A new passenger survey methodology: a model of Parsimonious AHP

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Introduction

In every preference type survey, the researchers face the dilemma of acquiring the most valuable data by the questionnaire asking many and complex questions, or keeping the evaluations as simple as possible for reaching high response rate. The Analytic Hierarchy Process (AHP) is an example for the first option, the questionnaire is complex, containing several pairwise comparisons, while the evaluation time is long and severe cognitive effort is required from the respondents. Simple Additive Weighting (SAW) or Multi-attribute Value Theory (MAVT) represent the second option avoiding pairwise comparisons and hierarchy in their process.

However, the AHP has a clear advantage compared to any other survey methods. It provides a hierarchical decision structure to the respondents, thus leads them by the questions and also motivates the consistent evaluation by the checking procedure. It is an asset in case of layman respondents because the risk of getting confusing or uncertain scoring is reduced. This paper presents a decision problem in which keeping the nature of AHP has been a clear objective because of the public participants. Further, even due to the non-expert evaluators, the application of a more simple and understandable questionnaire has also been among the objectives. Completing both of these – at first sight – contradictory goals has led the author of this paper to create the multi-level Parsimonious AHP model.
Parsimonious AHP (PAHP) is a recently created (Abastante et al, 2018) methodology which aims to unburden decision makers in the Analytic Hierarchy Process by requiring less pairwise comparisons than in the classical AHP procedure. As being a fresh technique, there are only few applications of it and some questions still remained open in terms of the conditions and limitations related to the usage of PAHP.

This paper aims to investigate the question; how to apply PAHP for a multi-level decision problem in which public respondents participate in the survey. Multi-level decision problems are very common in AHP applications, this is one of the advantages of the method that it can handle complex problems with several levels of criteria. All the PAHP models so far investigated the method on alternatives (which are obviously on the same, last level of the decision structure) the recent objective is to examine the applicability on criteria in an arbitrary level of the decision tree.

There are only two articles currently available in the scientific literature introducing and applying the PAHP technique. The first related paper (Abastante et al, 2018) published in this topic, demonstrated the method for evaluating social housing project initiatives in which a board (Programma Housing) - connected to the Italian Bank Foundation - played the role of the decision maker. Although Programma Housing as an operating entity consisted experts from several different fields (e.g. engineers, architects, financial experts, psychologists), the board was considered as one homogeneous decision maker group and the overall result was gained by making no distinction among evaluators. In this demonstrative application, 10 criteria was selected (five social and five technical) in a uni-level decision structure (not including the level of alternatives). The other known example is the paper of Abastante et al, 2019, in which the effectiveness of PAHP was examined by an experiment with 100 university students. During the frames of this scientific project, students were asked to estimate the area of different geometrical figures and the evaluation was made both by classical and parsimonious AHP. In this study the authors also provided an example in which a dean selects appropriate students based on their estimated results of three subjects by PAHP. The weights of the subjects can be considered as criteria level but there were no sub-levels except for the alternatives.

This paper endeavors to fill this scientific gap by examining the application of PAHP for a multi-level decision problem. Based on the survey experience this application is not trivial and a slightly modified model has to be created for gaining the appropriate results. Moreover, a previous complete AHP study from 2018 has been available, so the direct comparison of AHP and PAHP results has also been possible. Finally, an unprecedented immanent comparison is also demonstrated for getting deeper insight to the PAHP methodology and outcome.

Methodologically, Parsimonious AHP and Sparse AHP have got common roots. Sparse AHP refers to decision problems in which only a limited amount of information is available (Oliva et al, 2017) and not all the values of pair wise comparisons. For these cases, Sparse AHP applies the Sparse Eigenvector Method (SEM) (Oliva et al, 2017) in which the unknown utilities are approximated via the dominant eigenvector even if perturbations occur. There are other papers in AHP literature coping with missing entries of the ratio matrix e.g. Fedrizzi, M., Giove, S. (2007), Menci et al (2018). The main difference between Sparse and Parsimonious AHP is that the reference points in PAHP are systematically selected and further interaction between the respondents and survey instructors are provided compared to Sparse AHP.

Although parsimonious Analytic Hierarchy Process was originally created for decision problems with many alternatives (Abastante et al, 2018), the core objective of the method also exists in case of many decision criteria. In the demonstrated case study of public transport system development, 24 hierarchical criteria describe the supply quality of public bus service. That means that for a classical entire AHP procedure (23×24)/2 = 276 pairwise comparison evaluations would be necessary if all criteria would be on the same level. Considering the created three levels and seven branches of the examined decision problem (see Figure 1), the number of the required pairwise comparisons for AHP is 27. Note that elements of the decision tree are not compared if they are situated on different levels or branches. For public participants completing even this number of comparisons (27) might be too demanding if they are not committed enough to the actual development. The largest comparison matrix is 5×5 in the decision structure and the tolerably consistent filling of this sized matrix requires significant cognitive effort from non-expert evaluators. Thus, urban transport system improvement is a suitable case study for demonstrating multi-level PAHP.

The proposed multi-level Parsimonious AHP technique

Let us have m criteria structured in a decision problem into l levels. Thus we have k = 1,…, l levels in the decision, $k \epsilon K$, and the m criteria is distributed to the l levels. Note that in this model there are no alternatives applied, only the weights of the criteria are important to be determined. Let us denote p the criteria on a certain level of the decision, so c_{kp} denotes a criterion on a certain level in which if we have g criteria, p=1,…, g. $p \epsilon M$ and M is the set of all criteria of the decision, thus M = 1,…, m. Consequently, c_{kp} denotes the p-th criterion of the k-th level, so that, for example c_{11} is the first criterion of the first level in the hierarchical structure of the decision.

The first step of the suggested method is to select level or levels in the decision for which the Parsimonious AHP will be conducted. It is proposed to select that level(s) which have $g \geq 9$ that can be considered as enough number worth unburdening the evaluators from numerous pairwise comparisons. Moreover, it is recommended (following Saaty's 7 ± 2 rule for a PCM) to select level(s) for which larger or equal to 5×5 pairwise comparison matrices should be evaluated. Based on own experience the pair wise comparisons for a 5×5 matrix might be demanding for layman evaluators.

As second step, direct evaluations have to be made for the chosen level(s) for the c_{kp} criteria with respect to the goal of the decision problem on a scale from 0 to 100.

Then *s* reference elements have to be selected on the chosen level(s) *k*.

Afterwards, the original AHP pairwise comparisons are conducted for the chosen criteria, obtaining the normalized AHP scores for the *s* criteria: $u(c_{ks})$, for all s =1,…,t_p.

Following the PAHP procedure by Abastante et al (2018), consistency and monotonicity are being checked and the required modifications are made.

Finally, the formula (8) is applied for all criteria (c_{kp}) existing on the Parsimonious level(s) using:

$$u(c_{kp}) = u(c_{ke}) + \frac{u(c_{ke+1}) - u(c_{ke})}{\gamma_{ke+1} - \gamma_{ke}} (r_{kp} - \gamma_{ke}). \quad (1)$$

With respect to e= 1,…,t_p and c_{kp} has the importance between the two reference criteria c_{ke} and c_{ke+1}, thus $r_{ke} < r_{kp} < r_{ke+1}$. Having finished with the Parsimonious level(s) the decision structure should be reconstructed in order to gain the final weight and alternative scores and ranking. Consequently, all $u(c_{kp})$-s have to be multiplied by the weight score of its respective element from the previous level *k*-1. Also, due to the characteristics of AHP, for the lower levels, the new $u(c_{kp})$ weight scores have to be applied for multiplying the scores of the respective lower elements.

Testing the new model on a real-world transport development problem

The proposed multi-level Parsimonious AHP model has been tested on a real decision making problem: the possible improvement of a public bus transport system in an emerging city: Mersin, Turkey. From methodological point of view the chosen case study seemed fortunate, since previous experience of this problem has been available, published in Duleba and Moslem 2018 and 2019. The applied decision structure of criteria has become the same as used in other different AHP surveys (Duleba et al, 2012) as exhibited in Figure 1.

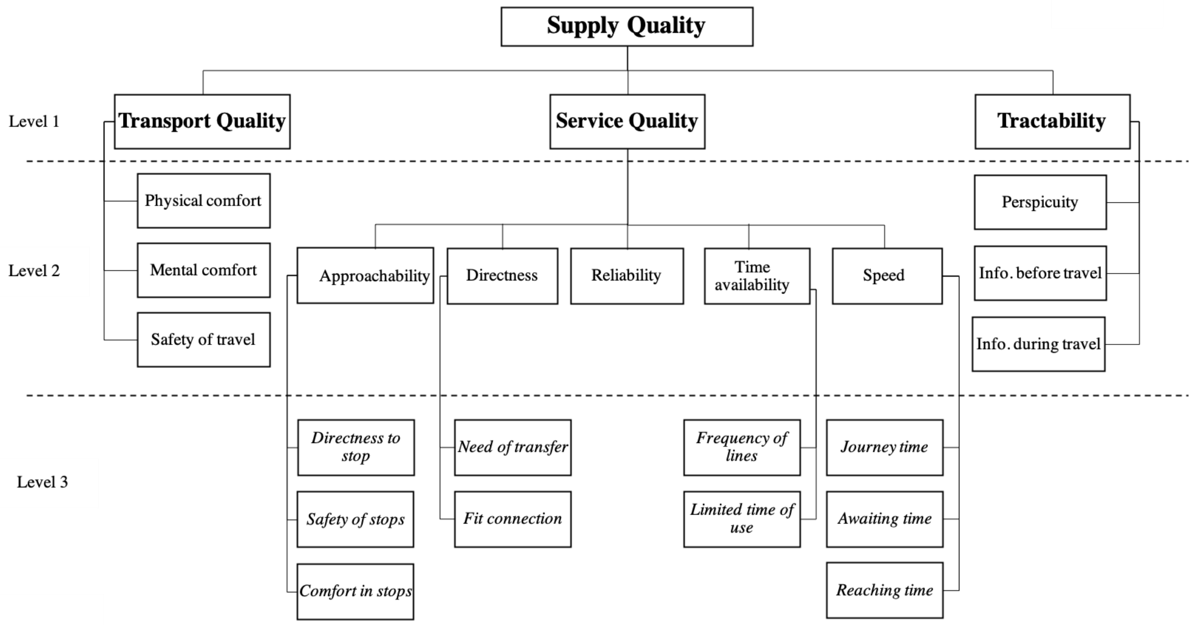

Figure 1. The hierarchical decision structure for public transport development
(Source: Duleba et al., 2012)

For testing the multi-level Parsimonious AHP model, first the Parsimonious level or levels has to be created. As visible on Figure 1., the most appropriate level is Level 2., since it contains most of the criteria (11) and there is a 5×5 PCM containing the Approachability, Directness, Reliability, Time Availability and Speed factors. Based on this, the Parsimonious model for decision making can also be constructed (Figure 2.).

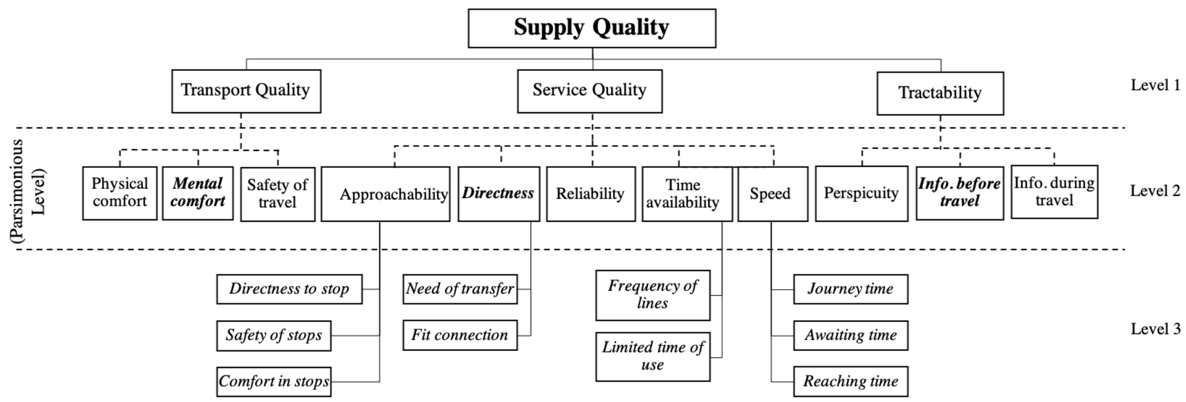

Figure 2. The decision structure reflecting the Parsimonious criteria

Figure 2 demonstrates the logic of a multi-level PAHP model. The criteria in the selected Parsimonious level(s) are handled regardless their position in the decision tree, ignoring their branches (exhibited by dotted line), thus their direct evaluation can be conducted. From this phase, all the 5 steps of PAHP can be followed as introduced in the Methodology section (in bold, the reference elements are exhibited for step 2). Having conducted all steps, for computing the final scores even for the Parsimonious level criteria, the structure is rebuilt so branch connections are considered again by multiplying the scores by the respective previous level element scores.

As a real life pattern, 42 passengers were interviewed in the survey in Mersin for the Parsimonious AHP evaluations both the direct and pairwise part

For each interview, approximately 10 minutes were spent in explaining and filling up the questionnaire and it can be stated that PAHP is far less time consuming than the classical AHP. By using the multi-level PAHP model, altogether 6 PCM-s had to be filled: a 3×3 on the first level, another 3×3 for the Parsimonious level for the reference criteria, and two 3×3 and two 2×2 matrices for the third level elements. All other efforts of the evaluators had just been direct evaluations for the second level criteria which meant a much easier job for them than participating in a classical AHP survey. Taking this decision structure the evaluators should have filled 8 PCM-s including a 5×5 large pairwise comparison matrix. That would have meant altogether 21 pairwise evaluations while in the PAHP survey only 14 had to be done with avoiding the confusing 5×5 PCM. The reflections of the public participants verified the simplicity and the effort and time consuming characteristics of the PAHP process highlighting the better understandability compared to the 2018 AHP survey in which the respondents were complaining about the complexity of the questionnaire.

A direct comparison of an AHP and a PAHP survey

Since the results of a previous AHP survey in the same city using the same decision structure is available it is advisable to conduct a rank comparison between the two surveys. In both research, 42 evaluators were directly interviewed, however, not the same persons, so different scoring could have be expected even if they evaluated the same public transport system without any changes during the one year difference in time. For the comparison, Spearman's rank correlation technique has been applied. The 'R' value of the calculation shows the degree of correlation among the two different rankings. Above 0.5, the correlation can be considered as strong and positive between the two surveys. The following formula has been applied for the calculation:

$$R = 1 - \left(\frac{6\sum d^2}{m^3 - m}\right) \quad (12)$$

In which d is the difference between the ranks and m is the number of elements to be compared.

Table 1: Spearman's Rank correlation coefficient for Level 1.

Criteria	Rank of 2018 AHP survey	Rank of 2019 PAHP survey	d_i	$(d_i)^2$
Service quality	2	2	0	0
Transport Quality	1	1	0	0
Tractability	3	3	0	0
m = 3		R = 1		

Table 2: Spearman's Rank correlation coefficient for Level 2.

Criteria	Rank of 2018 AHP survey	Rank of 2019 PAHP survey	d_i	$(d_i)^2$
Approachability	5	9	-4	16
Directness	3	6	-3	9
Time availability	4	8	-4	16
Speed	11	7	4	16
Reliability	8	3	5	25
Physical comfort	2	1	1	1
Mental comfort	7	4	3	9
Safety of travel	1	2	-1	1
Perspicuity	10	10	0	0
Information before travel	6	5	1	1
Information during travel	9	11	-2	4
m =11		R = 0.5545		

Table 3: Spearman's Rank correlation coefficient for Level 3.

Criteria	Rank of 2018 AHP survey	Rank of 2019 PAHP survey	d_i	$(d_i)^2$
Directness to stops	3	4	-1	1
Safety of stops	6	9	-3	9
Comfort in stops	7	10	-3	9
Need of transfer	2	1	1	1
Fit connection	4	5	-1	1
Frequency of lines	1	2	-1	1
Limited time of use	5	6	-1	1
Journey time	9	7	2	4
Awaiting time	10	8	2	4
Time to reach stops	8	3	5	25
m = 10		R = 0.6606		

The direct comparison resulted in strong rank correlation for all three levels. Note that the third level has been affected by the previous levels and even with this impact, over 66% correlation can be detected, which is remarkable. Selecting the first two most important elements is also noticeable in both surveys. The first level ranking is totally identical, and for the second level, both research identified Physical Comfort and Safety of Travel as most significant criteria, for the third, Frequency of lines and Need of Transfer have been seeded as the most important ones.
Thus, it can be concluded that the direct comparative analysis verified the PAHP methodology.

Conclusion

Applying Parsimonious AHP model instead of the conventional AHP methodology has caused less evaluation time and cost, better understandability for participants, while resulted a very similar final ranking of the decision criteria, for all levels as the Spearman index indicated (strong rank correlation). The immanent logic of AHP has not been strictly kept, but conducting the checking procedure, a so far hidden dominant criterion could be detected and the outcomes of PAHP could be explained and verified. Thus, it can be suggested that the immanent checking step should be integrated to the Parsimonious AHP methodology in case of multi-level models.

As limitation, it can be stated that the unique immanent analysis revealed that the multi-level PAHP method is highly sensitive to the phase of pairwise comparing the reference criteria of the parsimonious level. Consequently, not only the Consistency Ratio should be checked as suggested so far in the scientific literature of the technique, but also the extreme high or extreme low weight scores gained in this phase should be negotiated with the evaluators to ensure the real intention of scoring.

Remarking the further research, many other parsimonious applications are necessary to get familiar with all characteristics of this new methodology. The objective benefits are clear, it provides faster and cheaper survey process and undoubtedly, the survey pattern can more easily be extended by this technique than applying the complex pairwise comparison questionnaire of the conventional AHP. But are the results of PAHP as trustworthy as AHP which has been applied many times by many researchers successfully? This paper merely provided one example but only many other applications can verify the technique ultimately.

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