The value of travel time information

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1. Introduction

In a series of papers, Noland and Small (1995), Fosgerau and Karlström (2010), and Engelson (2011) have derived valuations of travel time variability from scheduling preferences and a random distribution of travel time, assuming that travellers choose departure time to maximise their expected scheduling utility. Applications of this model assume that the pertinent travel time distributions can simply be estimated from a sample of travel time measurements.

The traveller may, however, obtain information in some form, a signal, about the travel time before the trip. She may then consider the distribution of travel time conditional on the signal to increase her expected utility by making a better informed choice of departure time. The signal does not have to be a perfect prediction of the travel time. It is sufficient that it carries information about the travel time.

The aim of this study is to derive the value of travel time information obtained prior to the choice of departure time. This knowledge may facilitate design and cost-benefit analysis of traveller information systems and policies decreasing travel time variability. We show that a signal always increases the expected utility compared to the situation without any signal. Furthermore, we demonstrate that even perfect travel time information does not necessarily eliminate the cost of travel time variability and we establish necessary and sufficient conditions for when it does. We find that the predictable part of travel time variability may or may not be costly, depending on the shape of the traveller's scheduling utility at the origin of the trip. Using estimates of scheduling preferences from the literature, we show that the cost of predictable travel time variability may constitute a substantial part of the total cost of travel time variability. In a particular case of scheduling preferences, travel time distribution and noise distribution we establish an analytic relationship between the strength of the signal and the expected utility of the trip and evaluate the cost of signal weakness.

2. The model

Let $u(r,t)$ be the utility of travel given departure at time r and travel duration t. The travel duration is a random variable T with distribution independent of departure time r where $r \in [t_0, t_1]$. In general, the traveller does not know the travel duration in advance but she knows the distribution of random variable T. The departure time is chosen at time $t_c < t_0$. If no other information is available to the traveller, she chooses the departure time maximising the expected utility of travel, $Eu(r,T)$.

The traveller may receive some information related to the travel duration. If this information is received before t_c, it may affect the decision. We specify this information in the form of another random variable S called a "signal". For example, S may be the travel duration between the origin and the destination of the intended trip at some time before t_c. In order to carry information on the travel duration T the signal

S has to have some dependence with T. We assume for tractability that there is a relationship $S = T + \sigma X$ where X is white noise that is independent of T and with $EX = 0$ and $E(X^2) = 1$. The traveller is assumed to know the parameter σ and the distribution of X but not its realised value before the planned trip. A low value of σ means that the signal is strong. Then we may say that parameter σ is the weakness of the signal, i.e. the larger is σ the lower is the correlation between S and T and the more difficult it is for the traveller to infer the travel duration from the signal. Knowing the realized value $S = s$, the traveller selects optimal departure time $r^*(s,\sigma) = \underset{r}{\operatorname{argmax}}\, E[u(r,T)|T + \sigma X = s]$.

Given a non-degenerate distribution of travel duration T, two extreme situations are possible. If $\sigma = 0$ then $T = S = s$ is known to the traveller at time t_c and she chooses the departure time $r^*(s,0)$ obtaining the utility maximising the function $u(r,T)$. The average utility accrued is then $u_p = E\left[\max_r u(r,T)\right] = E[u(r^*(T,0),T)]$ (where subscript p means perfect information). On the other hand, in the absence of any signal or if the signal is uncorrelated with the travel duration (which corresponds to $\sigma = +\infty$), then the choice of departure time is based on the *a priori* distribution of travel duration and the expected utility is $u_r = \max_r Eu(r,T)$. With a positive and finite σ, the traveller obtains on average the expected utility $U_S(\sigma) = E_S\left\{\max_r E[u(r,T)|T + \sigma X = s]\right\}$. Denoting $f_S(s)$ the probability density of S and $f(t|s)$ the conditional density of T with respect to $T + \sigma X = s$, the average expected utility can be expressed as

$$U_S(\sigma) = \int \left[\max_r \int u(r,t) f(t|s) dt\right] f_S(s) ds = \iint u(r^*(s,\sigma),t) f_{T,S}(t,s) dt ds,$$

where $f_{T,S}$ is the joint density distribution.

3. Value of perfect and imperfect information

In this section, we investigate the value of information about travel duration obtained before the choice of departure time. In particular, we consider the situation with perfect information ($\sigma = 0$) and compare it to the case of constant travel duration. Even if the traveller knows the exact travel duration when choosing the departure time, the travel duration still varies from day to day. The mean utility obtained in this case is therefore $u_p = E\left[\max_r u(r,T)\right]$. This should be compared to the utility obtained when the travel duration has the same mean value but does not vary, $u_f = \max_r u(r,ET)$. If $u_p = u_f$, then perfect information eliminates the cost of travel time variability. On the other hand, if $u_p < u_f$ then even the predictable part of the travel time variability has a positive cost.

Proposition 1. For any travel time distribution and for any $\sigma \geq 0$, $u_r \leq U_S(\sigma) \leq u_p \leq u_f$.

This proposition establishes that the traveller using a signal to correct the travel time distribution may increase the utility of travel up to a threshold corresponding the perfect information. However even perfect information cannot increase the utility of travel above the case with constant travel time.

In order to establish the next result, we need to assume (like Vickrey, 1973) that the utility of the trip is separable in time spent at the origin and time spent at the destination, i.e.

$$u(r,t) = \int_0^r h(\tau) d\tau - \int_0^{r+t} w(\tau) d\tau,$$

where h and w are rates of utility obtained at the origin and at the destination of the trip, respectively, $h' \leq 0 < w'$, and $h(0) = w(0)$.

Proposition 2. Assume that the travel duration T is non-negative and its support $\mathrm{supp}\, T$ is an interval. Then $u_p = u_f$ if and only if there exists such $t \geq 0$ that $h(r) = w(t)$ for any $r \in t - \mathrm{supp}\, T$.

Proposition 2 shows that, with strictly decreasing rate of utility at origin and non-degenerate travel time distribution, even the perfect travel time information cannot eliminate the cost of travel time variability, i.e. even predictable variability of travel time has a positive cost. The only case when the traveller is indifferent between the situation with perfect travel time information and the situation with constant travel time is when the rate h of utility of time at origin is constant on a certain interval depending on the support of travel time distribution and the function w. In this case, the traveller's arrival time is the same for any possible travel duration and her utility is an affine function of travel duration. In particular, the constant h on interval $]-\infty, 0]$ guarantees $u_p = u_f$ for any non-negative random T.

4. Value of perfect travel time information and predictable travel time variability: an example

In order to illustrate the cost of predictable travel time variability consider now an example with strictly decreasing utility rate at origin of the trip. Following Engelson (2011) assume the exponential form of scheduling preferences: $h(t) = h_0 + \alpha \exp(\beta t)$ and $w(t) = w_0 + \gamma \exp(\beta t)$ with $\beta > 0, \alpha < 0, \gamma > 0$. The utility without signal, with perfect information and with constant travel time are then

$$u_r = \frac{\alpha - \gamma}{\beta} \ln \frac{\gamma E e^{\beta T} - \alpha}{\gamma - \alpha} - w_0 ET$$

$$u_p = \frac{\alpha - \gamma}{\beta} E \ln \frac{\gamma e^{\beta T} - \alpha}{\gamma - \alpha} - w_0 ET,$$

and

$$u_f = \frac{\alpha - \gamma}{\beta} \ln \frac{\gamma e^{\beta ET} - \alpha}{\gamma - \alpha} - w_0 ET.$$

The total cost of travel time variability is $u_f - u_r$ while the cost of predictable travel time variability is $u_f - u_p$ and the value of perfect travel time information is $u_p - u_r$.

As an illustration, let us assume that T is uniformly distributed on $[A, B]$ as in Hjorth et al (2015) where parameters for the exponential scheduling preferences were estimated based on SP data from Stockholm. Then the involved expectations are $ET = (A + B)/2$, $Ee^{\beta T} = \frac{\exp(\beta B) - \exp(\beta A)}{\beta(B - A)}$, and $E\ln\left(\gamma e^{\beta T} - \alpha\right) = \ln(-\alpha) - \frac{1}{\beta(B-A)}\left[\mathrm{Li}_2\left(\frac{\gamma e^{\beta B}}{\alpha}\right) - \mathrm{Li}_2\left(\frac{\gamma e^{\beta A}}{\alpha}\right)\right]$ where Li_2 is Spence's dilogarithm (Lewin, 1958). Substitute $A = 15, B = 55, \alpha = -3.4, \beta = 0.00287, \gamma = 6.28, w_0 = -5.3$ from Hjorth et al (2015; Tables 1 and 5, All trips) to obtain $u_r = -39.38, u_p = -38.48$, and $u_f = -38.13$. This gives a cost of travel time variability of 1.24, a value of perfect travel time information of 0.90 and a cost of predictable travel time variability of 0.34, which constitutes 27% of the total. A similar result (21%) is obtained by using exponential scheduling parameters for the subsample of travellers with fixed working time estimated in Hjorth et al (2015). We can then conclude that the share of the cost of predictable travel time variability that cannot be eliminated by travel time information may be substantial.

5. Value of signal strength: an example

In general, utilities u_r, u_p and u_f depend on travel time distribution (u_f just on the mean travel time) and the scheduling preferences while $U_s(\sigma)$ depends on distribution of the noise as well. Although calculation of $U_s(\sigma)$ often needs simulation, it can be calculated analytically at least for some combinations of scheduling preferences, travel duration and noise distribution.

Similar to Vickrey (1969), assume a constant utility rate $h > 0$ at the origin and a step utility rate at destination, $w(t) = h - \alpha$ for $t < 0$ and $w(t) = h + \beta$ for $t > 0$, with $\alpha > 0$ and $\beta > 0$. If travel duration T is uniformly distributed on $[0,1]$ and noise X is uniformly distributed on $[-½, ½]$ then the optimal departure time based on the signal $S = T + \sigma X$ can be shown to equal

$$r^*(s,\sigma) = -\left[\alpha \max\left(0, s - \frac{\sigma}{2}\right) + \beta \min\left(1, s + \frac{\sigma}{2}\right)\right] / (\alpha + \beta),$$

while the expected utility of the trip is

$$U_s(\sigma) = \begin{cases} \dfrac{\alpha\beta}{2(\alpha+\beta)}\left(\dfrac{\sigma^2}{3} - \sigma\right) - \dfrac{h}{2} & \text{if } \sigma \leq 1, \\ \dfrac{\alpha\beta}{2(\alpha+\beta)}\left(\dfrac{1}{3\sigma} - 1\right) - \dfrac{h}{2} & \text{if } \sigma > 1. \end{cases}$$

The value $-U_s{}'(\sigma)$ of signal strength can now be easily calculated as $\frac{\alpha\beta}{6(\alpha+\beta)}(3 - 2\sigma)$ for $\sigma \leq 1$ and $\frac{\alpha\beta}{6(\alpha+\beta)}\sigma^{-2}$ for $\sigma > 1$, and it is always positive. Hence the traveller obtaining stronger signal incurs a higher utility on average. In particular, with perfect travel time information the mean utility is $U_s(0) = -h/2 = u_p$ which is equal to u_f in accordance with Proposition 2. Hence the predictable travel time variability has no cost in this example. Without any signal, the choice of departure time is based on the *a priori* travel time distribution and the expected utility is $u_r = \lim_{\sigma\to\infty} U_s(\sigma) = -\frac{\alpha\beta}{2(\alpha+\beta)} - h/2$.

References

L Engelson (2011) Properties of expected travel cost function with uncertain travel time. *Transportation Research Record: Journal of the Transportation Research Board*, **2254**, 151–159.

M Fosgerau and A Karlström (2010) The value of reliability. *Transportation Research, Part B*, **44**(1), 38–49.

K Hjorth, M Börjesson, L Engelson and M Fosgerau (2015) Estimating exponential scheduling preferences. *Transportation Research, Part B*, **81**, 230-251.

L Lewin (1958) Dilogarithms and associated functions. London: Macdonald.

R B Noland and K Small (1995) Travel-time uncertainty, departure time choice, and the cost of morning commutes. *Transportation Research Record: Journal of the Transportation Research Board* **1493**, 150–158.

W S Vickrey (1969) Congestion theory and transport investment. *American Economic Review*, **59**(2), 251-261.

W S Vickrey (1973) Pricing, metering, and efficiently using urban transportation facilities. *Highway Research Record*, **476**, 36-48.