

Model predictive perimeter control of MFD networks with guaranteed stability

Short paper for hEART 2020, Lyon

Isik Ilber Sirmatel and Nikolas Geroliminis

Urban Transport Systems Laboratory (LUTS)

École Polytechnique Fédérale de Lausanne (EPFL), Switzerland

ABSTRACT

Traffic management for large-scale urban road networks remains a challenging problem. Aggregated dynamical models of city-scale traffic, based on the macroscopic fundamental diagram (MFD), enable development of model-based control schemes for use with the perimeter control method involving actuation over macroscopic traffic flows, specifying an efficient and practicable congestion control solution. In this paper we propose a nonlinear model predictive control scheme with guaranteed closed-loop stability for a two-region MFDs system. Extensive macroscopic simulations demonstrate the performance and region of attraction properties of the proposed method.

INTRODUCTION

Management of large-scale urban road traffic presents substantial challenges in development of modeling and control techniques. Inadequate infrastructure, lack of coordination of different parts, spatiotemporal congestion propagation, the sheer size of the urban network, and the interactions of drivers with the traffic management system, among others, constitute the complications which arise when developing urban road network models and control schemes. Although development of real-time traffic control methods received significant attention in the literature especially in the last decades (see [Papageorgiou et al. \(2003\)](#) for a review), city level traffic modeling and control design for heterogeneously congested networks remains a challenging problem.

Many works in the literature focused on modeling and control of urban traffic, which usually considered link-level traffic flows via mesoscopic models and local control strategies (see [Aboudolas et al. \(2010\)](#); [Diakaki et al. \(2002\)](#); [Kouvelas et al. \(2014\)](#); [Wongpiromsarn et al. \(2012\)](#)). Mesoscopic traffic models facilitate accurate simulations with high level of detail, however they might complicate control design for large scale networks due to high model complexity. Furthermore, heavily congested conditions might create problems for local control strategies as congested regions upstream are not protected. Highly detailed traffic information, which is difficult to measure or estimate, might be needed for local controllers, further complicating their use.

Perimeter control approach for large-scale networks appeared as a practicable and possibly complementary option to local traffic signal control methods. In perimeter control, the idea is to manipulate macroscopic traffic flows exchanged between neighborhood-sized sections (i.e., regions) of the urban network by changing the green times of the traffic lights on the boundaries (i.e., perimeters) between the regions. Feedback control systems, using the perimeter flow manipulation as actuator, can then be constructed by instrumenting the road network with traffic sensors.

Model-based control design for perimeter controlled urban networks is possible using the macroscopic fundamental diagram (MFD) of urban traffic. First proposed by Godfrey (1969) and, for large-scale urban networks, experimentally proven to exist by Geroliminis and Daganzo (2008), the MFD enables modeling of an urban region with homogeneously distributed congestion by providing a unimodal, low-scatter, and demand-insensitive relationship between accumulation and trip completion flow Geroliminis and Daganzo (2008).

The MFD, despite being a powerful tool for building aggregated models, might also face difficulties that can hamper its performance in traffic modeling accuracy. Heterogeneously distributed congestion, for example, can lead to high scatter in the MFD (see Geroliminis and Sun (2011); Knoop et al. (2012); Mazloumian et al. (2010)). Although it can have its challenges, it is possible to considerably reduce traffic model complexity by using the MFD, thus enabling model-based network-level control design with aggregated, low-dimensional dynamical models. Analysis, modeling, and control with MFD thus attracted increasing interest in the traffic management literature. Many researchers proposed MFD-based control strategies for single-region (Daganzo, 2007; Haddad and Shraiber, 2014; Keyvan-Ekbatani et al., 2012) and multi-region (Aboudolas and Geroliminis, 2013; Haddad and Geroliminis, 2012) urban networks.

Enabled via MFD-based modeling approaches, model-based control design methods also received increasing interest: Nonlinear model predictive control (MPC) for a two-region network actuated with perimeter control (Geroliminis et al., 2013), hybrid MPC with perimeter control and switching signal timing plans (Hajiahmadi et al., 2015), dynamical modeling of heterogeneity and hierarchical control with MPC on the upper level (Ramezani et al., 2015), MPC with MFD-based travel time and delays as performance measures (Csikós et al., 2017), two-level hierarchical MPC with MFD-based and link-level models (Zhou et al., 2017), multimodal MFDs network model-based MPC of city-scale ride-sourcing systems (Ramezani and Nourinejad, 2018), MPC with perimeter control and regional route guidance (Sirmatel and Geroliminis, 2018) and extensions with a path assignment mechanism (Yildirimoglu et al., 2018), combined operation of state estimation and MPC (Sirmatel and Geroliminis, 2019). Detailed literature reviews of MFD-based modeling and control can be found in Ramezani et al. (2015) and Yildirimoglu et al. (2015).

The problem of stabilization for MFD-based feedback perimeter control schemes received relatively low interest in the literature. Although there are some works examining stability of MFDs systems (see Haddad and Geroliminis (2012) and Zhong et al. (2018)), closed-loop stability under nonlinear MPC remains unexplored. In this paper we aim to examine stability properties of MFD-based nonlinear MPC for perimeter control, and propose application of a nonlinear MPC scheme with guaranteed closed-loop stability for a two-region MFDs system.

MODELING

We consider an urban road traffic network, with heterogeneous distribution of congestion, which can be partitioned into 2 homogeneously congested regions. Each region i , $i \in \{1, 2\}$, has a well-defined outflow MFD, which provides a static nonlinear relationship between accumulation $n_i(t)$ and outflow (i.e., trip completion flow) $G_i(n_i(t))$ (veh/s). The inflow demand $q_{ij}(t)$ (veh/s) represents flow of vehicles entering the network from region i destined to region j (i.e., origin-destination (OD) demand), whereas the accumulation $n_{ij}(t)$ (veh) specifies the number of vehicles in region i with destination j , while $n_i(t)$ (veh) is the regional accumulation at time t ; $n_i(t) = \sum_{j=1}^2 n_{ij}(t)$. Perimeter control actuators, modeled via the control inputs $v_{12}(t)$ and $v_{21}(t) \in [v_{\min}, v_{\max}]$ (with $0 \leq v_{\min} < v_{\max} \leq 1$), can manipulate the flows transferring between the re-

gions. We can then write the dynamics of a 2-region MFDs network as ([Geroliminis et al., 2013](#)):

$$\dot{n}_{11}(t) = q_{11}(t) + M_{21}(t) - M_{11}(t) \quad (1a)$$

$$\dot{n}_{12}(t) = q_{12}(t) - M_{12}(t) \quad (1b)$$

$$\dot{n}_{21}(t) = q_{21}(t) - M_{21}(t) \quad (1c)$$

$$\dot{n}_{22}(t) = q_{22}(t) + M_{12}(t) - M_{22}(t), \quad (1d)$$

where $M_{ii}(t)$ and $M_{ij}(t)$ (veh/s) represent the exit flows (i.e., vehicles disappearing from the network) and transfer flows (i.e., vehicles transferring between regions), respectively:

$$M_{ii}(t) = \frac{n_{ii}(t)}{n_i(t)} G_i(n_i(t)) \quad \forall i \in \{1, 2\} \quad (2a)$$

$$M_{ij}(t) = v_{ij}(t) \frac{n_{ij}(t)}{n_i(t)} G_i(n_i(t)) \quad \forall i \in \{1, 2\}, j \neq i. \quad (2b)$$

It is assumed that trip lengths inside a region are similar (i.e., the distance traveled by a vehicle is not affected its origin and destination).

The MFD can be approximated by an asymmetric unimodal curve skewed to the right (i.e., the critical accumulation n_i^{cr} , for which $G_i(n_i(t))$ is at maximum, is less than half of the jam accumulation n_i^{jam}), as suggested by simulation and empirical results ([Geroliminis and Daganzo, 2008](#)). Thus, a third degree polynomial in $n_i(t)$ can be used to express the outflow MFD $G_i(n_i(t))$:

$$G_i(n_i(t)) = a_i n_i^3(t) + b_i n_i^2(t) + c_i n_i(t), \quad (3)$$

where a_i , b_i , and c_i are model parameters (which are to be estimated from historical data).

CONTROL

Nonlinear Model Predictive Control

Defining vectors of accumulations, inflow demands, and control inputs as follows

$$n(t) = [n_{11}(t) \ n_{12}(t) \ n_{21}(t) \ n_{22}(t)]^T \quad (4)$$

$$q(t) = [q_{11}(t) \ q_{12}(t) \ q_{21}(t) \ q_{22}(t)]^T \quad (5)$$

$$v(t) = [v_{12}(t) \ v_{21}(t)]^T, \quad (6)$$

it is possible to write the dynamics in equation (1) in compact form:

$$\dot{n}(t) = f_n(n(t), q(t), v(t)). \quad (7)$$

Assuming a constant inflow demand $q(t) = \bar{q}$ that is feasible (see [Zhong et al. \(2018\)](#)), and the corresponding accumulation state and perimeter control input equilibrium points \bar{n} and \bar{v} (with $f_n(\bar{n}, \bar{q}, \bar{v}) = \mathbf{0}$), the origin of the system f_n can be shifted to (\bar{n}, \bar{v}) by defining the state $x(t) \in \mathbb{R}^n$ (with $x(t) \triangleq n(t) - \bar{n}$) and control input $u(t) \in \mathbb{R}^m$ (with $u(t) \triangleq v(t) - \bar{v}$), leading to the dynamics:

$$\dot{x}(t) = f(x(t), u(t)). \quad (8)$$

We consider here the regulation problem, which involves steering the system state to the origin. From the traffic engineering point of view, this corresponds to steering the accumulation of the urban network from an arbitrary initial (possibly highly congested) state to the equilibrium. The regulation problem can be formulated as a nonlinear MPC problem as follows:

$$\begin{aligned} & \underset{u_k}{\text{minimize}} \quad \sum_{k=0}^{N_c-1} \|x_k\|_Q^2 + \|u_k\|_R^2 \\ & \text{subject to} \quad x_0 = x(t) \\ & \quad \text{for } k = 0, \dots, N_c - 1 : \\ & \quad \quad x_{k+1} = F(x_k, u_k) \\ & \quad \quad u_{\min} \leq u_k \leq u_{\max}, \end{aligned} \tag{9}$$

where x_k and u_k are the state and control input vectors internal to the MPC, respectively, for time interval k , N_c is the prediction horizon, Q and R are positive definite, symmetric weighting matrices, $x(t)$ is the information on the actual system state at time step t , $F(\cdot)$ is the time-discretized version of the dynamics $f(\cdot)$ in equation (8).

Stabilizing Nonlinear Model Predictive Control

As is well-known in the literature (see Bitmead et al. (1990)), the MPC problem in equation (9) does not guarantee closed-loop stability. One of the standard approaches for establishing stability of the closed-loop under nonlinear MPC is the quasi-infinite horizon nonlinear MPC (QIH-NMPC) method proposed in Chen and Allgöwer (1998). This method involves adding suitable terminal cost and a terminal state constraint to the standard nonlinear MPC formulation (9) as follows:

$$\begin{aligned} & \underset{u_k}{\text{minimize}} \quad \sum_{k=0}^{N_c-1} (\|x_k\|_Q^2 + \|u_k\|_R^2) + \|x_{N_c}\|_P^2 \\ & \text{subject to} \quad x_0 = x(t) \\ & \quad \text{for } k = 0, \dots, N_c - 1 : \\ & \quad \quad x_{k+1} = F(x_k, u_k) \\ & \quad \quad u_{\min} \leq u_k \leq u_{\max} \\ & \quad \quad x_{N_c} \in \Omega, \end{aligned} \tag{10}$$

where the positive definite and symmetric matrix P is for specifying the cost on the terminal state, while Ω specifies the terminal state constraint (i.e., terminal region). Provided that P and Ω are chosen in such a way that Ω has an invariance property, the closed-loop system is guaranteed to have asymptotic stability if there is a feasible solution to the problem (10) at time step $t = 0$ (see Chen and Allgöwer (1998) for details).

RESULTS

Simulation Setup

We consider a two-region MFDs network, with the peripheral region (region 1) having jam accumulation of $n_1^{\text{jam}} = 26800$ veh, a critical accumulation of $n_1^{\text{cr}} = 8933$ veh, and a capacity flow of $G_1(n_1^{\text{cr}}) = 20.15$ veh/s, while the central region (region 2) has a jam accumulation of $n_2^{\text{jam}} = 22000$

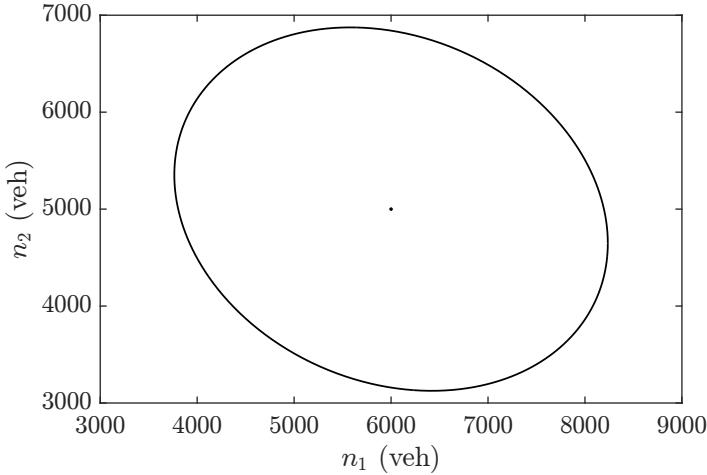


FIGURE 1: Terminal region for the two-region MFDs system.

veh, a critical accumulation of $n_2^{\text{cr}} = 7333$ veh, and a capacity flow of $G_2(n_2^{\text{cr}}) = 14.4$ veh/s. The perimeter control input constraints are $v_{\min} = 0.1$ and $v_{\max} = 0.9$. These values correspond to data extracted from microscopic simulation based studies on a two-region partitioning of a part of the urban road network of the city of Barcelona in Spain (see [Kouvelas et al. \(2017\)](#) for details on this network with four-region partitioning). Assuming a constant inflow demand vector as follows $\bar{q} = [6 \ 5 \ 4 \ 2]^T$ (veh/s), and equilibrium regional accumulations $\bar{n}_1 = 6000$ veh and $\bar{n}_2 = 5000$ veh, the equilibrium accumulation state and perimeter control input can be obtained as $\bar{n} = [3269 \ 2731 \ 2348 \ 2652]^T$ (veh) and $\bar{v} = [0.6 \ 0.65]^T$. Note that these values are chosen arbitrarily to obtain a scenario with an equilibrium state satisfying the $f_n(\bar{n}, \bar{q}, \bar{v}) = 0$ condition.

Numerical Computation of Terminal Cost and Constraint

We can consider the terminal region Ω to be in the form of an ellipsoid (see [Chen and Allgöwer \(1998\)](#) or [Chen et al. \(2003\)](#)):

$$\Omega = \{x \in \mathbb{R}^n | x^T P x \leq \alpha\}, \quad (11)$$

which means that for the QIH-NMPC method we need to compute offline the matrix P and the positive scalar α based on the problem data (i.e., dynamics $f(\cdot)$, weighting matrices Q and R , input constraints u_{\min} and u_{\max}). This can be done, for example, by solving convex optimization problems as suggested by [Chen et al. \(2003\)](#). Considering the numerical values of the problem data given in the previous section, together with the weighting matrices $Q = I_n$ and $R = 0.01 \cdot I_m$ (with $n = 4$ the state and $m = 2$ the input dimension, respectively), and a fixed value of $\alpha = 1000$ (to ensure that the terminal cost is not too large relative to the stage cost), the convex optimization version of the problem in equation (26) of [Chen et al. \(2003\)](#) can be solved for a series of state bounds to maximize the volume of the resulting ellipsoid representing the terminal region. The resulting ellipsoid (projected onto regional accumulations n_1 - n_2 space) is depicted in figure 1.

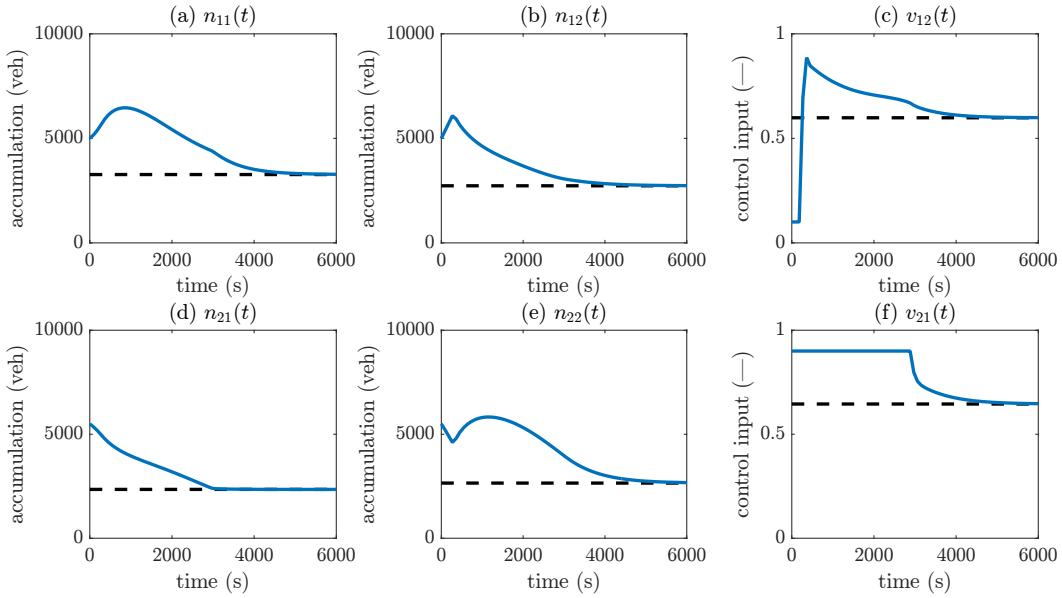


FIGURE 2: Accumulation state and perimeter control input trajectories for a congested scenario using QIH-NMPC with $N_c = 40$.

Congested Scenario

Here we examine a single simulation scenario, where the network is congested initially, having an initial state of $n(0) = [5000 \ 5000 \ 5500 \ 5500]^T$ (veh) (with both initial regional accumulation higher than the critical accumulations, i.e., $n_1(0) = 10000$ veh and $n_2(0) = 11000$ veh). The accumulation state n_{ij} and perimeter control input v_{ij} trajectories of a closed-loop system simulated with a QIH-NMPC having a prediction horizon of $N_c = 40$ is given in figure 2, for a time period corresponding to 100 minutes of real time.

From the figure it can be seen that the QIH-NMPC is successful in steering the system state to the equilibrium. However, for dynamical systems subject to control input constraints, it is impossible to guarantee stability for all possible initial states. In the next section we examine, via simulations, the region of attraction for the closed-loop system under QIH-NMPC.

Simulation-based Construction of the Region of Attraction

To investigate the region of attraction properties of the QIH-NMPC for the two-region MFDs system, we conducted a series of simulation experiments on a grid of initial accumulation state values $n(0)$. The region of attraction of a control system is the set of initial states for which the closed-loop system trajectories eventually reach the equilibrium state. The set of initial states that are important for the traffic point of view for the two-region MFDs system is those that have a regional accumulation between 0 and jam accumulation, which correspond to $0 \leq n_1(0) \leq 26800$ and $0 \leq n_2(0) \leq 22000$ for the considered problem data. To be able to consider a two-dimensional region of attraction for visualization purposes, the initial states of each simulation experiment is chosen to be extracted as half of the initial regional accumulation (e.g., for a scenario with $n_1(0) = 8000$ veh and $n_2(0) = 12000$, the initial state is $n(0) = [4000 \ 4000 \ 6000 \ 6000]^T$ (veh)).

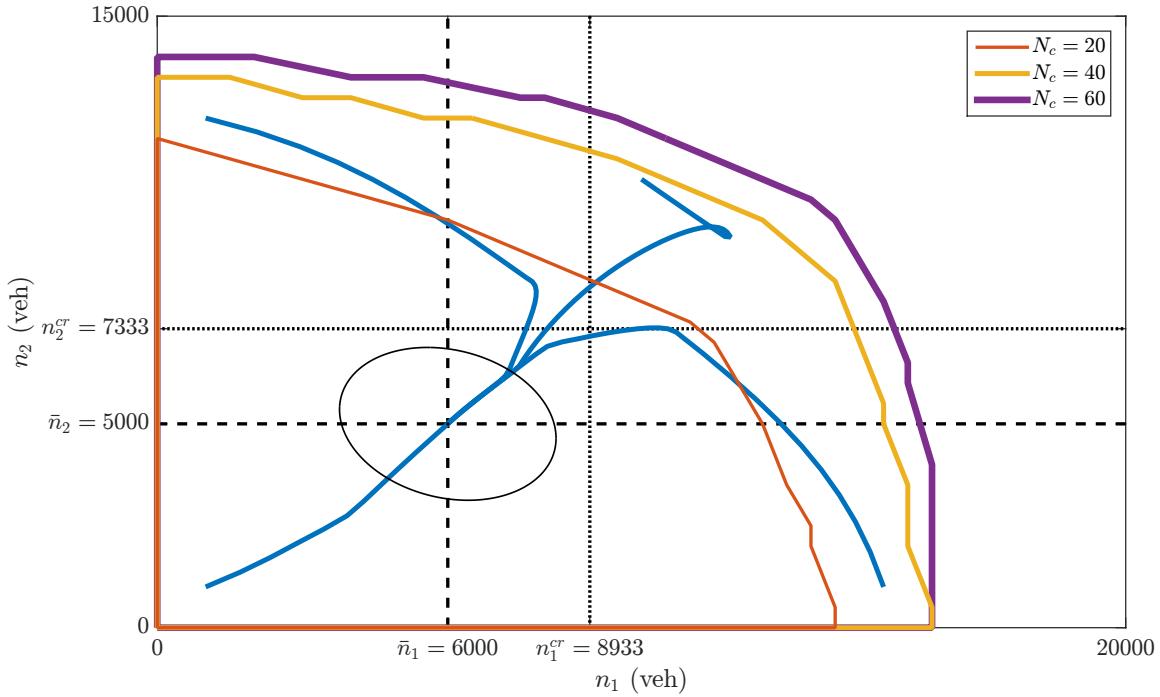


FIGURE 3: Region of attraction boundaries of QIH-NMPC with various N_c values and four simulation trajectories with $N_c = 40$ (blue), shown with the terminal region (black).

Each grid point on the n_1 - n_2 space of initial states, with increments of 500 veh, up to the jam accumulation for each region, is simulated with QIH-NMPC for various prediction horizon values. The results are shown in figure 3, depicting boundaries of the region of attraction for various values of prediction horizon (with all initial states and state trajectories projected from the 4D n -space onto 2D n_1 - n_2 space for visualization purposes).

These results show that it is possible to stabilize a substantial section of the n_1 - n_2 space of initial states using the QIH-NMPC approach, and that the region of attraction can be enlarged by increasing the prediction horizon, as expected.

CONCLUSION

In this paper we proposed application of the quasi-infinite horizon nonlinear MPC method that can guarantee closed-loop stability to the problem of regulation of a two-region MFDs system to an equilibrium state. Performance and region of attraction properties of the scheme is analyzed via macroscopic simulation experiments. Future research could include tests with microscopic simulations, comparisons with other feedback perimeter control methods, and considering measurement and modeling uncertainty in the evaluations.

REFERENCES

- Aboudolas, K., Geroliminis, N., 2013. Perimeter and boundary flow control in multi-reservoir heterogeneous networks. *Transportation Research Part B: Methodological* 55, 265–281. doi:[10.1016/j.tra.2013.02.011](#).

[1016/j.trb.2013.07.003](#).

Aboudolas, K., Papageorgiou, M., Kouvelas, A., Kosmatopoulos, E., 2010. A rolling-horizon quadratic-programming approach to the signal control problem in large-scale congested urban road networks. *Transportation Research Part C: Emerging Technologies* 18, 680–694. doi:[10.1016/j.trc.2009.06.003](#).

Bitmead, R.R., Gevers, M., Wertz, V., 1990. Adaptive Optimal Control — The Thinking Man's GPC. Prentice Hall New York.

Chen, H., Allgöwer, F., 1998. A quasi-infinite horizon nonlinear model predictive control scheme with guaranteed stability. *Automatica* 34, 1205–1217. doi:[10.1016/S0005-1098\(98\)00073-9](#).

Chen, W.H., O'Reilly, J., Ballance, D.J., 2003. On the terminal region of model predictive control for non-linear systems with input/state constraints. *International journal of adaptive control and signal processing* 17, 195–207. doi:[10.1002/acs.731](#).

Csikós, A., Charalambous, T., Farhadi, H., Kulcsár, B., Wymeersch, H., 2017. Network traffic flow optimization under performance constraints. *Transportation Research Part C: Emerging Technologies* 83, 120–133. doi:[10.1016/j.trc.2017.08.002](#).

Daganzo, C.F., 2007. Urban gridlock: Macroscopic modeling and mitigation approaches. *Transportation Research Part B: Methodological* 41, 49–62. doi:[10.1016/j.trb.2006.03.001](#).

Diakaki, C., Papageorgiou, M., Aboudolas, K., 2002. A multivariable regulator approach to traffic-responsive network-wide signal control. *Control Engineering Practice* 10, 183–195. doi:[10.1016/S0967-0661\(01\)00121-6](#).

Geroliminis, N., Daganzo, C.F., 2008. Existence of urban-scale macroscopic fundamental diagrams: Some experimental findings. *Transportation Research Part B: Methodological* 42, 759–770. doi:[10.1016/j.trb.2008.02.002](#).

Geroliminis, N., Haddad, J., Ramezani, M., 2013. Optimal perimeter control for two urban regions with macroscopic fundamental diagrams: A model predictive approach. *IEEE Transactions on Intelligent Transportation Systems* 14, 348–359. doi:[10.1109/TITS.2012.2216877](#).

Geroliminis, N., Sun, J., 2011. Properties of a well-defined macroscopic fundamental diagram for urban traffic. *Transportation Research Part B: Methodological* 45, 605–617. doi:[10.1016/j.trb.2010.11.004](#).

Godfrey, J., 1969. The mechanism of a road network. *Traffic Engineering & Control* 8.

Haddad, J., Geroliminis, N., 2012. On the stability of traffic perimeter control in two-region urban cities. *Transportation Research Part B: Methodological* 46, 1159–1176. doi:[10.1016/j.trb.2012.04.004](#).

- Haddad, J., Shraiber, A., 2014. Robust perimeter control design for an urban region. *Transportation Research Part B: Methodological* 68, 315–332. doi:[10.1016/j.trb.2014.06.010](https://doi.org/10.1016/j.trb.2014.06.010).
- Hajiahmadi, M., Haddad, J., De Schutter, B., Geroliminis, N., 2015. Optimal hybrid perimeter and switching plans control for urban traffic networks. *IEEE Transactions on Control Systems Technology* 23, 464–478. doi:[10.1109/TCST.2014.2330997](https://doi.org/10.1109/TCST.2014.2330997).
- Keyvan-Ekbatani, M., Kouvelas, A., Papamichail, I., Papageorgiou, M., 2012. Exploiting the fundamental diagram of urban networks for feedback-based gating. *Transportation Research Part B: Methodological* 46, 1393–1403. doi:[10.1016/j.trb.2012.06.008](https://doi.org/10.1016/j.trb.2012.06.008).
- Knoop, V., Hoogendoorn, S., Van Lint, J., et al., 2012. Routing strategies based on macroscopic fundamental diagram. *Transportation Research Record: Journal of the Transportation Research Board* 1, 1–10. doi:[10.3141/2315-01](https://doi.org/10.3141/2315-01).
- Kouvelas, A., Lioris, J., Fayazi, S.A., Varaiya, P., 2014. Maximum pressure controller for stabilizing queues in signalized arterial networks. *Transportation Research Record: Journal of the Transportation Research Board* 2421, 133–141. doi:[10.3141/2421-15](https://doi.org/10.3141/2421-15).
- Kouvelas, A., Saeedmanesh, M., Geroliminis, N., 2017. Enhancing model-based feedback perimeter control with data-driven online adaptive optimization. *Transportation Research Part B: Methodological* 96, 26–45. doi:[10.1016/j.trb.2016.10.011](https://doi.org/10.1016/j.trb.2016.10.011).
- Mazloumian, A., Geroliminis, N., Helbing, D., 2010. The spatial variability of vehicle densities as determinant of urban network capacity. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences* 368, 4627–4647. doi:[10.1098/rsta.2010.0099](https://doi.org/10.1098/rsta.2010.0099).
- Papageorgiou, M., Diakaki, C., Dinopoulou, V., Kotsialos, A., Wang, Y., 2003. Review of road traffic control strategies. *Proceedings of the IEEE* 91, 2043–2067. doi:[10.1109/JPROC.2003.819610](https://doi.org/10.1109/JPROC.2003.819610).
- Ramezani, M., Haddad, J., Geroliminis, N., 2015. Dynamics of heterogeneity in urban networks: Aggregated traffic modeling and hierarchical control. *Transportation Research Part B: Methodological* 74, 1–19. doi:[10.1016/j.trb.2014.12.010](https://doi.org/10.1016/j.trb.2014.12.010).
- Ramezani, M., Nourinejad, M., 2018. Dynamic modeling and control of taxi services in large-scale urban networks: A macroscopic approach. *Transportation Research Part C: Emerging Technologies* 94, 203–219. doi:[10.1016/j.trc.2017.08.011](https://doi.org/10.1016/j.trc.2017.08.011).
- Sirmatel, I.I., Geroliminis, N., 2018. Economic model predictive control of large-scale urban road networks via perimeter control and regional route guidance. *IEEE Transactions on Intelligent Transportation Systems* 19, 1112–1121. doi:[10.1109/TITS.2017.2716541](https://doi.org/10.1109/TITS.2017.2716541).
- Sirmatel, I.I., Geroliminis, N., 2019. Nonlinear moving horizon estimation for large-scale urban road networks. *IEEE Transactions on Intelligent Transportation Systems* doi:[10.1109/TITS.2019.2946324](https://doi.org/10.1109/TITS.2019.2946324).

- Wongpiromsarn, T., Uthaicharoenpong, T., Wang, Y., Frazzoli, E., Wang, D., 2012. Distributed traffic signal control for maximum network throughput, in: 15th IEEE Conference on Intelligent Transportation Systems, IEEE. pp. 588–595. doi:[10.1109/ITSC.2012.6338817](https://doi.org/10.1109/ITSC.2012.6338817).
- Yildirimoglu, M., Ramezani, M., Geroliminis, N., 2015. Equilibrium analysis and route guidance in large-scale networks with MFD dynamics. *Transportation Research Part C: Emerging Technologies* 59, 404–420. doi:[10.1016/j.trc.2015.05.009](https://doi.org/10.1016/j.trc.2015.05.009).
- Yildirimoglu, M., Sirmatel, I.I., Geroliminis, N., 2018. Hierarchical control of heterogeneous large-scale urban road networks via path assignment and regional route guidance. *Transportation Research Part B: Methodological* 118, 106–123. doi:[10.1016/j.trb.2018.10.007](https://doi.org/10.1016/j.trb.2018.10.007).
- Zhong, R., Huang, Y., Chen, C., Lam, W., Xu, D., Sumalee, A., 2018. Boundary conditions and behavior of the macroscopic fundamental diagram based network traffic dynamics: A control systems perspective. *Transportation Research Part B: Methodological* 111, 327–355. doi:[10.1016/j.trb.2018.02.016](https://doi.org/10.1016/j.trb.2018.02.016).
- Zhou, Z., De Schutter, B., Lin, S., Xi, Y., 2017. Two-level hierarchical model-based predictive control for large-scale urban traffic networks. *IEEE Transactions on Control Systems Technology* 25, 496–508. doi:[10.1109/TCST.2016.2572169](https://doi.org/10.1109/TCST.2016.2572169).