A Linear Quadratic Integral Regulator for Local Ramp Metering in the Case of Distant Downstream Bottlenecks

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Abstract

This work develops and investigates the performance of a Linear Quadratic Regulator augmented with integral action (LQI) for the freeway ramp metering control problem in the case of far downstream bottlenecks. Simulation results demonstrate that the proposed methodology is less sensitive compared to previously proposed control strategies, when the bottleneck is located several kilometers downstream from the metered on-ramp.

keywords: ramp metering, distant downstream bottlenecks, linear quadratic integral regulator.

I. Introduction

One of the most effective control measures for freeways is Ramp Metering (RM). RM, when driven by an opportune control strategy, succeeds in reducing traffic congestion and improving traffic conditions [1]. During the last decades, several local RM control strategies have been proposed in the literature, with ALINEA [2], an I-type feedback regulator, being the most popular and efficient [3]. ALINEA aims to maximize freeway throughput in the ramp merging area and utilizes occupancy measurements collected from a mainstream section located at most a few hundred meters downstream of the metered on-ramp.

However, there exist cases where a bottleneck with smaller capacity than the merging area may exist further downstream for various reasons (e.g., lane drop, tunnel, speed-limit area, uncontrolled on-ramp etc.). This suggests using measurements from that downstream bottleneck. Although ALINEA is not suitable for handling RM in such cases, its extended version PI-ALINEA [4] can serve the need satisfactorily. However, in cases where the bottleneck is located many kilometers downstream, PI-ALINEA's performance may also deteriorate.

This study develops and investigates the application of a Linear Quadratic regulator with Integral action (LQI) for RM [5], in the case of bottleneck located many kilometers downstream. A simulation study is performed where both the hypothetical networks and the simulation set-up are kept the same with those utilized in [4] for comparison purposes. The second-order model METANET [6] is utilized as ground truth for the control application. The results demonstrate that the LQI regulator acts effectively even in the (rare but realistic) case of 5 km downstream from the ramp bottleneck.

II. LQI Formulation

In order to derive the LQI regulator for local RM (single on-ramp control), we assume that the freeway system is described by a set of nonlinear difference equations $\boldsymbol{\rho}(k+1) = F(\boldsymbol{\rho}(k), r(k))$, where F is a nonlinear function reflecting the discretized LWR model [7], [8], [9], [10], $\boldsymbol{\rho}(k) \in \Re^n$ and $r(k) \in \Re$ denote the densities (corresponding in freeway sections between the on-ramp and the bottleneck) and the on-ramp flow, respectively, at time step k. Linearization of this system around a desired steady-state $(\boldsymbol{\rho}^d, r^d)$ yields

$$\Delta\boldsymbol{\rho}(k+1) = \boldsymbol{A}\Delta\boldsymbol{\rho}(k) + \boldsymbol{B}\Delta r(k), \tag{1}$$

where $\Delta\boldsymbol{\rho}(k) = \boldsymbol{\rho}(k) - \boldsymbol{\rho}^d$ and $\Delta r(k) = r(k) - r^d$ are the linearized state vector and control input, while $\boldsymbol{A} \in \Re^{n\times n}$ and $\boldsymbol{B} \in \Re^{n\times 1}$ are the state and input matrices, respectively. Consider now the state equation (1) augmented by use of

$$y(k+1) = y(k) + \boldsymbol{H}\Delta\boldsymbol{\rho}(k), \tag{2}$$

where $y \in \Re$ and $\boldsymbol{H} \in \Re^{1\times n}$ is a horizontal vector, which has an 1 at the n^{th} component (so that the bottleneck density is integrated in (2)) and 0's elsewhere. The control goal is to minimize the quadratic criterion

$$J = \frac{1}{2}\sum_{k=0}^{\infty}\Big[||\Delta\boldsymbol{\rho}(k)||^2_{\boldsymbol{Q}} + ||\Delta r(k)||^2_R + ||y(k)||^2_S\Big], \tag{3}$$

where $\boldsymbol{Q} \in \Re^{n\times n}$ is a symmetric positive definite weighting matrix and $R, S \in \Re$ correspond to positive constants. Considering (1), (2) and (3), the following augmented problem matrices are obtained:

$$\tilde{\boldsymbol{A}} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{0} \\ \boldsymbol{H} & 1 \end{bmatrix}, \tilde{\boldsymbol{B}} = \begin{bmatrix} \boldsymbol{B} \\ 0 \end{bmatrix}, \tilde{\boldsymbol{Q}} = \begin{bmatrix} \boldsymbol{Q} & \boldsymbol{0} \\ \boldsymbol{0} & S \end{bmatrix}, \tilde{R} = R, \tag{4}$$

leading to the following time-invariant solution

$$r(k) = -\tilde{\boldsymbol{K}} \begin{bmatrix} \boldsymbol{\rho}(k) \\ y(k) \end{bmatrix}, \tag{5}$$

where $\tilde{\boldsymbol{K}}$ is a horizontal vector that may be calculated by the backward integration of the augmented Riccati matrix $\tilde{\boldsymbol{P}}(k)$ starting form any terminal condition $\tilde{\boldsymbol{P}}(K) \geq 0$ until convergence towards a unique stationary value $\tilde{\boldsymbol{P}} \geq 0$ is obtained. Decomposing $\tilde{\boldsymbol{K}} = \begin{bmatrix} \boldsymbol{K_x} & K_y \end{bmatrix}$ and after some algebra, we get the final LQI controller

$$r(k) = r(k-1) - \boldsymbol{K_P}(\boldsymbol{\rho}(k) - \boldsymbol{\rho}(k-1)) + K_I(\rho_n^d - \rho_n(k)), \tag{6}$$

where $\boldsymbol{K_P} = \boldsymbol{K_x} - \boldsymbol{H} K_y$, $K_I = K_y$ and $\rho_n(k)$, ρ_n^d indicate the bottleneck's state and desired state, respectively. Similarly to the application of PI-ALINEA, the calculated $r(k)$ is truncated if it exceeds a range $[r^{min}, R^{max}(k)]$, where $R^{max}(k) = \min(r^{max}, r^m(k-1)+400)$, r^{min} and r^{max} are the minimum and maximum admissible on-ramp flow, respectively, $r^m(k-1)$ is the measured ramp inflow during the last control time interval with 400 being an empirical value [4]. Note that, if $n = 1$, i.e. the bottleneck is the merging area, then (6) reduces to PI-ALINEA.

III. Simulation Results

METANET [6] is utilized with a simulation time step $T = 5$ sec. The simulated 3-lane freeway network is 8 km long and is divided into N =32 cells ($L_i = 0.25$ km for $i = 1, ..., N$, see Fig. 1(a)). The freeway has one main entrance (cell 1), one main exit (cell 32) and an on-ramp located at the upstream boundary of cell 9, while a 1km-long bottleneck is placed in different locations downstream of the on-ramp for considering different Bottleneck Cases (BCs) (grey areas in Fig. 1(a)). Within METANET, bottleneck and non-bottleneck cells are considered to have different Fundamental Diagrams (FDs) with the first being characterized by FDs with lower capacity compared to the second (see Fig. 1(b)). Fig. 1(c) presents the entrance and on-ramp demand scenarios employed. The rest of the simulation settings are the same as in [4].

Within the LQI framework, the considered (linearized) system extends from the on-ramp location (here cell 9) to the first cell of the bottleneck section (circled areas in Fig. 1(a)). Therefore, the dimension of the vector $\boldsymbol{\rho}$ (and the corresponding matrices $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{Q}$, etc) considered in Section II depend on the bottleneck location, with the index n indicating the bottleneck cell. In particular, the matrices $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{H}$ are as follows: $a_{i,i} = (1 - \frac{T}{L_i}\mathrm{v}_i^d)$ $(i = 1, ..., n)$, $a_{i,j} = \frac{T}{L_i}\mathrm{v}_j^d$ for $i = j+1$ $(j = 1, ..., n-1)$, $a_{i,j} = 0$ otherwise, $b_1 = \frac{T}{L_i}$, $b_i = 0$ for $i \neq 1$, $h_i = 0$ for $i = 1, ..., n-1$ and $h_n = 1$. Here, v_i^d denotes the slope of the FD at the desired state (here $\mathrm{v}_i^d = 72$ km/h for $i = 1, ..., n-1$, and $\mathrm{v}_n^d = 54$ km/h). For deriving the gains $\boldsymbol{K_P}$, K_I, an appropriate selection of the weights $\boldsymbol{Q}$, R, S should be performed. After extensive simulation tests, in this study we used $R = 1$, $S = 5000$ and $\boldsymbol{Q}$ such as $q_{i,i} = \frac{10^4}{n}$ $(i = 1, ..., n-1)$ and $q_{n,n} = \frac{10^6}{n}$. $\boldsymbol{K_P}$, K_I obtained using the above weights lead to smooth and efficient control performance. However, similar performance can be obtained using constant gains (which are close to the optimal solution) for the elements of $\boldsymbol{K_P}$ and K_I. All the simulation tests for the different BCs (presented herein or not) have been derived using $K_{P,i}$ =200 km*lane/h $(i = 1, ..., n)$ and K_I =60 km*lane/h, indicating the low sensitivity in gain selection of the LQI regulator. Finally, we set $r^{min} = 300$ veh/h and $r^{max} = 6000$ veh/h.

For BCs (1) and (2) (see Fig. 1(a)), no-control and control results with LQI and PI-ALINEA are given. The set-point used for both regulators is the factual critical density of each bottleneck cell, which - due to METANET model dynamics - is different with the corresponding FD parameter (here, $\rho_{20}^d = \rho_{29}^d = 42$ veh/km/lane for corresponding BCs (1) and (2)).

No-Control Case: Figs. 2(i)(a), 2(ii)(a) depict the density and flow trajectories in the no-control case for BCs (1), (2), respectively. In both cases, a congestion is created in the freeway mainstream resulting accordingly to capacity drop. The congestion initially builds in the bottleneck cell and gradually spills back to the upstream cells.

RM Case: As several simulation tests demonstrate, in cases where the bottleneck is located at most 2 km downstream from the ramp, the performance of LQI and PI-ALINEA (using the gains proposed in [4]) is pretty much the same; both regulators succeed to avoid mainstream congestion and to achieve a higher throughput. However, when the bottleneck is moved further downstream (beyond 2 km, including BCs (1), (2) presented herein), the control task becomes increasingly difficult; using the same gains for PI-ALINEA leads to aggressive and oscillatory behavior. Thus, in order to obtain satisfactory performance for PI-ALINEA the gains must change for every different BC by selecting more and more conservative gains as the botleneck moves further downstream. Figs. 2(i)(b) and 2(ii)(b) depict the density and flow trajectories for BCs (1) and (2), respectively, under the LQI action. Accordingly, Figs. 2(i)(c) and 2(ii)(c) for PI-ALINEA's action but with $K_P = 60$, $K_I = 1$ km*lane/h for BC (1) and $K_P = 30$, $K_I = 0.3$ km*lane/h for BC (2), respectively. PI-ALINEA's performance is similar with LQI's (a bit less-damped in most of the cases), however, the overall study indicates that LQI is much less sensitive in terms of the gains selection, hence easier to deploy.

IV. Conclusions

The presented results suggest that LQI regulator handles efficiently the local RM task in the case of very distant downstream bottlenecks. Future work will focus on integrating LQI within a random-located bottleneck framework [11].

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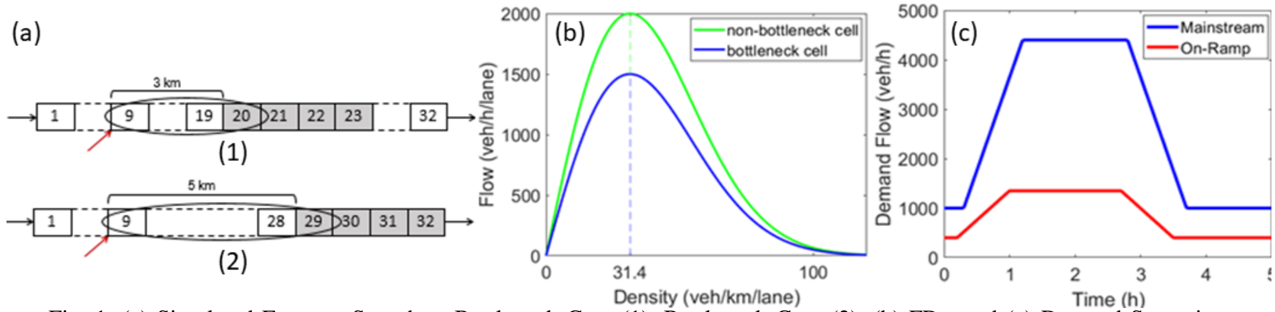

Fig. 1: (a) Simulated Freeway Stretches: Bottleneck Case (1), Bottleneck Case (2), (b) FDs, and (c) Demand Scenarios.

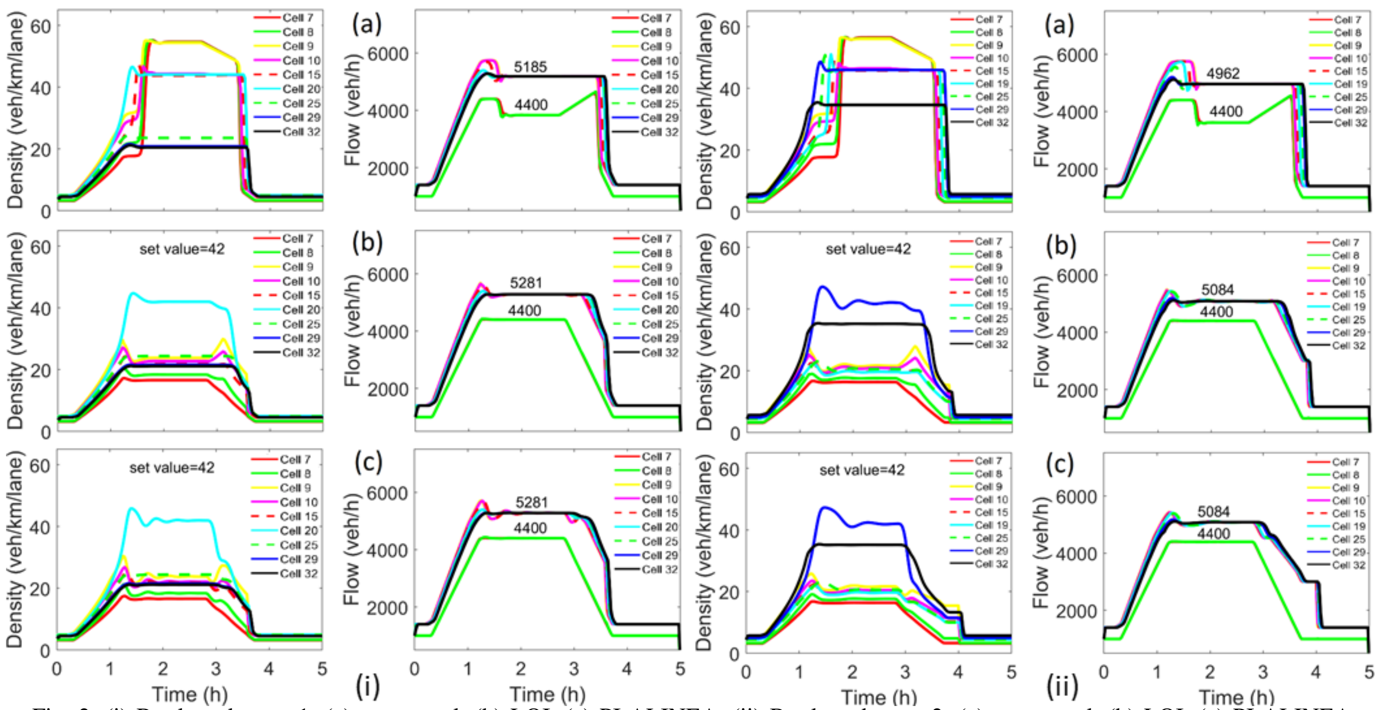

Fig. 2: (i) Bottleneck case 1: (a) no control, (b) LQI, (c) PI-ALINEA, (ii) Bottleneck case 2: (a) no control, (b) LQI, (c) PI-ALINEA

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