model is based on an assumption that CSS users arriving at a location at the same interval would wait together until the demand is satisfied by incoming shared cars at a later stage, which hold only when the unit of one time interval is extremely small. Summarily, the FSFC principle has not sufficiently addressed and discussed in the study of CSS.

Therefore, this study aims to compare four different FCFS principle of business-to-customer CSS, namely, *no waiting* FCFS (NW-FCFS), *aggregate* FCFS (A-FCFS), *disaggregate* FCFS (D-FCFS), and VIP (very important person) membership D-FCFS (VD-FCFS), in a multi-modal transport system. For each variant, we formulate the supply-demand dynamics of CSS and generalized costs of using shared car. Moreover, a boundedly rational dynamic traffic assignment model is embedded with CSS and formulated as a variational inequalities (VI) problem to study the supply-demand interactions and the effects of different FCFS principles.

The remainder of the paper is organized as follows. Section 2 provides the space-time multi-modal supernetwork representation and notations. Section 3 formulates four variants of FCFS service mechanism and the supply-demand dynamics of shared cars. In Section 4, the boundedly rational dynamic user equilibrium conditions and properties of the solutions are analyzed. Numerical examples are given in Section 5 to illustrate the essential ideas of the proposed model. Finally, conclusions are provided in Section 6.

2. Network representation and notations

In this study, we consider a multi-modal supernetwork $SNK(N, A)$, where N and A denote the sets of locations and links respectively. Travel links, involving private car link set (A_{PC}) and shared car link set (A_{SC}), and transition links (A_{TS}) are two different types of link sets, $A = A_{PC} \cup A_{SC} \cup A_{TS}$. r and s are nodes representing the origin and destination respectively (see Liao et al. (2013) for the detailed explanations of multi-modal supernetwork representation).

3. FCFS mechanism

First-come-first-served (FCFS) is a service management principle that executes queued requests and processes by the order of arrival time. With FCFS in car-sharing services, traveler comes first is served first; a request in a service queue is executed once the previous one is complete. In the context of discrete time dimension, this section discusses four different variants of FCFS, namely, *no waiting* FCFS (NW-FCFS), *aggregate* FCFS (A-FCFS), *disaggregate* FCFS (D-FCFS), and VIP (very important person) membership D-FCFS (VD-FCFS). Using node a denote a CSS location, we defined the following notations to illustrate the FCFS mechanism. $S_a(k)$, $D_a(k)$, $h_a(k)$, and $g_a(k)$ are the supply, demand, stock and shortage of shared cars at node a during time interval k, respectively; $u_a(k)$ is the arrival flow that completes shared car trips at a during k, and $v_a(k)$ is the arrival flow that requests to use shared cars; $z_a(k, w)$ denotes the flow arriving at node a at k and being served with waiting time w. Unless otherwise explained below, l represents a shared car link starting from CSS location. $c_l^{\mathrm{W}}(k)$, $c_l^{\mathrm{D}}(k)$, and $c_l(k)$ are the waiting disutility, traveling disutility, and link disutility of link l respectively.

3.1. NW-FCFS

Due to the uncertain waiting time in a queue, NW-FCFS postulates that a traveler quits the request for a shared car if there is not enough supply of shared car at the moment of request. In other words, travelers either are served or seek alternatives. With the NW-FCFS principle, waiting is prohibited at the CSS location if no shared car are available (Chang et al., 2017). The supply-demand dynamics are formulated as

$$S_a(k) = h_a(k-1) + u_a(k) \tag{1}$$

$$D_a(k) = \min\{v_a(k), S_a(k)\} \tag{2}$$

$$h_a(k) = \begin{cases} S_a(k) - D_a(k), & S_a(k) \geq D_a(k) \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

$$g_a(k) = 0 \tag{4}$$

The zero waiting time results in Eq. (5), and hence the disutility of the shared car link l equals the duration disutility as formulated by Eq. (6).

$$z_a(k, w) = \begin{cases} D_a(k), & w = 0 \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

$$c_l(k) = c_l^{\mathrm{D}}(k), \quad l \in A_{SC} \tag{6}$$

3.2. A-FCFS

Travelers arriving at a service location during the same interval are regarded as an aggregate unit. Aggregate-FCFS (A-FCFS) suppose that an aggregate unit of travelers who come earlier than other units are served first. When there are sufficient supplies of shared car, travelers of the same aggregate unit are served all at a time; otherwise, they would either wait for the replenishment of shared cars together or seek alternatives (Li et al., 2018). A-FCFS has the same supply formulation as Eq. (1) but different stock formulation, compared with the NW-FCFS. Regarding the demand side, A-FCFS takes waiting time into consideration. The demand is equal to the summation of the existing shortage (unsatisfied demand) if any and the newly added service demand. The supply-demand dynamics under the A-FCFS principle are formulated as

$$D_a(k) = g_a(k-1) + v_a(k) \tag{7}$$

$$h_a(k) = \sum_{\tau=0}^{k} u_a(\tau) - \sum_{\tau=0}^{\bar{t}} v_a(\tau) \tag{8}$$

$$g_a(k) = \begin{cases} \sum_{\tau=0}^{k} v_a(\tau) - \sum_{\tau=0}^{\hat{t}} u_a(\tau), & S_a(k) < D_a(k) \\ 0, & \text{otherwise} \end{cases} \tag{9}$$

where $\bar{t}$ and $\hat{t}$ are the maximum time intervals with the stock and queue before k, formulated as $\bar{t} = \underset{t}{\operatorname{argmax}}\left\{\sum_{\tau=0}^{k} u_a(\tau) \geq \sum_{\tau=0}^{t} v_a(\tau),\ t \leq k\right\}$ and $\hat{t} = \underset{t}{\operatorname{argmax}}\left\{\sum_{\tau=0}^{k} v_a(\tau) \geq \sum_{\tau=0}^{t} u_a(\tau),\ t \leq k\right\}$.

With the A-FCFS principle, the waiting time $t_a^{\mathrm{W}}(k)$ and the flow being served at $k + w$ are expressed as Eq. (10) and (11) respectively.

$$t_a^{\mathrm{W}}(k) = \underset{t}{\operatorname{argmin}}\left\{D_a(k) \leq h_a(k-1) + \sum_{\tau=k}^{t} u_a(\tau)\right\} - k \tag{10}$$

$$z_a(k,w) = \begin{cases} v_a(k), & w = t_a^{\mathrm{W}}(k) \\ 0, & \text{otherwise} \end{cases} \tag{11}$$

Besides the waiting cost derived from the waiting time, the disutility of a shared car link contains the cost of using shared car.

$$c_l(k) = c_l^{\mathrm{W}}(k) + c_l^{\mathrm{D}}(k+w), \quad k, l \in A_{SC} \tag{12}$$

3.3. D-FCFS

In reality, shared car stock and shortage exist exclusively at a service location, reduced to the case of A-FCFS when the length of a time interval is extremely small. Rather than considering travelers arriving during the same interval as an aggregate unit throughout, the disaggregate FCFS (D-FCFS) principle enables serving them at different time intervals, no later than the time point starting to serve travelers who arrive later. The supply and demand under D-FCFS remain unchanged as Eq. (1) and (7) respectively. However, the disaggregation results in different shared car stock and shortage formulations as

$$h_a(k) = \begin{cases} S_a(k) - D_a(k), & S_a(k) \geq D_a(k) \\ 0, & \text{otherwise} \end{cases} \tag{13}$$

$$g_a(k) = \begin{cases} D_a(k) - S_a(k), & D_a(k) \geq S_a(k) \\ 0, & \text{otherwise} \end{cases} \tag{14}$$

With the D-FCFS principle, travelers arriving at the same interval may be divided into several groups and served with different waiting times. The minimum and maximum waiting times are expressed as

$$t_{a,\min}(k) = \max\left\{\underset{t}{\operatorname{argmin}}\left\{D_a(k-1) < h_a(k-2) + \sum_{\tau=k-1}^{t} u_a(\tau)\right\},\ k\right\} - k \tag{15}$$

$$t_{a,\max}(k) = \underset{t}{\operatorname{argmin}}\left\{D_a(k) \leq h_a(k-1) + \sum_{\tau=k}^{t} u_a(\tau)\right\} - k \tag{16}$$

When $t_{a,\min}(k) = t_{a,\max}(k)$, travelers arriving at k are served simultaneously, which is expressed by Eq. (11). Otherwise, $z_a(k,w)$ is formulated as

$$z_a(k,w) = \begin{cases} S_a(k+w) - \sum_{\tau \in \mathrm{R}(k)} z_a(\tau, m), & w = t_{a,\min}(k) \\ S_a(k+w), & w \in \left(t_{a,\min}(k), t_{a,\max}(k)\right) \\ v_a(k) - \sum_{\tau = t_{a,\min}(k)}^{t_{a,\max}(k)-1} z_a(k,\tau), & w = t_{a,\max}(k) \end{cases} \tag{17}$$

where $\mathrm{R}(k) = \{t \mid t < k,\ k+w = t+m\}$.

Weight coefficient $\lambda_a(k,w)$ determines the proportion of travelers being served at time interval $k + w$, which can be formulated in terms of the ratio of $z_a(k,w)$ to $v_a(k)$ as Eq. (18). Accordingly, the disutility of the shared car link can be formulated by a weighted summation as Eq. (19).

$$\lambda_a(k,w) = \frac{z_a(k,w)}{v_a(k)} \tag{18}$$

$$c_l(k) = \sum_{w=t_{a,\min}(k)}^{t_{a,\max}(k)} \left[\lambda_a(k,w) \cdot \left(c_l^{\mathrm{W}}(k) + c_l^{\mathrm{D}}(k+w)\right)\right], \quad k,l \in A_{SC} \tag{19}$$

3.4. VD-FCFS

"VIP membership" D-FCFS (VD-FCFS) principle introduces the privilege service in D-FCFS to allow VIP members who pay extra membership fee to jump a queue. Depending on the service industry, pricing policy, and equity considerations, various privilege services exist. In this study, we suppose that a VIP-traveler is served immediately if there is a stock and no other VIP-travelers are waiting in the queue. Although VIP-travelers arriving later than the ordinary travelers at a service location may be served first, FCFS mechanism is maintained for the two-classes of travelers respectively. To keep consistency, the notations used above attached with $*$ and $'$ refer to the same entities for VIP and non-VIP travelers respectively. Under the VD-FCFS principle, the supply has the same formulation as Eq. (1). The demand, stock and shortage of shared cars are formulated as

$$D_a^*(k) = g_a^*(k-1) + v_a^*(k) \tag{20}$$

$$D_a'(k) = g_a'(k-1) + v_a'(k) \tag{21}$$

$$h_a(k) = \begin{cases} S_a(k) - D_a^*(k) - D_a'(k), & S_a(k) \geq D_a^*(k) + D_a'(k) \\ 0, & \text{otherwise} \end{cases} \tag{22}$$

$$g_a^*(k) = \begin{cases} D_a^*(k) - S_a(k), & D_a^*(k) \geq S_a(k) \\ 0, & \text{otherwise} \end{cases} \tag{23}$$

$$g_a'(k) = \begin{cases} D_a'(k), & D_a^*(k) \geq S_a(k) \\ D_a'(k) + D_a^*(k) - S_a(k), & D_a'(k) + D_a^*(k) \geq S_a(k) > D_a^*(k) \\ 0, & \text{otherwise} \end{cases} \tag{24}$$

Eqs. (20)-(23) have the similar expressions with the corresponding terms under the D-FCFS principle. The shared car shortage of non-VIP travelers does not only depend on the relation between the total demand and supply, but also relies on the VIP demand due to the privilege. The waiting times and flow being served of VIP travelers are not affected by non-VIP travelers, formulated as

$$t_{a,\min}^*(k) = \max\left\{\underset{t}{\operatorname{argmin}}\left\{D_a^*(k-1) < h_a(k-2) + \sum_{\tau=k-1}^{t} u_a(\tau)\right\},\ k\right\} - k \tag{25}$$

$$t_{a,\max}^*(k) = \underset{t}{\operatorname{argmin}}\left\{D_a^*(k) \leq h_a(k-1) + \sum_{\tau=k}^{t} u_a(\tau)\right\} - k \tag{26}$$

$$z_a^*(k,w) = \begin{cases} S_a(k+w) - \sum_{\tau \in \mathrm{R}(k)} z_a^*(\tau,\ m), & w = t_{a,\min}^*(k) \\ S_a(k+w), & w \in \left(t_{a,\min}^*(k), t_{a,\max}^*(k)\right) \\ v_a^*(k) - \sum_{\tau=t_{a,\min}^*(k)}^{t_{a,\max}^*(k)-1} z_a^*(k,\tau), & w = t_{a,\max}^*(k) \end{cases} \tag{27}$$

Accordingly, the disutility of the shared car link for VIP travelers is expressed as

$$c_l(k) = \sum_{w=t^*_{a,\min}(k)}^{t^*_{a,\max}(k)} \left[\frac{z^*_a(k,w)}{v^*_a(k)} \cdot \left(c^{\mathrm{W}}_l(k) + c^{\mathrm{D}}_l(k+w)\right)\right], \;\; k, l \in A_{SC} \tag{28}$$

Different from the VIP travelers, non-VIP travelers arriving at a at interval k begin to be served until the non-VIP demand of interval $k-1$, $D'_a(k-1)$, and the VIP demand at t, $D^*_a(t)$ $(t \geq k-1)$, are satisfied. Any VIP traveler arriving between $k-1$ and t jumps the queue and delays the CSS for non-VIP travelers. Similarly, the maximum waiting time depends on the inflow of cumulative VIP travelers from k, the VIP shortage at $k-1$, and the non-VIP demand at k. Hence, the bounded waiting times and the flow of served non-VIP travelers are

$$t'_{a,\min}(k) = \max\left\{\underset{t}{\operatorname{argmin}}\left\{D'_a(k-1) + D^*_a(t) < h_a(k-2) + \sum_{\tau=k-1}^{t} u_a(\tau)\right\},\, k\right\} - k \tag{29}$$

$$t'_{a,\max}(k) = \underset{t}{\operatorname{argmin}}\left\{D'_a(k) + g^*_a(k-1) + \sum_{\tau=k}^{t} v^*_a(\tau) \leq h_a(k-1) + \sum_{\tau=k}^{t} u_a(\tau)\right\} - k \tag{30}$$

$$z'_a(k,w) = \begin{cases} S_a(k+w) - \sum_{\tau\in \mathrm{H}(k)} z^*_a(\tau, m) - \sum_{\tau \in \mathrm{R}(k)} z'_a(\tau, m), & w = t'_{a,\min}(k) \\ S_a(k+w) - \sum_{\tau \in \mathrm{R}(k)} z^*_a(\tau, m), & w \in \left(t'_{a,\min}(k), t'_{a,\max}(k)\right) \\ v'_a(k) - \sum_{\tau = t'_{a,\min}(k)}^{t'_{a,\max}(k)-1} z'_a(k,\tau), & w = t'_{a,\max}(k) \end{cases} \tag{31}$$

where $\mathrm{H}(k) = \{t \mid k + w = t + m\}$.

The disutility of the shared car link for non-VIP travelers is a weight summation as

$$c_l(k) = \sum_{w=t'_{a,\min}(k)}^{t'_{a,\max}(k)} \left[\frac{z'_a(k,w)}{v'_a(k)} \cdot \left(c^{\mathrm{W}}_l(k) + c^{\mathrm{D}}_l(k+w)\right)\right], \;\; k, l \in A_{SC} \tag{32}$$

Eqs. (6), (12), (19), (28) and (32) formulate the disutilities of the first shared car link under different FCFS principles. Combined with the disutilities of transition links, PC links and other shared car links, the path disutility is usually calculated by the summation of disutilities of the associate links as

$$c^{rs}_p(k, \boldsymbol{f}) = \sum_{l\in A}\sum_{\tau} \delta^{rs}_{plk}(\tau) \cdot c_l(\tau), \quad \forall\, rs, p, k \tag{33}$$

where l denotes any link in set A, $c^{rs}_p(k, \boldsymbol{f})$ is the disutility of p with flow vector $\boldsymbol{f}$ during k of rs; $f^{rs}_p(k)$ denotes the flow on p of rs that enters the network during k and $\boldsymbol{f}$ is the vector of $f^{rs}_p(k)$; $\delta^{rs}_{plk}(\tau)$ is a 0-1 indicator variable, $\delta^{rs}_{plk}(\tau) = 1$ if a traveler between rs departs at k via path p and arrives at link l at time τ.

4. BR-DUE model

To model travel behavior more practically, we embed the bounded rationality (BR) into a dynamic user equilibrium (DUE) model for CSS. The BR-DUE condition is stated as: for each OD pair, the disutilities experienced by travelers fall within an acceptable tolerance. Formally, it is expressed as

$$c_p^{rs}(k,\boldsymbol{f}^*) \in \left[c_{\min}^{rs}(\boldsymbol{f}^*),\ c_{\min}^{rs}(\boldsymbol{f}^*)\cdot(1+\varepsilon^{rs})\right],\ \text{if}\ f_p^{rs*}(k)>0 \quad \forall\, p,\ rs,\ k \tag{34}$$

where $c_{\min}^{rs}(\boldsymbol{f})$ is the minimum disutility; $\varepsilon^{rs} \geq 0$ is the threshold of acceptable relative differences in travel disutilities experienced by travelers of rs.

The corresponding finite-dimensional variational inequality problem VI($\boldsymbol{f}, \boldsymbol{\Omega}$) of this condition is

$$\sum_{rs\in RS}\sum_{p\in P^{rs}}\sum_{k\in K}\tilde{c}_p^{rs}(k,\boldsymbol{f}^*)\cdot\left[f_p^{rs}(k)-f_p^{rs*}(k)\right]\geq 0 \tag{35}$$

where

$$\tilde{c}_p^{rs}(k,\boldsymbol{f}^*)=\max\left\{c_p^{rs}(k,\boldsymbol{f}^*),\ c_{\min}^{rs}(\boldsymbol{f}^*)\cdot(1+\varepsilon^{rs})\right\} \tag{36}$$

The BR-DUE problem is to find $\boldsymbol{f}^*$ such that Eq. (35) and $\boldsymbol{f}$ belongs to the feasible set Ω, which includes the following demand and non-negativity constraints.

$$\sum_{p\in P^{rs}}\sum_{k\in K} f_p^{rs}(k)=Q^{rs},\quad \forall rs \tag{37}$$

$$f_p^{rs}(k)\geq 0,\quad \forall p,\ rs,\ k \tag{38}$$

where Q^{rs} is the demand of rs.

5. Numerical examples

A six-node network (Fig. 1) is used to analyze the supply-demand relation at BR-DUE state under the D-FCFS and VD-FCFS mechanisms (due to space limitations, the analyses of other mechanisms and network settings are not presented). In Fig. 1, two OD pairs include (1, 3) and (2, 6) with the same demand of 3000. The numbers marked near each links are the corresponding link free flow times and capacities. The number of shared cars at nodes 1, 4 and 6 are initialized as 600, 400 and 200 respectively.

As an illustration of the supply-demand relation of shared cars, Fig. 2 depicts the supply-demand curves at nodes 1 and 6. In Fig. 2 (a), the orange curve is higher than red dotted curve before 7:50, meaning that the supply at node 1 is larger than the demand. In this case, travelers use the 600 shared cars without waiting. During time interval [8:30 8:45], there is an intersection between the demand and supply curves at node 6. Before the intersection, 200 shared cars are stored at node 6 and used directly. Subsequently, the demand is cumulated to 400. These travelers wait for the incoming shared cars used by travelers between OD pair (2, 6) from node 4 to 6. Taking VIP and non-VIP travelers into consideration, Fig. 2 (b) draws the demand and supply curves under VD-FCFS principle. As depicted, the non-VIP demands at both nodes are higher than VIP demand.

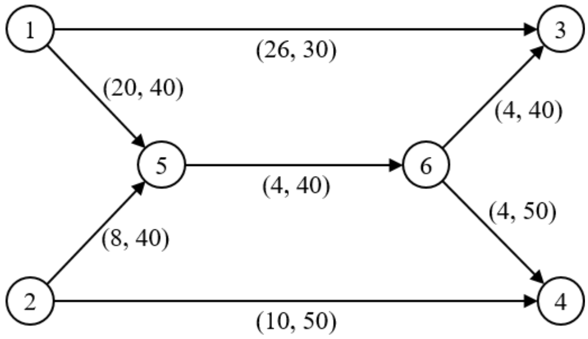
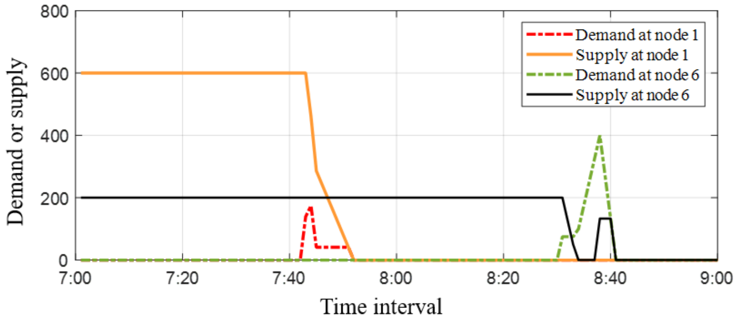
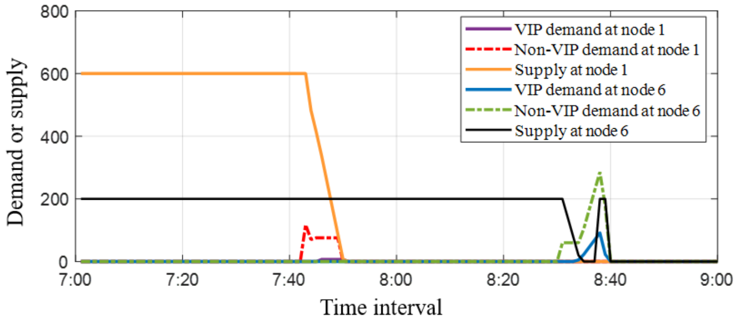


Fig. 1. The six-node network.



(a) Demand and supply results under D-FCFS principle



(b) Demand and supply results under VD-FCFS principle

Fig. 2. The supply and demand of shared cars at node 1 and 6.

6. Conclusions

The existing literature of car-sharing services (CSS) has focused on the travel preferences, demand forecasting, and supply management. This study addresses the supply-demand dynamics of CSS under four FCFS mechanisms in a multi-modal supernetwork. The effects of the service mechanisms are studied in a BR-DUE model. Numerical examples demonstrate that the FCFS mechanisms have significant effects on supply-demand dynamics.

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