Transportation network equilibrium in presence of autonomous vehicles

Michael Sorani, Shlomo Bekhor

Technion – Israel Institute of Technology

1. Introduction

Traffic assignment has been widely studied in the last century and many algorithms have been proposed to solve it. As stated by Wardrop's first principle (Wardrop, 1952), drivers are supposed to act rationally and selfishly and to choose their route in order to minimize their own travel time. This leads to the so-called User Equilibrium (UE) pattern. An alternative pattern proposed by Wardrop is the System Optimum (SO), where the total travel time of the system is minimized. In this case the collectivity will earn, while single users may be penalized.

SO assignment is used as a benchmark to measure the performance of a certain network under a certain demand; the ratio between the total system travel time of SO and UE is often referred to as the "price of anarchy" (Roughgarden and Tardos, 2002), reminding us that the wasted time is a consequence of the selfish behavior of drivers.

Haurie and Marcotte (1985) proposed a mixed traffic equilibrium for situation with both cooperation and competition between drivers. Similarly, Harker (1988) formulated an assignment model where UE and other equilibrium patterns (such as Cournot equilibrium) are mixed. Chen and Kempe (2008) assume that people not always act selfishly, and sometimes their behavior can be altruistic or malicious.

2. Reaching SO

The hypothesis of assigning traffic to the road network according to SO has been discussed in many studies. The SO may be ideally reached by applying a suitable toll, which mathematically represents the difference between UE and SO travel times, as the "incentive" to change route. Nevertheless this additional cost must be specific for each road segment and must reflect real time traffic conditions, making such a toll system not easily feasible for whole urban networks.

In the near future Autonomous Connected Vehicles (ACV) will probably compose a growing portion of traffic flow. Assuming this kind of vehicles will replace humans in both driving and navigation (i.e. route choice), the SO may be achieved if a hypothetical central traffic authority assigns each ACV to a route and minimizes the overall travel time on the road network. But the switch to ACV will not be immediate and we have to face with a mixed traffic scenario, where both ACV and regular vehicles coexist. Bagloe et al. (2017) depict a scenario in which ACV are assigned according to SO, while the residual traffic is still assigned according to UE. The authors formulate the model as a multiclass assignment, where the two classes of vehicles have different volume delay functions. Similarly, Sharon et al. (2017) and Zhang and Nie (2018) try to determine, for a given network and demand what is the minimum portion of ACV needed to lead the system to the optimum.

3. Will ACV really allow to reach SO?

Is the assumption that ACV will travel according to SO paradigm realistic? The introduction of ACV may bring a wide range of different possible scenarios, and we do not know which specific approach will be adopted. It will mainly depend on who will be the actor or the actors that will bring ACV to the public. On one hand, if a public authority (e.g. State, Local Authority, or City Governance) will have control upon ACV, the objective function will probably be to minimize the overall travel time (less traffic-jams) or the overall distance travelled (less air pollution) of all the road users. On the other hand, a private company owning a large ACV fleet (taxi service or car leasing) will assign its own vehicles in order to maximize its revenue or to minimize the travel time of its vehicles and customers. In other words, we have a wide range of possible objective functions and our algorithm has to be flexible enough to accommodate different scenarios.

4. Initial Results

In this paper, we first discuss the relatively simple case that each ACV acts as SO, and regular drivers as UE. The mathematical formulation of the problem is presented in Bagloe et al. (2017) for elastic demand. In this paper we assume fixed demand and monotonic volume-delay functions, which allow for reaching a unique flow pattern.

We developed a multi-class MSA algorithm for solving the mixed problem on larger networks as follows:

For each iteration:

- *Set step length alpha:* $\alpha = \frac{1}{n}$
- *Perform All-Or-Nothing (AON) assignment of ACV, based on marginal travel times. Store it as auxiliary SO solution* y_{SO}.
- *Update SO volumes of the network:* $x_{SO}^n = (1-\alpha)x_{SO}^n + \alpha y_{SO}$
- *Perform AON assignment of traditional vehicles, based on travel times. Store it as auxiliary UE solution* y_{UE}.
- *Update UE volumes of the network:* $x_{UE}^n = (1-\alpha)x_{UE}^n + \alpha y_{UE}$
- *Update total volumes of the network:* $x^n = x_{SO}^n + x_{UE}^n$
- *Update travel times (and marginal travel times)*
- *Check stopping criterion: relative gap convergence or max iterations*

The above algorithm was applied to solve well-known networks: a toy network composed of a single origin-destination pair and two links, the Sioux Falls network and the Winnipeg network. Figure 1 shows the overall travel time of the system, for different proportions of ACV; results has been normalized with respect to UE (0% ACV) and SO (100% ACV) cases, in order to compare the different networks.

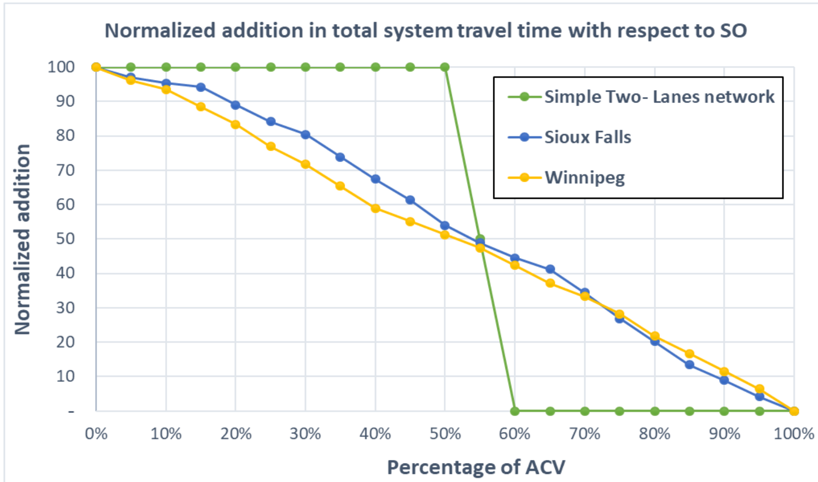

Figure 1: Total travel time as function of ACV proportion

The results indicate a monotonic decrease in travel time for both Sioux Falls and Winnipeg networks. In the case of the toy network, the solution can also be analytically calculated, and it reaches either the UE or SO solutions, because of the very simple network configuration.

The full paper will analyze additional formulations as discussed in Section 3 above.

References

Bagloee, S. A., Sarvi, M., Patriksson, M., & Rajabifard, A. (2017). A Mixed User Equilibrium and System Optimal Traffic Flow for Connected Vehicles Stated as a Complementarity Problem. Computer Aided Civil and Infrastructure Engineering.

Chen, P. A., & Kempe, D. (2008). Altruism, selfishness, and spite in traffic routing. In Proceedings of the 9th ACM conference on Electronic commerce (pp. 140-149). ACM.

Harker, P. T. (1988). Multiple Equilibrium Behaviors on Networks. Transportation Science, 22(1).

Haurie, A., & Marcotte, P. (1985). On the relationship between Nash-Cournot and Wardrop equilibria. Networks, 15(3), 295-308.

Roughgarden, T., & Tardos, É. (2002). How bad is selfish routing? Journal of the ACM (JACM), 49(2), 236-259.

Sharon, G., Albert, M., Rambha, T., Boyles, S., & Stone, P. (2017). Traffic Optimization For a Mixture of Self-interested and Compliant Agents. arXiv preprint arXiv:1709.09569.

Wardrop, J. G. (1952). Some theoretical aspects of road traffic research. Proceedings of the institution of civil engineers, 1(3), 325-362.

Zhang, K., & Nie, Y. M. (2018). Mitigating the impact of selfish routing: An optimal-ratio control scheme (ORCS) inspired by autonomous driving. Transportation Research Part C: Emerging Technologies, 87, 75-90.