Externalities and welfare oriented pricing in free-floating vehicle sharing systems

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Short paper submitted for presentation at the
Symposium of the European Association for Research in Transportation
Lyon, 2020

1 Introduction

Vehicle sharing is among the hot topics of transport policy. Technology enables that cars, bicycles, electric bikes, scooters and other micromobility tools can be rented in the modern city through user friendly smartphone applications, such that the costs of vehicle ownership, maintenance and storage are split within a growing community of users. Many stakeholders in the policy arena predict that personal vehicle ownership will not be required in the future at all. This tendency might be accelerated by the wider adoption of connected and autonomous vehicles, as driverless technology will provide a solution to one of the biggest challenges of shared vehicle operations: rebalancing the fleet of shared vehicles from low demand density areas to zones where many trips are intended to be initiated on a real-time basis. Policy making and regulation have to be accommodated to the changing landscape of transport technology.

This paper raises the traditional research questions of transport economics in the context of shared services. What is the socially optimal price of a car or bike sharing trip, and what is the optimal fleet size (i.e. capacity) in a free-floating system? Does the service require public subsidies if welfare maximising pricing and capacity setting is adopted? How much do these supply variables deviate from their social optima if the supplier's economic objective differs from welfare maximisation, e.g. it maximises profits, and what is the welfare effect of monopolistic supply? These questions have been raised decades ago in the context of private car use and public transport, and the consensual finding is that optimal supply, especially pricing, depends heavily on the externalities that users impose on each other and the rest of society.

We observe that externalities are present in free-floating vehicle sharing as well. For example, the time required to access the nearest vehicle of a car sharing service depends on the demand of

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fellow users who might begin their trips nearby. The spatial density of shared vehicles decreases with demand originating from a given area. We might call this an *access time externality*. The majority of supply-side studies are optimising pricing and vehicle allocation strategies in large-scale agent based simulators with numerical programming algorithms (see e.g. Boyacı et al., 2015, Jorge et al., 2015, Weikl and Bogenberger, 2013, Xu et al., 2018), hiding the link between such user externalities and the optimal price they derive. This paper intends to take a closer look the mechanisms behind user costs, the spatial distribution of shared vehicles and the set of prices that maximise the efficiency of service provision.

2 Unique features of free-floating vehicle sharing

With minor experience as a user of a car or bike sharing service, one can recover the major cost elements that consumers encounter when travelling. The journey is split into three main parts. First, some time is required to locate and access a shared vehicle, which is normally the parked the closest to the consumer's trip origin. This trip leg is performed by walking, in most cases. Second, the user drives or rides the vehicle close to her desired trip destination, which is very similar to a conventional ride with a privately owned vehicle. Third, the vehicle has to be parked, which normally doesn't require additional payments, as opposed to private car storage, and if the parking location does not coincide with the trip destination, then some walking is required again.

It is clear that the first journey phase is what makes vehicle sharing different from existing modes, and its cost (let us call it *access cost* from now on) is what requires increased attention. The introduction has already highlighted the importance of the spatial density of free-floating vehicles in a given geographical area, as it is the major determinant of the access cost. Thus, vehicle density defines the quality of service, more generally.

There is a straightforward way in which the supplier can improve the quality of service experienced by the average user: by increasing the fleet size, vehicle density also increases, and it becomes easer for users to find a vacant vehicle nearby. At this point, we find an important similarity with public transport and taxi services: it is likely that this mode is also subject to density economies in user costs, just as waiting time decreases with demand through adjustments in the frequency of public transport services (Mohring, 1972). Taxi networks feature the same property, with the exception that taxi drivers can individually affect the spatial density of vacant vehicles, while shared cars, bikes or scooters remain at the location where the previous user left them. Nevertheless, the presence of density economies is likely, and therefore, following the reasoning of Arnott (1996) for taxis, it is possible that shared services also require public subsidies to maximise their economic efficiency.

Unfortunately, the density of shared vehicles is not uniform throughout the urban space. This is a consequence of the unbalanced nature of travel demand itself. Car sharing users often experience that it is difficult to find vacant vehicles in the central district of a city by the end of the regular working hours, and the opposite happens in the suburbs in the morning peak. One might also observe that the directional imbalance in demand fluctuates between morning and afternoon peak. Thus, the split of vehicles by the end of the morning peak is expected to be similar to the beginning of the afternoon peak, and *vice versa*. We believe this is a crucial feature of free-floating systems, and therefore its appropriate modelling requires some spatial disaggregation to consider the impact of demand on the fleet's physical distribution.

Second, the evolution of access costs is dynamic in temporal terms as well. Assume that q_{ab} consumers travel from zone A to B in the morning peak. The peak begins with n_a vehicles located in zone A. Can we derive an average user cost that remains constant over the peak, as we often do in private car or public transport models? If inflows and outflows to and from zone A are unbalanced, then the access cost varies over time. In the extreme case when there is no inflow in the morning peak towards zone A at all, the access cost monotonously increases over time, and the service becomes inaccessible at some point during the peak if $q_{ab} > n_a$. Thus, the conventional static approach with an average user cost equated with the market's inverse demand function cannot be mobilised in the modelling exercise. The demand system must have a temporal dimension, and the market equilibrium must be derived considering the temporal evolution of access costs. To sum up, an appropriate vehicle sharing model requires some degree of spatial as well as temporal differentiation.

3 Modelling insights

For the sake of exposition, from now on we call the shared vehicles *cars*, but the model could equally represent a bike or scooter sharing service. In our initial modelling attempt we restrict the spatial and temporal resolution to the simplest possible setup. As discussed above, consider two zones connected by a bidirectional link. Let us assume first that morning and afternoon peaks are reduced to only one discrete point in time and users begin their trips simultaneously; later on we will increase the number of discrete points in time to allow for more temporal dynamics *within* the peaks. Let $c_a(n_a^0, q_{ab})$ denote the access cost of A→B customers, which depends on the share of the fleet located in zone A at the beginning of the peak, and demand from zone A to B. We can describe the physical movement of shared vehicles with three constraints:

$$n = n_a^0 + n_b^0 = n_a^1 + n_b^1; \tag{1a}$$

$$n_a^1 = n_a^0 - q_{ab} + q_{ba}; \quad n_b^1 = n_b^0 - q_{ba} + q_{ab}; \tag{1b}$$

$$n_b^0 = n_a^1; \quad n_a^0 = n_b^1. \tag{1c}$$

Constraint (1a) states simply that cars should not disappear from the system. The second constraint describes how the available fleet size evolves during the peak: by the end of this period, fleet size for market *ab* equals its initial endowment, minus the net outflow of shared vehicles. Constraint (1c) derives from the sequential alternation of morning and afternoon traffic flows: we assume that the fleet left in the origin of market *ab* by the end of the morning peak will serve as the initial endowment of vehicles for market *ba* in the afternoon peak. In equilibrium, morning and afternoon peaks follow each other such that all three constraints above are satisfied. By combining them, the initial endowments *in equilibrium* are

$$n_a^0 = \frac{n + q_{ab} - q_{ba}}{2}; \quad n_b^0 = \frac{n + q_{ba} - q_{ab}}{2}, \tag{2}$$

from which we will easily derive the marginal impact of car sharing demand in each direction on the split of shared vehicles, e.g. $\partial n_a^0/\partial q_{ab} = 0.5$ and $\partial n_a^0/\partial q_{ba} = -0.5$.

The demand system of the two car sharing markets modelled by linear inverse demand curves. Moreover, we assume that maximum willingness to pay is the same in both directions, and only the number of potential users differs between the busy and calm markets. Inverse demand in the former is $d_a(q_{ab}) = \alpha - \beta_a \, q_{ab}$, while the slope in the opposite direction is, equivalently, $-\beta_b$. In equilibrium, this inverse demand is equated with the sum of the access cost and the monetary payment:

$$d_i \equiv c_i(n_i^0, q_{ij}) + \tau_i; \quad i, j \in (a, b), \, i \neq j. \tag{3}$$

As the model's supply variables do not affect the second and third trip phases (see our discussion in Section 2), we normalise the in-vehicle and egress times to zero. We assign a linear functional form to the access cost as well: $c_i(n_i^0, q_{ij}) = \alpha q_{ij}/n_i^0$. This ensures that equilibrium demand can never exceed the available fleet size in either zone, because if $q_{ij} > n_i^0$, then even if the service is provided for free, the access cost is higher than the maximum willingness to pay, $c_i > \alpha$.

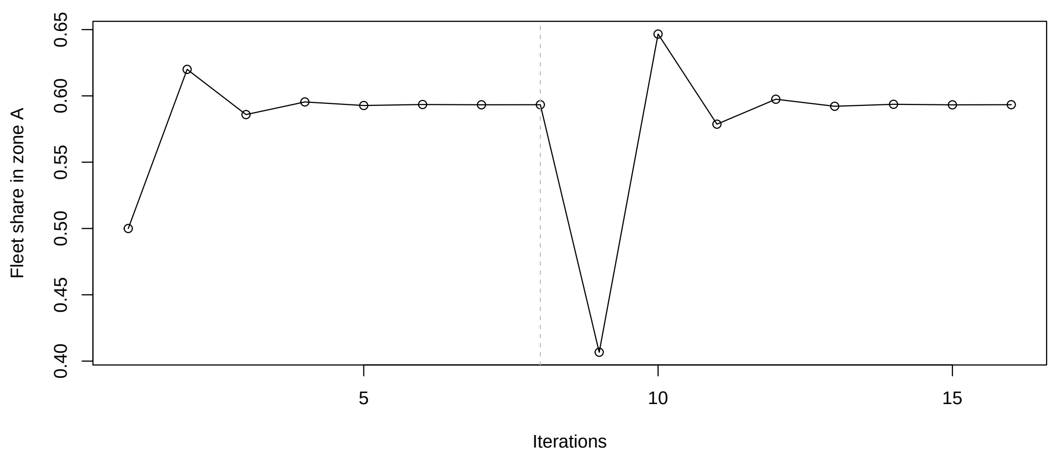

Figure 1: Convergence of the split of shared vehicle fleet between zone A and B.

For a given initial fleet distribution, equilibrium demand can be solved for by substituting the demand and user cost specifications into eq. (3). However, the simultaneous dependency between demand and the redistribution of the fleet prevents us from the analytical derivation of an equilibrium for any given supply. We apply the iterative process depicted in Figure 1. Note that convergence is reached very quickly, even after an artificial shock in the fleet distribution after the $8th$ iteration.

Optimal supply

We define two objective functions that the supplier might pursue. The first one is social welfare, i.e. the sum of consumers' and the supplier's surplus:

$$\max_{n, \tau_a, \tau_b} W = \sum_{ij \in (ab, ba)} \Big[\int_0^{q_{ij}} d_i(q'_{ij}) \, dq'_{ij} - q_{ij} \cdot c_i(n_i^0, q_{ij}) \Big] - z(n) \quad \text{s.t. (3)}, \tag{4}$$

where $z(n) = z_0 n$ is a linear fleet operational cost function. The second one the financial result (profit) of service provision.

$$\max_{n,\tau_a,\tau_b} \Pi = \sum_{ij\in(ab,ba)} (q_{ij} \cdot \tau_i) - z(\rho) \quad \text{s.t. (3)} \tag{5}$$

First order conditions of welfare maximising pricing yield the following expression for the optimal sharing fee between A and B:

$$\tau_a = \underbrace{q_{ab}\frac{\partial c_a}{\partial q_{ab}}}_{I_a} + \underbrace{q_{ab}\frac{\partial c_a}{\partial n_a^0}\frac{\partial n_a^0}{\partial q_{ab}}}_{II_a} + \underbrace{q_{ba}\frac{\partial c_b}{\partial n_b^0}\frac{\partial n_b^0}{\partial q_{ab}}}_{III_a}. \tag{6}$$

The optimal rental fee has four components. The first term is the straightforward marginal external access time cost imposed on fellow users in the same market. If someone pick up a car, then others may need to walk to more to reach a shared vehicle. The second and third elements of the optimal fare capture the effect of the marginal change in the equilibrium split of the car fleet between the two zones. The rearrangement due to the marginal trip in market ab has a beneficial impact on others in the same market, because at the beginning of the next peak period more vehicles will be available for use. By contrast, consumers are worse of in the opposite direction for the same reason.

From the welfare function in (4) we can express the optimality rule for the welfare maximising size of the car sharing fleet:

$$z'(n) = -\sum_{ij\in(ab,ba)} q_{ij}\frac{\partial c_i}{\partial n_i^0}\frac{\partial n_i^0}{\partial n}. \tag{7}$$

That is, the marginal cost adding more vehicles to the fleet must be equal to the marginal benefit of access cost reduction in both markets, which is realised through the increase in vehicle density right from the beginning of the peak period.

Numerical illustration

Once again, we have to turn to numerical methods to illustrate the mechanisms of the model. Let us set the following parameters values: $\alpha = 10$, $z_0 = 1$, and $\beta_a = 0.05$ such that market ab has 200 potential users, and we vary β_b between 0.05 and 0.55 to reveal how the degree of imbalance in demand between the two directions affects optimal supply. Demand in the busy market is kept constant, meanwhile. Numerical optimisation results are provided in Table 1.

Table 1: Welfare maximising supply in function of off-peak market size

β_b	n	τ_a	τ_b	q_{ab}	q_{ba}	n_a^0	n_b^0
0.05	465	3.16	3.16	74	74	0.500	0.500
0.15	284	3.34	2.84	58	31	0.547	0.453
0.25	235	3.39	2.64	53	21	0.567	0.433
0.35	211	3.42	2.49	50	16	0.580	0.420
0.45	197	3.43	2.37	48	13	0.588	0.412
0.55	187	3.44	2.27	47	11	0.595	0.405

The results suggest that service quality as well as the equilibrium demand in market *ab* with constant inverse demand function heavily depends on demand characteristic in the opposite direction. Fleet size decreases from 465 to 187 vehicles in the investigated range of β_b. Observe that even though there are more than four times as many *ab* users as *ba*, the shared fleet is still fairly equally split between the two zones in equilibrium. This contradicts reality in many cities where certain zones might be completely deserted in certain time periods. Apparently, the welfare maximising fare differentiation implies that $\tau_a > \tau_b$, but the difference is not striking, even in the most unbalanced scenario. To learn more about the reasons behind this outcome, we plot the components of the optimal fees, in line with the notation of equation (6).

Table 2: Components of the optimal vehicle sharing fee

	Busy direction			Calm direction		
β_b	I_a	II_a	III_a	I_b	II_b	III_b
0.05	3.16	-0.50	0.50	3.16	-0.50	0.50
0.15	3.75	-0.70	0.30	2.44	-0.30	0.70
0.25	3.96	-0.79	0.21	2.07	-0.21	0.79
0.35	4.08	-0.83	0.17	1.83	-0.17	0.83
0.45	4.16	-0.87	0.13	1.64	-0.13	0.87
0.55	4.22	-0.89	0.11	1.49	-0.11	0.89

This breakdown of the optimal fees reveals that the *direct* access cost externality is much greater in magnitude in the peak direction. In other words, a peak user imposes higher costs of fellow customers by reducing the spatial density of vehicles. However, this effect is somewhat compensated by the fact that peak user contributes to the redistribution of the fleet. For example, a morning peak trip towards the CBD implies that more cars will be available in the afternoon peak for those who want to drive back to the suburbs. Customers in the opposite direction, however, work against this desired distribution of the fleet, and therefore their fee is raised by component III which is substantial in magnitude. Note that components II and III are symmetric in the two directions, as *ab* and *ba* induce the opposite redistribution of the shared fleet.

Table 3: Welfare versus profit oriented supply

	Welfare maximisation					Profit maximisation				
β_b	n	τ_a	τ_b	W	Π	n	τ_a	τ_b	W	Π
0.05	465	3.16	3.16	270	0	232	5.00	5.00	203	135
0.15	284	3.34	2.84	159	0	142	4.80	5.20	119	79
0.25	235	3.39	2.64	126	0	118	4.72	5.29	94	63
0.35	211	3.42	2.49	109	0	106	4.67	5.33	82	54
0.45	197	3.43	2.37	98	0	99	4.64	5.37	73	49
0.55	187	3.44	2.27	90	0	94	4.61	5.39	68	45

Finally, we compare the welfare maximising outcomes with the profit oriented objective in Table 3, and highlight four key results. First, beside the fact the monopolist's fares are generally higher, it applies the opposite differentiation between the two directions: $\tau_b > tau_a$, as long as demand differs between the directions. The explanation is that B→A demand becomes more inelastic as we increase the (negative) slope of $d_b(q_{ba})$. This allows the monopolist to apply a higher monopoly mark-up. Second, note that the profit oriented fleet size is less than half of the socially optimal one. Third, we see that welfare maximising supply leads to zero profits, i.e. the service breaks even. This can be explained by the fact that the degree of homogeneity of the user cost function we specified is zero, while $z(n)$ is linear, and thus the Mohring–Harwitz Cost Recovery Theorem predicts full cost recovery (De Palma and Lindsey, 2007). Finally, Table 3 shows that positive profits can be achieved by the monopolist, but social welfare in this case is around 75% lower than its maximum.

4 Conclusions

The aim of this research is to uncover the fundamental economic mechanisms of one-way vehicle sharing systems, mechanisms that may remain hidden in large-scale numerical optimisation models often published in the contemporary literature. We assign importance to the spatially disaggregated nature of shared services, and the access cost externality which is unique to this mode. We show that the objective function of the service provider has substantial impact on the size of the fleet as well as the cost of the service. Both profit and welfare oriented operators have an interest to price discriminate between spatially separated markets, but the direction of differentiation may be of the opposite sign.

In subsequent steps of this research we intend to increase the temporal resolution of the model to allow for dynamic changes in vehicle densities *within* peak periods. This possibility would unlock a fourth type of externality: if vehicles can be recycled at the trip destination, then *ab* users impose a positive externality on *ba* ones, and *vice versa*. Another planned extension of the model is that we enrich the spatial pattern of demand by considering more complex but still transparent network setups.

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