Investigating heuristic algorithms for minimal controller location set problem in transportation networks

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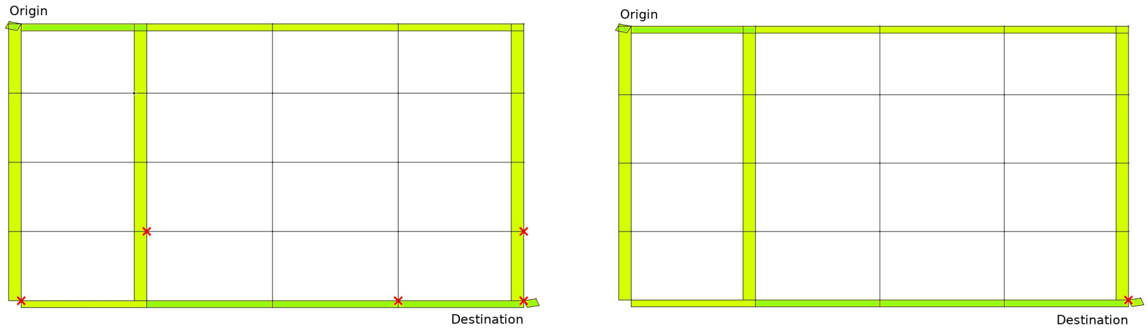
Abstract

The locations and types of controllers employed in a transportation network directly impact the level of performance reachable by traffic control policies. In order to guarantee that the highest level of performance is reachable, fully exploiting the available network capacity and reducing negative externalities, we explore methods to identify important locations in networks for controller while minimizing the number of controllers used. Previous work employed an exact method to identify controller locations on complex networks. However, this approach exhibits complications when applied to transportation networks containing bi-directional links. We therefore propose simple heuristic algorithms based on topological information to solve this problem, while avoiding heavy computation, thus being able to determine a solution to the minimal controller location set problem on large networks. Based on the existing framework, we aim to provide an experimental setup, with diverse network sizes and configurations to analyze the different heuristic methods performance, in order to develop an efficient algorithm.

Keywords: Transportation network, Controllability, Controller locations, Heuristic

1. Introduction

Transportation networks require control strategies to maximize efficiency and avoid delays for road users. To mitigate negative externalities, advanced control strategies are developed in order to use infrastructures at their full potential [1, 2, 3, 4]. These control strategies employ controllers to push road users toward less used routes, to have an more equitable use of the network, thus reducing marginal cost and societal externalities such as congestion and pollutant emissions. In a previous work (Rinaldi, 2018 [5]), a relationship between control policies performance and both locations and amount of installed controllers was established. In said work a framework was also introduced adapting control theory principles to transportation networks. With it, we are able to represent the impact of two types of controller on a transportation network: pricing controllers, bearing a direct impact on the location they are placed on, by adding a cost to it; traffic lights, which manage conflicting flows at a given intersection by distributing total available capacity over time, thus inducing indirect costs (in the form of delays) for road users. In this work we are going to solely focus on the former controller type, due to their direct impact on the position they are placed on and their capability to directly affect road users by increasing direct cost for them. The previously described framework also brought an adaptation of the controllability Gramian matrix, introduced by Kalman et al.(1963) [6], to the instance of transportation networks, which allows us to compute the level of controllability yielded by a set of controllers placed on a given network. We aim to find a collection of controllers that reach full controllability, which is defined as a



(a) All links mono-directional (from NW to SE).

(b) All links bi-directional.

Figure 1: Yuan's method applied on a 25 nodes graph. Controller locations marked in red.

set of controllers that, through a given control action, can steer from the current network state, whichever it may be, toward any other state. With this property, we can define our problem as finding the minimal set of controllers that satisfy the constraint of full controllability.

2. Research contribution

In a previous research an exact method, provided by the work of Yuan et al. (2013) [7], was adapted to the specific instance of transportation networks. This method is able to identify a set of location for pricing controller capable to fully control the network. (Fig.1a) However, while using our controllability framework, the method exhibited difficulties in producing sensible solutions when dealing with complex networks featuring bi-directed links. (Fig.1b) In this case, the addition of bi-directed links introduces self-dependencies during the computation of the controller locations which violate the algebraic assumptions behind the method. This is exemplified in Fig. 1, based on a simple twenty-five node grid-like network. The introduction of bi-directionality completely changes the pricing controller set found by the exact method, yielding a clearly insufficient controller set to be able to control the whole network (the a-posteriori measure of controllability confirming that the network is indeed not fully controllable). This raises a necessity to develop an heuristic approach which is able to find a satisfying solution to the minimal controller location set problem, leveraging topological information, thus avoiding the algebraic issues raised by the aforementioned exact methodology.

3. Methodology

In this work, a given transportation network is represented by a directed graph $G(N, L)$ comprising of a set N of nodes and a set $L : l \in L = (i, j)$, $i, j \in N$ of directed links connecting them. On said graphs, origin and destination nodes are included; on top of it, we consider user behavior in the form of a given route set, enumerated through a K-shortest path heuristic (Yen (1971)[8]). Every node $i \in N$, except origin and destination nodes, is a potential location for a pricing controller. To solve the minimal controller location set problem we propose four simple heuristics. All methods stop once full controllability is reached.

- The first approach is an improved greedy algorithm, adapted to the problem of determining controller locations in transportation networks. For each route forming the previously defined route set, one controller is placed on the node that brings the highest increase in the level of controllability. (Alg.1)

- The second method weighs each node by its degree, and successively selects locations with the highest weights. Moreover, to have a better consideration of the routes information, we propose a second version of the method, by only considering nodes that do appear in the route set when computing node degrees. (Alg.2)
- The third method weighs nodes by their respective topological distance to the closest origin, following existing routes. Controllers are then placed individually on the node bearing the smallest weight (Alg.3)
- The last method is based on the intuition that locations where routes pertaining to a given origin-destination pair are either splitting or merging are critical positions in order to be able to control movement of road users. Following this intuition, we place controllers on every splitting and merging node until reaching full controllability. If the method is unable to reach full controllability by itself, for example due to the network's shape, we then complement the set of controllers following one of the previous methods to reach full controllability. (Alg.4)

Algorithm 1 Best controller per route

```
for each route in the route set do
  get node i with max(level of controllability)
  place controller on node i
  if current level of controllability = full controllability then
    stop
  end if
end for
```

Algorithm 2 Node degree weighted

```
for each node do
  node weight = node degree
end for
while current level of controllability < full controllability do
  place controller on node with max(weight)
end while
```

Algorithm 3 Origin distance weighted

```
for each node do
  node weight = topological distance to closest origin
end for
while current level of controllability < full controllability do
  place controller on node with min(weight)
end while
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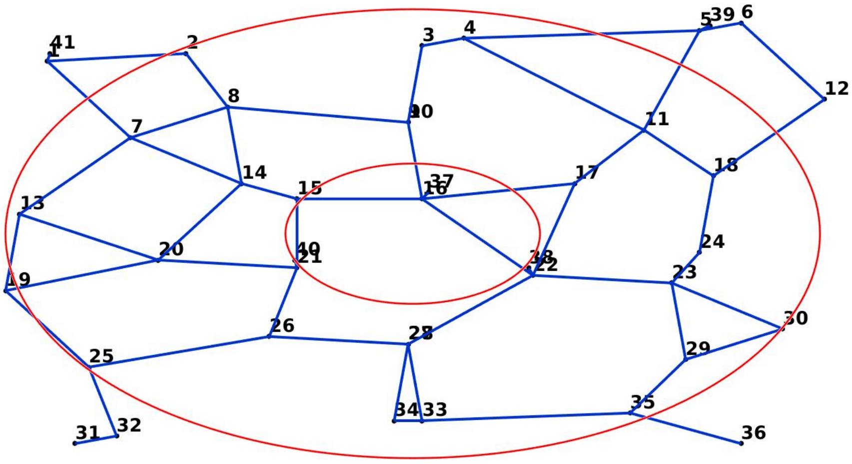

Figure 2: Example of generated network graph.

Algorithm 4 Splitting or merging nodes

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for each origin-destination pair **do**
  for each merging or splitting node of the origin-destination pair routes **do**
    if current level of controllability $<$ full controllability **then**
      place controller on node
    end if
  end for
end for

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4. Experimental setup

To generate graphs representative of typical urban transportation networks, we utilized a graph generator introduced in De Villafranca et al.(2019) [9]. This graph generator starts from a square grid network, thereafter randomly perturbing node locations and generating **bi-directional** links between sufficiently close node couples. (Fig.2). The graph is subsequently divided in three concentric zones (as exemplified by the red circles in Fig.2), for each of which a given amount of origin and destination nodes are placed. In this work, the number of origin/destination nodes per zone is chosen equal to two. Each origin node is thereafter connected to all destinations belonging to a different zone, by a collection of routes. For the first part of our experimental setup, the number of routes per origin/destination pair is set to $k = 3$.

In order to compare the efficiency of the previously defined methods, we employed a set of variably sized networks, ranging from sixteen up to one hundred nodes. For each network size, we generate one hundred graphs each featuring a different random seed, which produces graphs with similar characteristics but different shapes. For each instance, all heuristic methods are applied in order to obtain the corresponding candidate controller locations. The approach of Yuan is also applied on each network, to provide a baseline comparison.

5. Results

To begin, we consider the percentage of instances where the different methods managed to reach full controllability, thus satisfying the key constraint of our problem. In Figure

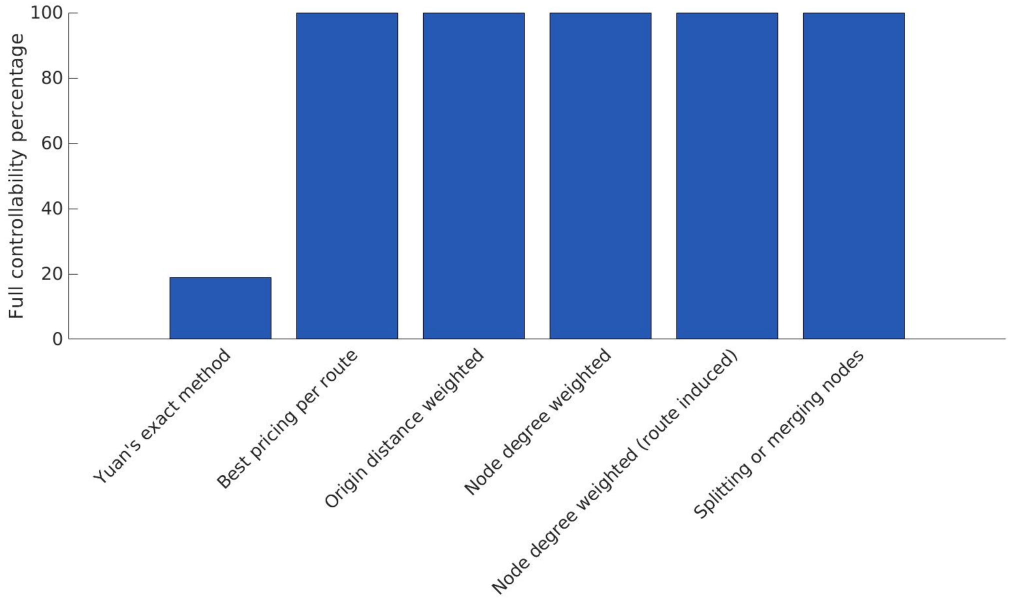

Figure 3: Percentage of solutions reaching full controllability on a 25 nodes graph over 100 random draws.

(3) we showcase the percentage of solutions that manage to reach full controllability over all iterations for a network of twenty-five nodes. We can see that the heuristic methods manage to fulfill the full controllability constraint for each instance, as opposed to the exact method, who only seldom succeeds in identifying an appropriate solution in bi-directional networks. We obtained similar results for every network sizes we tested.

Furthermore, to asses the heuristics' respective degrees of efficiency, we studied the total number of controllers required by the different methods in order to reach full controllability. The results showcased in Fig. (4) represent the number of controllers required by the different methods over one hundred distinct (but equally-sized) networks. These results suggest that indeed some methods perform better in general than others for the given experimental setup, mainly the greedy "Best controller per route" heuristic. As expected, the solutions found by the exact method tend to yield a very small controller set, indeed failing to reach full controllability. Similar trends were highlighted when considering other network sizes. Finally, we also investigated the evolution of the number of controllers needed by each method when increasing the number of routes per each origin/destination pair k.. The results of Figure (5) show the evolution of the mean number of controllers required by the different methods with the increase in routes number, over one hundred randomised generations of twenty-five node graphs. The number of controllers needed by all methods initially increase with the number of routes, although a plateau effect appears beyond a certain threshold. The methods' relative performances are however in line with previous findings. In future works we will carry out further experiments and comparison with other adaptable methods to our problem, so to validate the proposed simple heuristics' capabilities.

6. Conclusion

In this work we developed several heuristics in order to solve the problem of minimum pricing controller locations set. Over a various range of network size and configuration the proposed methods managed to consistently satisfy the constraint of full controllability,

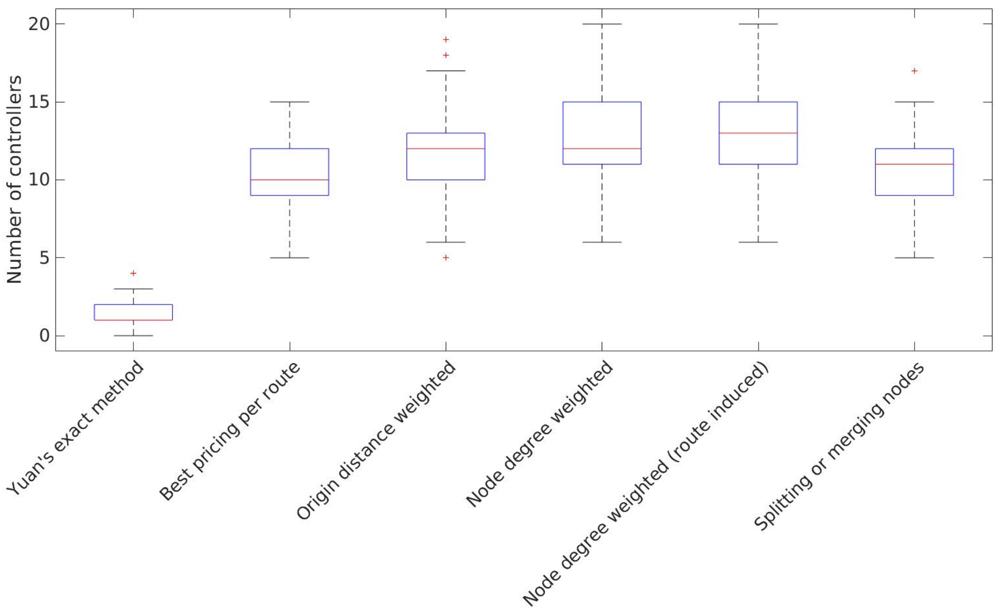

Figure 4: Number of controllers used to reach full controllability on a 25 nodes graph over 100 random draws.

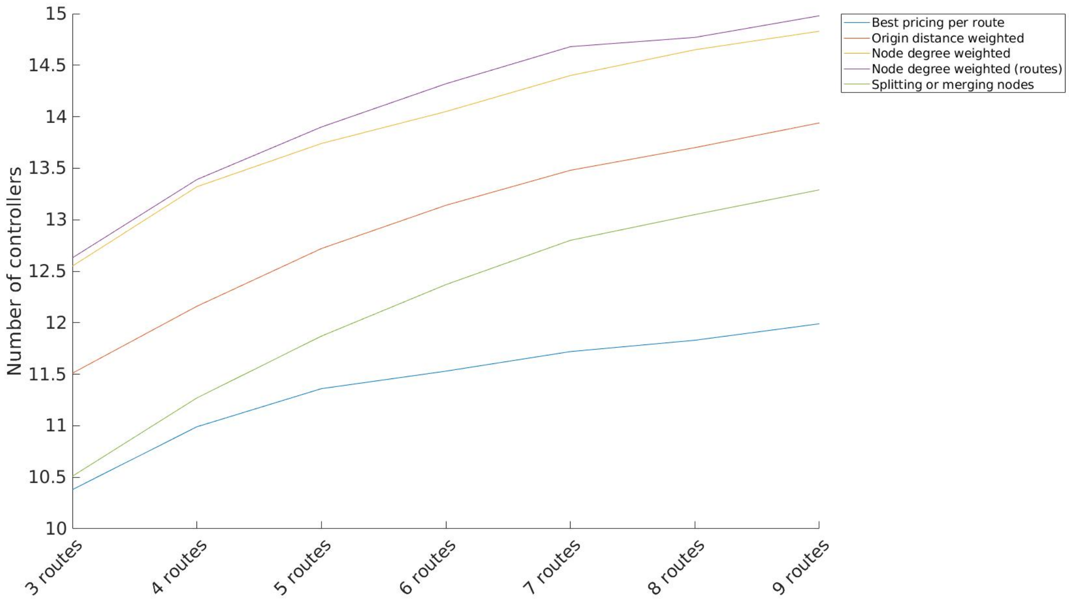

Figure 5: Evolution of the number of controllers used to reach full controllability for increasing k.

while employing a sufficiently low number of controllers.

To advance in this direction, we plan to improve the proposed heuristics, as well as develop extensions to the exact approach proposed by Yuan so to effectively tackle networks bearing bi-directed links. Moreover to have a better representation of the efficiency of the controller set produce by the respective methods, we plan to employ flow-based simulation to asses their capacity to actually redirect flows and highlight an efficient methods for the minimal controller location set problem.

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