1. Introduction

Up to now, literature has identified and quite deeply addressed two ways how the introduction of AVs may affect future traffic flows and congestion. First, reduced inconvenience of travel (captured in a penalty or value of travel time) may cause people to desire more car travel, longer distances and increase traffic volume and congestion. Second, reduced headways between AVs may help to increase the capacity and ease congestion, especially in the high penetration scenarios. Much discussion has been devoted to which of these two effects will dominate in which scenarios (e.g., Wadud et al. 2016), and both effects have been modelled for their impact on traffic congestion and congestion pricing policy (van den Berg & Verhoef 2016, Simoni et al. 2019).

This paper proposes that there is a third, so far not considered way how automated vehicles may influence future traffic congestion: via changed preferences for departure times. Future travellers may be able to engage in advanced activities in the AV and therefore may shift the origin or destination activity to the automated vehicle. That might result in a desire to depart from origin earlier (and shift origin-type activities to the travel episode) or depart and arrive at the destination later (and shift destination-type activities to the travel episode). In the context of morning commute, traveller may choose to perform in the AV ‘home activities’, such as getting ready (brushing teeth, getting hair done, applying make-up), preparing and eating breakfast, getting a little more sleep, or ‘work activities’, such as replying emails, planning the day, adjusting meeting schedule, checking news.

This paper offers a simple model that captures a shift of origin or destination activities into the automated vehicle in the context of dynamic bottleneck congestion. I adopt a setting where a number of travellers desire to travel from a single origin to a single destination and have the same preferences for the timing of the travel. Given this setting, I obtain results for three cases:

1. The case of a single traveller or many travellers who do not create a congestion (the bottleneck capacity is larger than the number of travellers). I observe that, depending on whether the automated vehicle is better equipped for home or work activities, traveller(s) desire to travel to the work place earlier or later, respectively. This result is obtained analytically using a very generic type of scheduling preferences.

2. The case of many travellers who create a congestion (the bottleneck capacity is smaller than the number of travellers) and use the same type of AVs. I obtain that home- and work-facilitating AVs skew the congestion towards early and late times, respectively, and intensify it
– the queueing times with AVs are longer than with conventional vehicles and longest with AVs that facilitate home activities. This result is obtained analytically assuming the so-called $\alpha - \beta - \gamma$ scheduling preferences (Vickrey, 1969; Arnott et al., 1990).

3. The case of many travellers with two types of AVs – such that facilitate home and work activities. I combine the results from point 2 and find that AV-congestion is more intense than with conventional vehicles, but, in many scenarios, less intense than when all travellers have home-facilitating AVs. This result applies, as before, to the $\alpha - \beta - \gamma$ scheduling preferences.

This work is policy-relevant: it provides clues that having heterogeneous AVs in the population – some that facilitate work- and some that facilitate home-type activities – could mitigate the increasing congestion levels with AVs. Clearly, much work still needs to be done to investigate more complex scenarios and to balance the effect due to departure time changes with the two established effects of potentially induced travel and increased capacity. Yet, this paper offers a first step towards a richer understanding of the mobility in the AV-era.

The remainder of the short paper is structured as follows: section 2 introduces the scheduling preferences that capture the shift of home or work activities to the vehicle. Sections 3 – 5 demonstrate the departure time and congestion impacts of the new scheduling preferences for cases 1 – 3 above. Section 6 concludes.

2. The new scheduling preferences

Let us assume that marginal home and work utilities $h(x)$ and $w(x)$ are monotonously decreasing and increasing, respectively, in a morning time interval $[0, \Omega]$. This is the most generic form of scheduling functions (Vickrey, 1973). Conventionally, it is assumed that the travel episode ‘cuts out’ utility of either home or work activities, whichever is higher at any time moment. In the context of AVs¹ however, I assume that the travel episode does not cut out, but reduces the utility of either home or work activities with multiplicative efficiency factors $0 \leq e_h, e_w \leq 1$, respectively.² From these conditions follows that the optimal time for on-board home activity (if any) is at the start of the trip, and similarly the optimal time for the on-board work activity (if any) is at the end of the trip. I further assume that marginal home and work utilities $h(x)$ and $w(x)$ are positive for $x \in [0, \Omega]$. From here follows that the traveller wants to continually engage in on-board activities and that there is an optimal switching point between the two activities, which I express as a share of the trip duration $k \in [0,1]$. See Figure 1 for the illustration of the model setup.

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¹ This could also apply to travel in public transport, albeit to a lesser extent.
² Future work could explore scenarios where $e_h$ or $e_w$ exceed 1 (AVs facilitating activities better than stationary locations) as well as alternative specifications for the utility of on-board activity.
Figure 1 Model setup, including the utility obtained from home and work activities on board

Total home, work and total utilities are as follows:

\[ H(t, k) = \int_{0}^{t} h(x)dx + e_h \int_{t}^{t+kT(t)} h(x)dx, \]  \hspace{1cm} (1)

\[ W(t, k) = \int_{t+T(t)}^{\Omega} w(x)dx + e_w \int_{t+kT(t)}^{t+T(t)} w(x)dx, \]  \hspace{1cm} (2)

\[ V(t, k) = H(t, k) + W(t, k). \]  \hspace{1cm} (3)

Departure time is \( t \), and travel time depends on it: \( T(t) \). Every traveller tries to maximise the total utility function \( V(t, k) \) by choosing the departure time \( t \) and the switching point between the on-board activities \( k \).

3. Case of no congestion

Having introduced the scheduling preferences, it is easiest to investigate, what would be the optimal departure time of a single traveller. The derivation is the same as for multiple travellers that do not create congestion (when the bottleneck capacity is the same or larger than the number of travellers). Formally, this situation can be summarised as travel time being independent from the departure time and constant: \( T(t) = T \). We can jointly obtain the optimal departure time \( t_o \) and the optimal switching point \( k_o \) between the home- and work-type activities on board by taking the derivatives of the total utility function (3), with respect to the switching point \( k \) and departure time \( t \).

\[ 0 = \frac{\partial}{\partial k} V(t, k) = e_h T * h(t + kT) - e_w T * w(t + kT) \]  \hspace{1cm} (4)

\[ 0 = \frac{\partial}{\partial t} V(t, k) = h(t) + e_h (h(t + kT) - h(t)) - w(t + T) \]
\[ + e_w (w(t + T) - w(t + kT)) \]  \hspace{1cm} (5)
Inserting (4) in (5), we obtain

\[ t_o: h(t_o)(1 - e_h) - w(t_o + T)(1 - e_w) = 0, \]  
\[ k_o: e_h h(t_o + k_o T) - e_w w(t_o + k_o T) = 0. \]

(6)
(7)

The first equation determines the optimal departure time \( t_o \), and the second determines the optimal switching point \( k_o \) between the home- and work-type activities on board.\(^3\) In order to demonstrate the change in the optimal departure time depending on the type of AV, we should distinguish between three cases:

a) AV facilitates work- and home-type activities equally well: \( e_h = e_w \);

b) AV facilitates work-type activities better than the home-type activities: \( e_h < e_w \);

c) AV facilitates home-type activities better than the work-type activities: \( e_h > e_w \).

a) If \( e_h = e_w \neq 1 \), optimal departure time from (6) is

\[ t_1 \text{ s.t. } h(t_1) = w(t_1 + T). \]

This result holds for a conventional car \( (e_h = e_w = 0) \), as well as any AV, where home and work activities are equally well (but not perfectly) facilitated. The optimal departure times are equivalent for all these vehicles. In case of an ideal AV – where home and work activities are perfectly facilitated: \( e_h = e_w = 1 \) – every departure time in the interval \([x^* - T, x^*]\), where \( x^*: h(x^*) = w(x^*) \), is optimal. There is an indifference interval for departure times. This interval follows from equation (7) and from \( 0 \leq k_o \leq 1 \).

b) If work is better facilitated in the AV \( e_h < e_w < 1 \), equivalently \( 1 - e_h > 1 - e_w \), we can observe from (6) that optimal departure time is

\[ t_2 \text{ s.t. } h(t_2) < w(t_2 + T). \]

Therefore \( t_1 < t_2 \): the optimal departure time is later than with conventional car.

c) If home-type activities are better facilitated in AV than work \( 1 > e_h > e_w \), equivalently \( 1 - e_h < 1 - e_w \), we can in the same way observe from (6) that the optimal departure time is

\[ t_3 \text{ s.t. } h(t_3) > w(t_3 + T). \]

Therefore \( t_1 > t_3 \): the optimal departure time is earlier than with conventional car.

Hereby, we have observed that travellers who perform home or work activities in an AV would shift their optimal departure time to earlier and later, respectively. This is the first result.

\(^3\) Note that for some values of \( e_h, e_w \) and some functions \( h(x) \) and \( w(x) \) the solutions do not exist. For example, if \( e_w = 0 \) and \( e_h > 0 \), \( h(\cdot) \) would need to equal 0 due to (7). However, by definition \( h(\cdot) > 0 \). Treatment of these special cases is still work-in-progress.
4. Case of congestion – single AV-type

In order to analytically study the changes in congestion pattern, if the number of travellers exceed the bottleneck capacity, we need an analytical function of the flow rate – the number of travellers departing at every time moment. However, such analytical function has been obtained (to the best of author’s knowledge) only for the so-called $\alpha - \beta - \gamma$ model (called also ‘step model’ or ‘const-step model’) by Arnott et al. (1990). Therefore, I adopt this form of scheduling functions from now on.

The home and work utility functions in the $\alpha - \beta - \gamma$ model are as follows:

$$h(t) = \alpha,$$

$$w(t) = \begin{cases} \alpha - \beta, & \text{if } t \leq t^* \\ \alpha + \gamma, & \text{if } t > t^* \end{cases},$$

where $\alpha, \beta, \gamma$ are positive constants and are assumed to have the relationship $\beta < \alpha < \gamma$; $t^*$ is the preferred arrival time at work. Because many travellers like to arrive at $t^*$, congestion arises – travel time is longer for trips that end around $t^*$. Eventually, it is assumed that the scheduling and travel time costs at all departure times are perfectly balanced (in terms of their disutility to travellers): this condition corresponds to the Nash equilibrium. From this condition, Arnott et al. (1990) obtain the flow rates $r(t)$:

$$r(t) = \begin{cases} \frac{as}{\alpha - \beta}, & \text{if } t \in [t_q, \bar{t}] \\ \frac{as}{\alpha + \gamma}, & \text{if } t \in (\bar{t}, t_q'], \end{cases}$$

where $t_q$ and $t_q'$ are times at which congestion begins and ends, respectively, $\bar{t}$ is the departure time that leads to arrival at $t^*$, and $s$ is the capacity of the bottleneck. Arnott et al. (1990) further derive the three times characterising congestion:

$$t_q = t^* - \frac{(\beta + \gamma)N}{s},$$

$$t_{q'} = t^* + \frac{\beta N}{s},$$

$$\bar{t} = t^* - \frac{\beta \gamma N}{s},$$

where $N$ is the number of travellers.

I follow the same steps as Arnott et al. (1990) to establish the results for AVs. The change from conventional cars to AVs however complicates the derivations. Whereas departure time with conventional cars affects only scheduling and travel time costs, in AVs the departure time also influences the time available for on-board activities. All utility components should be balanced at all departure times to ensure Nash equilibrium. Furthermore, depending on the $\alpha - \beta - \gamma$ constants and efficiency parameters $e_h$ and $e_w$, it is optimal for the traveller to engage in different activities on board. Three AV-types can be distinguished depending on their home or work activity facilitation levels (as
defined in the first two rows of Table 1). The results for these AVs are in Table 1; the derivations are
omitted for the sake of brevity.

<table>
<thead>
<tr>
<th>Optimal activity before $t^*$</th>
<th>AV-type</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td>Home</td>
<td>Home</td>
</tr>
</tbody>
</table>

| Optimal activity after $t^*$ | Home | Work |

<table>
<thead>
<tr>
<th>Equilibrium flow rate $r(t)$</th>
<th>In departure time interval $t \in [t_q, \tilde{t}]$:</th>
</tr>
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<tbody>
<tr>
<td>$\frac{\alpha(1 - e_h)}{\alpha(1 - e_h) - \beta}$</td>
<td>$\frac{\alpha(1 - e_h)}{\alpha(1 - e_h) - \beta}$</td>
</tr>
</tbody>
</table>

<table>
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<tr>
<th>In departure time interval $t \in [\tilde{t}, t^*]$:</th>
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<tbody>
<tr>
<td>$\frac{\alpha(1 - e_h)}{\alpha(1 - e_h) + \gamma}$</td>
</tr>
</tbody>
</table>

<table>
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<tr>
<th>In departure time interval $t \in [t^*, t_{q'}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\alpha(1 - e_h)}{\alpha(1 - e_h) + \gamma}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Congestion start time $t_q$</th>
<th>$t^* - \frac{\gamma}{\beta + \gamma}$N</th>
</tr>
</thead>
</table>

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<thead>
<tr>
<th>Undelayed departure time $\tilde{t}$</th>
<th>$t^* - \frac{\beta \gamma N}{\alpha(1 - e_h)(\beta + \gamma)}$</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Congestion end time $t_{q'}$</th>
<th>$t^* + \frac{\beta N}{\beta + \gamma}$</th>
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<table>
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<tr>
<th>Conditions</th>
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<tbody>
<tr>
<td>$e_h &lt; \frac{\alpha - \beta}{\alpha}$</td>
</tr>
<tr>
<td>$e_w &lt; \frac{\alpha - \beta}{\alpha + \gamma}$</td>
</tr>
</tbody>
</table>

Table 1 shows the flow rates at each departure time interval and the start, on-time and end times of the congestion. The on-time departure time is earlier for all AV-types than for the conventional vehicles, and more early, if the AV facilitates home activities. The last row shows that the results require some relationship between facilitation levels $e_h$, $e_w$ and the scheduling parameters $\alpha$, $\beta$, and

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4 The fourth case would be to engage in work activities before $t^*$ and home activities after $t^*$. However, no $e_h$ and $e_w$ values make this case optimal, given the $\alpha - \beta - \gamma$ model.

5 Type I AV can equivalently be interpreted as reducing travel time penalty $\alpha$, which is the most common approach in modelling AV-effects.
These conditions ‘cover’ the cases with low facilitation levels $e_h$ and $e_w$, but not cases with high facilitation levels. This is analogous to the condition in van den Berg and Verhoef (2016, p.48).

Full impression of the congestion shape is given by Figure 2 – it illustrates the development of queueing time for all AV-types and the base conventional vehicle. The queueing time (which equals the travel time, because the free flow travel time has been normalised to 0) is a function of the departure time rate (Table 1):

$$T(t) = \int_0^t \frac{r(u) - s}{s} du. \quad (17)$$

The figure is obtained for the following parameter values:

**Table 2 Parameter settings for Figure 2**

<table>
<thead>
<tr>
<th></th>
<th>Conventional vehicle</th>
<th>Type I AV</th>
<th>Type II AV</th>
<th>Type II I AV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_h$</td>
<td>0</td>
<td>0.4</td>
<td>0.4</td>
<td>0</td>
</tr>
<tr>
<td>$e_w$</td>
<td>0</td>
<td>0</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>$N$</td>
<td>200</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$s$</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t^*$</td>
<td>50</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Figure 2 Development of queueing times in AV Cases I-III and base case (conventional vehicle).**

It can be seen that the congestion is most skewed to earlier times for the home-activity facilitating type I AVs, followed by type II. For work-facilitating AVs (type III), not only the uptake of the
congestion is faster, but also the initial decline of the congestion is flatter than in the base case. The latter effect skews the congestion towards the opposite direction than the home-facilitating AVs.

5. Case of congestion – two AV-types

Given that all AV types intensify the congestion, but possibly in different directions (as seen in the previous section), it is interesting to see the net congestion effect of having different AVs in the population. Such net effect can be obtained as the so-called Travel Equilibrium Frontier (TEF), as in Arnott et al. (1994). The TEF consists of fragments of the equilibrium queueing times (Figure 2 or Figure 3). These queueing times can be seen as the cost which the travellers in each group are willing to pay to travel in that time slot. Having a mixture of the groups, the one who pays most, ‘wins’ the departure time slot, which is represented by the TEF. Given different sizes of the groups, the respective curves need to be up- or down-scaled, such that the ‘winning segments’ contain the required number of travellers.

As a first exercise in this direction, I select two AV-types (I and III) from the previous section and analyse the congestion effect of having both in the population. The original curves in Figure 2 would correspond to a TEF with a traveller split of 169:31 in groups I and III. Adjusting it to a half-half (100:100) split results in Figure 3. For any lower share of home-facilitating AVs than 169/200, the curve of type I AV is downscaled and the maximum queueing time is reduced. This demonstrates the congestion-mitigating effect of heterogeneous AVs in the population.

![Figure 3 Development of queueing times with half-half split between home- and work- facilitating AVs](image)

6. Conclusion

This short paper has proposed that, in addition to the more-widely known AV-effects on mobility, AVs could influence departure time decisions of future travellers. This mechanism would be activated, if home or work activities can be shifted from their original locations to the AV. This paper presents scheduling preferences capturing such a shift and analyse the congestion impacts of the changed scheduling preferences. It can be concluded that the type of AV – (better) facilitating home or work
activities – has a large impact on the congestion shape and most congested fragments of it. In addition, a mixture of AVs in population seem to have a beneficial impact by reducing the highest ‘peaks’ of the congestion (in terms of the queueing delays). Nevertheless, this work presents only the first steps of the analysis of the new scheduling preferences. I hope that it will invite many more studies to investigate the preferences in more complex contexts and together contribute to a richer understanding of the fully automated future.

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