AHP-Pareto Model for Evaluating Public Bus Transport

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According to recent findings in the applied operations research, the eigenvectors derived from pairwise comparison matrices are not Pareto-efficient in all cases. Thus, efficient outcomes may exist for characterizing the decision makers' opinion than the traditional eigenvector method of Saaty and the Pareto-efficient modified vector may lead to better decisions. The objective of the paper is to dissect Pareto-efficiency on real data pairwise comparison matrices gained from a public bus transport analytic hierarchy process survey. Moreover, digging out the repercussion of weight score modification on the whole analytic hierarchy process structure and thus the significance of Pareto test is also a scope of the current study. A detailed description of the general process of Pareto-efficient AHP is also included. The application has been conducted in Mersin city, Turkey with the purpose of highlighting public preference on the importance of developing supply quality elements in public bus transportation service.

Keywords: Public transport, Supply quality, Analytic Hierarchy Process (AHP), Pareto-efficiency, Decision making.

1. Introduction

Recently, public transport (PT) system has been the most significant issue for many experts and decision makers in all over the world, as being a significant part of the public's daily life. However, revaluating PT system would improve not only the supply quality of PT, but also it would ameliorate public life standard quality. The impact of the improving PT could be distinctly detected from many aspects; by reducing congestion problems (Cats et al., 2017), reducing carbon footprint (Cavallaro et al., 2018), decreasing gasoline consumption (Ercan et al., 2016). The experts in the related field conducted many researches in order to enhance PT supply quality in different countries in entire continents.

The pairwise comparison (PC) technique is the basic step of the Analytic Hierarchy Process (AHP), which are one of the most popular multi criteria decision making approaches. The PC and its improvements were conducted not only in the transportation researches (Farooq and Moslem, 2019; Ghorbanzadeh et al., 2019; Moslem and Duleba, 2018; Duleba and Moslem, 2018), but also in different research fields (Awasthi et al., 2018; Ishizaka and Labib, 2010; Ivanco et al., 2017) in order to solve the complex decision problems. However, many researchers improved the original AHP results in order to get more efficient outcomes for the complex problems. The eigenvector approach is the main step of AHP approach (Saaty, 1977) and it is based on the consistent pairwise comparison matrices. The researchers used four different methods to find the eigenvector (Saaty, 1977; Crawford and Williams, 1985; Fülöp, 2008; Bozóki and Tsyganok, 2017). The aim of the paper is to highlight Pareto efficiency test on the original AHP weight scores gained from a real public transport development survey, analyse the inefficient cases, propose optimized weight scores and indicate the modification of final AHP scores of the decision structure. It is also an aim to suggest Pareto efficiency (Pareto, 1906) of eigenvectors for other appliers of AHP approach in order to get more reliable outcomes and more effective final decisions.

2. Research Methodology

Recently, the researchers in the decision and operation fields spotted the light on the efficiency issue of the derived eigenvector from the classic AHP approach, which is a popular approach for analysing the complex problems. Thus, the Pareto efficiency was recommended in order to get more reliable and robust decisions (Blanquero et al., 2006; Bozóki and Fülöp, 2018; Duleba and Moslem, 2019). The paper aims to use AHP-Pareto Model to enumerate and ameliorate public bus transport supply quality system in Mersin, Turkey.

The process of applying the new AHP-Pareto Model composed of eleven steps: (I) Setting up the hierarchical structure of decisions and alternatives (II) Crating pairwise comparison matrices (PCM-s); (III) Constructing the questionnaire survey; (IV) Checking the consistency of the PC-s; (V) Aggregating by geometric mean; (VI) Driving weight vectors; (VII) Selecting (4×4) matrices or bigger ones; (VIII) Pareto efficiency test for the eigenvector of selected matrices; (IX) if it is efficient then the matrix has to be modified; (X) Calculation the final scores; (XI) Sensitivity analysis.

The first step of AHP approach done by Duleba et al., (2012), in the second step the following matrices were constructed: five (3×3) PC matrices, two (2×2) matrices and one (5×5) matrix, the judgement matrix denoted as:

$$A = [a_{ij}], \quad i,j = 1,2,\ldots,n. \tag{1}$$

The entries a_{ij} are governed by the following equations:

$$a_{ij} > 0, \quad a_{ji} = \frac{1}{a_{ij}}, \quad a_{ii} = 1. \text{ for all } i. \tag{2}$$

Table 1. Judgment scale of relative importance (Saaty's 1-9 scale)

Numerical values	Verbal scale	Explanation
1	Equal importance of both factors	Two elements contribute equally
3	Moderate importance of one factor over another	Experience and judgment favour one factor over another
5	Strong importance of one factor over another	An factor is strongly favoured
7	Very strong importance of one factor over another	An factor is very strongly dominant
9	Extreme importance of one factor over another	An factor is favoured by at least an order of magnitude
2,4,6,8	Intermediate values	Used to compromise between two judgments

The experts are supposed to specify their judgments of the relative importance of each contribution of criteria towards achieving the overall goal in the third step. In the fourth step the consistency check should be done after computing the weight eigenvectors $W = (W_1, W_2, \ldots\ldots W_n)$, and the multiplication of the matrix pairwise ratios with W is $n.W$, where n is the number of the criteria in the judgement matrix however, Saaty's approach calculates W as the principal right eigenvector of A (Saaty, 1997):

$$A\,W = \lambda_{max}\,W, \tag{3}$$

Where λ_{max} is the maximum eigenvalue of A, and the eigenvector can be computed by the following equation when A is a consistency matrix:

$$(A - \lambda_{max}\,I)W = 0. \tag{4}$$

Saaty (1990) defined the consistency index (CI) and the consistency ration (CR) by the following equations:

$$\text{CI} = \frac{\lambda_{max}-n}{n-1}, \tag{5}$$

$$\text{CR} = \text{CI} / \text{RI} \tag{6}$$

Where (RI) is random index that represents the average consistency index over numerous random entries of same order reciprocal matrices. A value is considered acceptable when the consistency ratio (CR) is smaller than 0.1, otherwise the evaluator has to revise his judgements (Saaty, 1994). Aggregating the experts judgements come in the fifth step by applying the following equation as Aczél and Saaty (1983) recommended, If "h" evaluators take part in the procedure an aggregated matrix is to be created as the following equation:

$$A = \left[\sqrt[h]{\textstyle\prod_{k=1}^{n} a_{ijk}}\right] i,j = 1, \ldots, n \tag{7}$$

where a_{ijk} denotes entries, in the same position (i,j), of PCM-s, filled in by the k-th decision maker. The following steps include driving weight vectors and computing final scores. The final step of the approach done by detecting sensitivity analysis which enables in understanding the effects of changes in the main criteria on the sub criteria ranking and help the experts to check the robustness throughout the operation.

However, it has been proven that the eigenvectors and the final scores of the criteria and alternatives are not necessarily Pareto-efficient (Bozóki and Fülöp, 2018) in case of (4×4) or larger sized matrices. Therefore w_{Ai}-s can be tested and improved with dominant weight vectors that give better approximation to a_{ij}-s than the eigenvectors.

In the traditional AHP approach, the eigenvector method deems that a_{ij} values in the PC process which provided by the evaluators approximate well the w_i/w_j ratio which are intern the consistent PC matrix values. From here, we can conducted the Perron eigenvector by taking $a_{ij} - s$ as $\frac{w_i}{w_j} - s$ with considering the acceptable consistency ratio (CR). That is why the deriving weight scores from PC matrices can be considered a multi-objective optimization problem where the difference between $a_{ij} - s$ and $\frac{w_i}{w_j} - s$ have to be minimized.

So, from the methodological point of view it is a multi-objective optimization problem and the general formula is (Ehrgott, 2000):

$$\min\{f_1(y), f_2(y), f_2(y), \ldots \ldots f_m(y), \ldots \ldots f_M(y)\}$$
$$\text{while } y \in S,$$

, where $M \geq 2$, the number of objective functions and $f_m : \mathbb{R}^n \longrightarrow \mathbb{R}$ for at least $1 \leq m \leq M$, the variables are $y = (y_1, y_2, y_3, \ldots \ldots . y_n)$, and the set of possible solutions: $S \subseteq \mathbb{R}^n$. An $y \in S$ vector is Pareto optimal if there is no $y' \in S$, for which $f_m(y') \leq f_m(y)$ and $1 \leq m \leq M$, $f_m(y') \leq f_m(y)$.

Let **A** be a pairwise comparison matrix, $A = [a_{ij}]_{i,j=1,\ldots,n} \in PCM_n$ and $w = (w_1, w_2, w_3, \ldots . w_n)^{\top}$ is a positive weight vector (for $S = \mathbb{R}^n_{++}$, the positive orthant), and n is the number of criteria in the examined matrix. The objective function $f_{ij}(w) := \left|a_{ij} - \frac{w_i}{w_j}\right|$ for all $i \neq j, M = n^2 - n$ is the total number of the objective functions.

Based on this, the following definition can be constructed (Blanquero et al, 2006):
A positive weight vector **w** is Pareto optimal if there is no other positive weight vector $w' = (w'_1, w'_2, w'_3, \ldots . w'_n)^{\top}$, for which

$$\left|a_{ij} - \frac{w'_i}{w'_j}\right| \leq \left|a_{ij} - \frac{w_i}{w_j}\right|, \quad \text{for all} \quad 1 \leq i, j \leq n \quad \text{and}$$

$$\left|a_{kl} - \frac{w'_k}{w'_l}\right| < \left|a_{kl} - \frac{w_k}{w_l}\right|, \quad \text{for some} \quad 1 \leq k, l \leq n .$$

However, the objective function for *n* criteria in the tested matrix is:

$$f_{ij}(w) := \left|a_{ij} - \frac{w_i}{w_j}\right| \quad \text{for all} \quad \mathrm{i} \neq \mathrm{j}, M = n^2 - n$$

Where M is the total number of the objective functions, and the following function can be conducted to create the minimum difference between $a_{ij} - s$ and $\frac{w_i}{w_j} - s$ values:

$$f_{ij}: \mathbb{R}^n_{++} \longrightarrow \mathbb{R}, i, j = 1, \ldots, n, \qquad (8)$$
$$\min \mathrm{f(x)}$$

This optimization problem is nonlinear but it can be transformed to a linear program by taking the logarithm of the objective function and constraints, the details about the conducted technique description is explained in (Blanquero et al., 2006; Bozóki and Fülöp, 2018).

Assuming a w' weight vector could be found that dominates the w eigenvector from Pareto aspect so can be considered as more effective than w. In Pareto-efficient AHP w have to be replaced by w' and in case there are lower level elements connected with w, their related weight coordinate are also to be modified by the respective w' coordinate, thus this modification flows through all the connected elements of w.

$$w_{A_i}{}' = \frac{w'_j}{w} \frac{w_{ij}}{\sum_{k=1}^{n} w_{ik}} = \left(\frac{w'_j}{w} \frac{1}{\sum_{k=1}^{n} w_{ik}}\right) k_{ij} \qquad (8)$$

where w'_j is the respective coordinate of the modified eigenvector from the previous level, $w_{Ai}{}'$ is the modified final score of the lower level element.

3. Conducted Results

In August 2017, an AHP surveys were conducted in Mersin Metropolitan area (Turkey) to examine public bus transport system supply quality. The employed hierarchical structure of criteria in a public bus transport development decision was developed by Duleba and his colleagues (Duleba et al., 2012).

As stated in this paper it has been proven that Pareto efficiency is always refer at least (4 × 4) matrices or bigger. In the presented hierarchy model there is only one possible non efficient matrix that is bigger than (4 × 4) matrix and suitable for Pareto efficiency test. This matrix locates in level 2 and it has 5 coordinates.

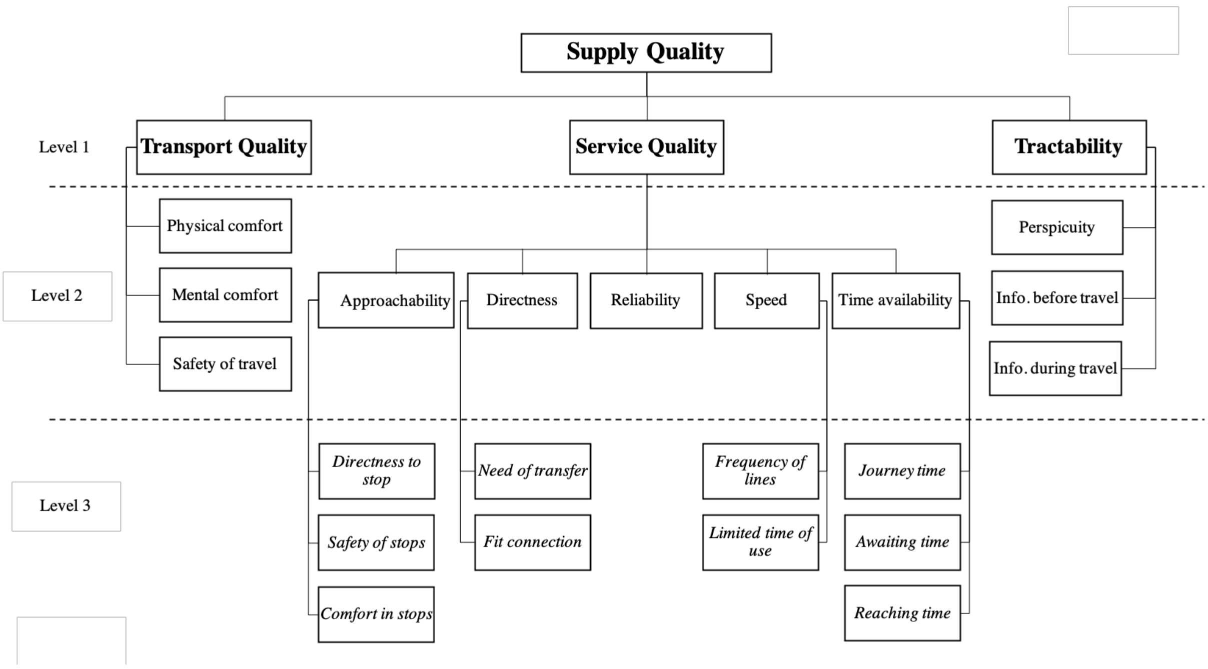

Figure 1. Title The hierarchical structure of criteria in a public bus transport development decision (Duleba et al., 2012).

Based on the findings, the most critical issue was “Service quality” followed by “Transport quality” and “Tractability”. The optimized scores in the first level from the original AHP scores was the same (Table 2), because in our case study the detected inefficient eigenvector was the in the second level for the (5×5) matrix. However, only the scores in the second and third level will be modified based on Pareto optimality.

Table 2. Final preference ranking of criteria by evaluators for level 1.

Ranking of criteria by the original scores			Ranking of criteria by the optimized scores		
Rank	**Criteria**	**Score**	**Rank**	**Criteria**	**Score**
1	Service Quality	0.5086	1	Service Quality	0.5086
2	Transport Quality	0.3215	2	Transport Quality	0.3215
3	Tractability	0.1698	3	Tractability	0.1698

The optimized score values in the second and third level was slightly different from their original score values. However, the criteria ranking was stable in the third level, where the most important criteria was the “Fit connection” followed by the “Frequency of lines”, “Directness to stops” and “Safety of stops” (Table 3).

Table 3. Final preference ranking of criteria by evaluators for level 2.

Ranking of criteria by the original scores			Ranking of criteria by the optimized scores		
Rank	**Criteria**	**Score**	**Rank**	**Criteria**	**Score**
1	Reliability	0.1996	1	Reliability	0.1991
2	Safety of travel	0.1891	2	Safety of travel	0.1891
3	Mental comfort	0.1033	3	Mental comfort	0.1033
4	Directness	0.0873	4	Approachability	0.0888
5	Approachability	0.0856	5	Directness	0.0881
6	Time availability	0.0824	6	Time availability	0.0785
7	Information during travel	0.0600	7	Information during travel	0.0600
8	Perspicuity	0.0561	8	Perspicuity	0.0561
9	Speed	0.0538	9	Speed	0.0540
10	Information before travel	0.0537	10	Information before travel	0.0537
11	Physical comfort	0.0292	11	Physical comfort	0.0292

In the second level, a slight change was detected where, in the original ranking the “Directness” was in the fourth level followed by the “Approachability”, but in the optimized outcomes the “Approachability” was in the fourth level followed by the “Directness” (Table 4).

Table 4. Final preference ranking of criteria by evaluators for level 3.

Ranking of criteria by the original scores			Ranking of criteria by the optimized scores		
Rank	**Criteria**	**Score**	**Rank**	**Criteria**	**Score**
1	Fit connection	0.0740	1	Fit connection	0.0747
2	Frequency of lines	0.0558	2	Frequency of lines	0.0532
3	Directness to stops	0.0346	3	Directness to stops	0.0359
4	Safety of stops	0.0330	4	Safety of stops	0.0343
5	Limited time of use	0.0329	5	limited time of use	0.0313
6	Awaiting time	0.0200	6	Awaiting time	0.0201
7	Comfort in stops	0.0179	7	comfort in stops	0.0186
8	Time to reach stops	0.0174	8	Time to reach stops	0.0175
9	Journey time	0.0164	9	Journey time	0.0165
10	Need of transfer	0.0133	10	Need of transfer	0.0134

4. Conclusions

Within this research, we illustrated how applying Pareto-efficiency test could provide more reliable outcomes by optimizing the non-efficient eigenvectors which is one of the main step of the AHP process. Moreover, the optimized outcome would have a slight changes comparing with the original ones, not only a change with the criteria scores, but also a change with criteria ranking. From here, Pareto optimality test, which does not require much conduction, is recommended to be added as a part of the original AHP approach where the hierarchy includes 4×4 or bigger PC matrices. For future study, Pareto optimality would be included in the analytic network process to generate more reliable results,

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