A unified public transport assignment model for mixed schedule- and frequency-based networks

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Introduction

Many public transport systems throughout the world consist of schedule-based (SB) as well as frequency-based (FB) lines. The public transport system of the Greater Copenhagen Area is an example of such a system, where e.g. the Metro operates as frequency-based while most bus lines operate according to a published timetable. Modelling of such mixed systems in which both schedule-based (SB) and frequency-based (FB) services are present have for a long time been considered a topic which deserves further research, as neither of the current modelling approaches of schedule- or frequency-based public transport assignments models can adequately predict the passengers' behaviour (Gentile *et al.*, 2016). In this paper we propose a novel joint modelling approach combining SB- and FB-modelling, which in a behaviourally realistic manner aims to capture the uncertainty of waiting time when using a FB service and the probabilities of catching a service when transferring between FB and SB services. We exemplify the model using a real-life case-study and compare observed route choices collected from smart card data with the flows obtained from applying the model. The motivation for proposing such a unified SB and FB assignment model is three-fold;

Firstly, the presence of both SB and FB services impacts the route choice behaviour of passengers, and the model should be able to reflect these different route choice preferences in a realistic manner (Gentile *et al.*, 2016). Frequency-based models are typically used for networks where passengers' arrival at stops can be assumed to be randomly distributed and, passengers' are in most frequency-based models assumed to adopt a strategy-approach in their route choice (Nguyen and Pallottino, 1988; Spiess and Florian, 1989). However, for headways of more than 10 minutes schedule-based models are usually preferred, as passengers' tend to arrive more timely to the first stop and therefore time their route choice to specific departures (Tong and Wong, 1999; Gentile *et al.*, 2016). A recent study from the Greater Copenhagen Area has found that timely arrival are observed when headways are as low as five minutes (Ingvardson *et al.*, 2018).

Secondly, large-scale models primarily serve as a tool for evaluating major changes in the network, such as new timetables or future infrastructure investments. For SB models this involves a major task for the modeller in implementing new timetables. The task of setting up such scenarios will become significantly simpler if the modeller can choose to simply set a frequency for a line instead of setting up a full timetable, while keeping SB lines that are unaffected the same as the base scenario.

Thirdly, a unified SB and FB assignment model will be able to include delays, as all paths in the model comes with a probability of realisation which is dependent on the headway distribution.

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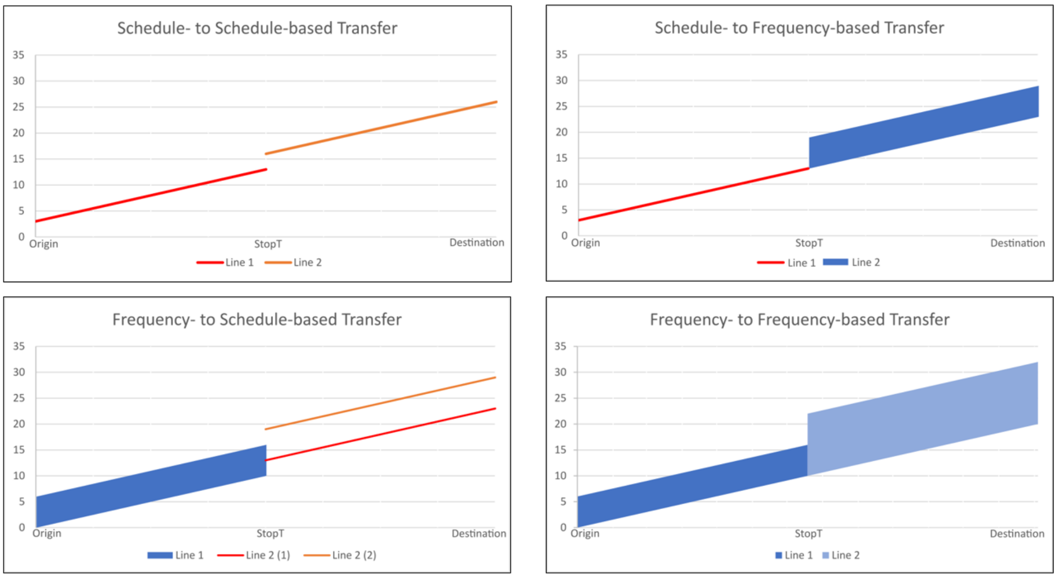

Figure 1 - The four different transfer scenarios in a mixed network

Methodology

Four possible transfer scenarios can be identified when considering a network with both SB and FB services and ignoring congestion effects (Figure 1), and these has to be treated differently. The "traditional" cases from the original two types of models are presented in the top left and bottom right corner, and the two "special" cases are in the opposite corners. When transferring from a SB service to a FB service, the arrival time at the destination is simply the headway distribution of the FB service. The most difficult cases to handle is the transfers between two FB services and transfers between a FB service and a SB service; in the first case, convolution of probability distributions is required. In the second case there exists a probability to catch the first possible schedule-based departure, and if this departure run is not reached with certainty, the passenger needs to wait until the next departure of the line.

The identification of the above mentioned transfer scenarios is important, as the computation of the set of available alternatives (choice set) heavily relies on these. Differently from most FB models (e.g., Nguyen and Pallottino (1988) and Spiess and Florian (1989)), the proposed model facilitate a strategy-approach. The choice set generation method proposed instead adopts a slightly modified version of the event dominance principle from Florian (2004). Because the arrival time of FB services is defined by a statistical distribution and the fact that a path is not always feasible given the transfers from FB services to SB services, a threshold parameter is included in the event dominance. This allows non-optimal paths to be kept in the search, and the non-optimal routes are kept if their costs are below a user defined threshold, for example a maximum of 20% extra cost compared to the best path to the same stop. The network is dynamically built in a time expanded graph, and Algorithm 1 shows the idea behind the choice set generation methodology.

```
while any unchecked event is below threshold do
   Find cheapest event in heap, which has not been checked
      for all possible event from node considered do
              Check if event can be inserted based on dominance criteria;
        if event can be inserted then
        Insert event
        Check if any event at to node is dominated and remove dominated events
       end
end
```

Algorithm 1 – Pseudo code for choice set generation algorithm

The events in the algorithm are the physical movements in the network. In the paper, these events are access/egress, in vehicle time and transfers. Waiting at the stop is handled implicitly because the graph is built dynamically. An earlier arriving event at a node is denoted E_1 compared to the later arriving event E_2. A new event (E_1) is inserted if both of the two following criteria are not violated:

$$E_1 TotalCost + (E_2 EndTime - E_1 EndTime)\beta_{\mathrm{Wait}} \leq E_2 TotalCost \ (1)$$
$$E_1 TotalCost + E_1 TotalCost * \beta_{slack} \leq E_2 TotalCost \ (2)$$

In the current implementation, the flow is distributed using a standard MNL model with the following utility specification:

$$V_{kn} = \sum_m \beta_{IVT,m} IVT_m + \beta_{HW} HWT + \beta_{Con} ConT + \beta_{Walk} WalkT + \beta_{Wait} WaitT + \beta_{TP} TP$$

where IVT_m is the in vehicle time for submode *m*[1], *HWT* is hidden waiting time, *ConT* is access/egress time and *TP* is number of transfers.

Results

The model proposed has been applied to a real-life case study shown in Figure 2. The case-study is a simplified network for travelling from the Technical University of Denmark (DTU) to Copenhagen Airport (CPH). It consists of 5 lines, two FB lines and three SB services.

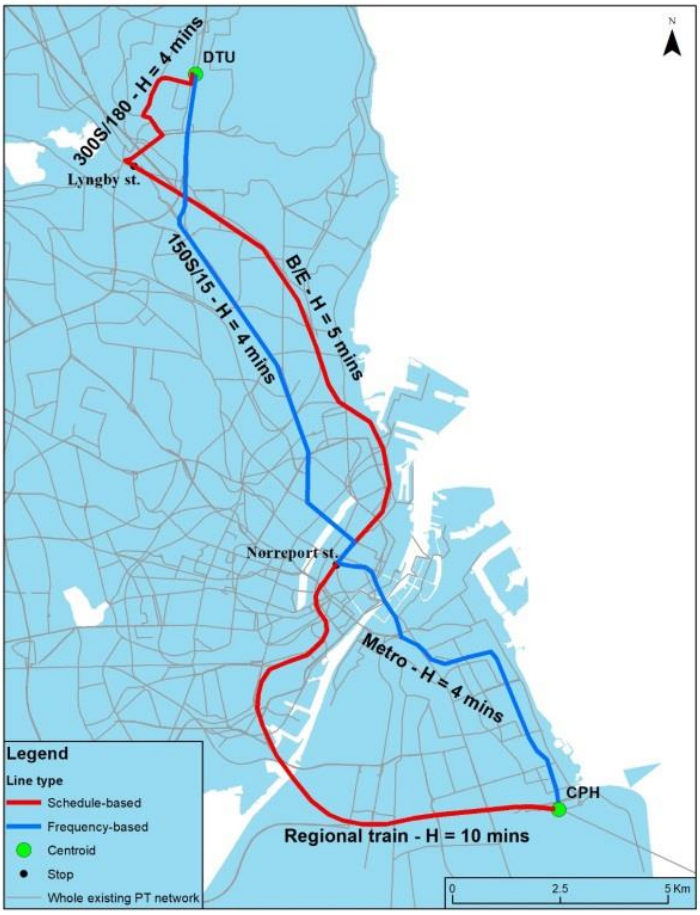

Figure 2 - Test network from Technical University of Denmark (DTU) to Copenhagen Airport (CPH)

[1] Submodes are bus, S-train, Regional train and Metro

In the current implementation of the model, the parameters for the utility specification above have been taken directly from the Danish National Transport Model and the threshold parameter has been set to 20%. The choice set generation methodology was able to enumerate 3 alternatives, excluding the alternative via Lyngby St. and taking the regional train from Nørreport, as. Figure 3 shows on the left figure the observed shares obtained from Smart Card Data (Rejsekort A/S, 2017) and the modelled shares on the right figure. The sample size for the observed trips is 206 trips undertaken by 190 unique individuals. As the flows in Figure 3 show, the model predicts the flows on each segment within a maximum absolute error of around 10% without having re-estimated nor re-calibrated the model parameters for the current model and the larger data sample.

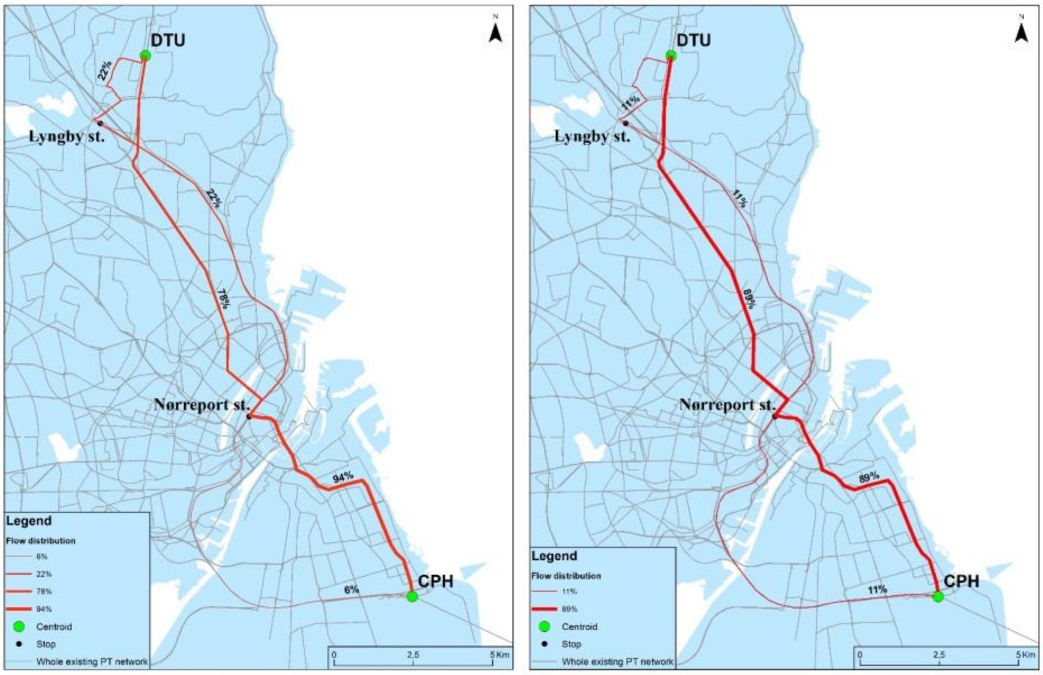

Figure 3 - Left: Flows obtained from observed routes from smart card data;
Right: The estimated flows using the proposed model

Conclusion

This paper presents a novel approach for modelling passenger route choice in mixed schedule- and frequency-based public transport networks. The paper demonstrates the basic ideas behind the proposed model and initial results on a small test case indicate its capability of replicating passenger flows. Future work will include using the behaviourally more realistic path size correction (PSC) logit model as choice model (Prato, 2009), the inclusion of congestion, as well as calibration of the parameters to the large amount of smart card data available.

Keywords

Public transport assignment, mixed schedule- and frequency-based networks, passenger route choice behaviour, choice set generation, discrete choice models

References

Florian, M. (2004) 'Finding Shortest Time-dependent Paths in Schedule-based Transit Networks: A Label Setting Algorithm', in Wilson, N. and Nuzzolo, A. (eds) *Schedule-Based Dynamic Transit Modeling: theory and applications*. Kluwer Academic Publishers, pp. 43–52.

Gentile, G. *et al.* (2016) 'The Theory of Transit Assignment: Basic Modelling Frameworks', in Gentile, G. and Nuzzolo, A. (eds) *Modelling Public Transport Passenger Flows in the Era of Intelligent Transport Systems*. 1st edn. Springer, pp. 287–386.

Ingvardson, J. B. *et al.* (2018) 'Passenger arrival and waiting time distributions depending on train service frequency and station characteristics: A smart card data analysis', *Transportation Research Part C*, In press.

Nguyen, S. and Pallottino, S. (1988) 'Equilibrium traffic assignment for large scale transit networks', *European Journal of Operational Research*, 37, pp. 176–186.

Prato, C. G. (2009) 'Route choice modeling: past, present and future research directions', *J. Choice Modelling*, 2(1), pp. 65–100. doi: 10.1016/S1755-5345(13)70005-8.

Rejsekort A/S (2017) *What is Rejsekort?* Available at: http://www.rejsekort.dk/om-rejsekort/hvad-er-rejsekort.aspx (Accessed: 22 December 2017).

Spiess, H. and Florian, M. (1989) 'Optimal Strategies: A New Assignment Model for Transit Networks', *Transportation Research Part B*, 23B(2), pp. 83–102.

Tong, C. O. and Wong, S. C. (1999) 'A schedule-based time-dependent trip assignment model for transit networks', *Journal of Advanced Transportation*. John Wiley & Sons Ltd, 33(3), pp. 371–388. doi: 10.1002/atr.5670330307.