A Lagrangian relaxation technique for the demand-based benefit maximization problem

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1 Introduction

The integration of discrete choice models in Mixed Integer Linear Programming (MILP) models provides a better understanding of the preferences of the customers to the operators while planning for their systems. Despite the clear advantages of this integration, the formulations associated with choice models are highly nonlinear and non convex, and therefore difficult to include in MILP.

In Pacheco et al. (2017), we overcome this limitation by defining a general framework that allows to characterize almost any discrete choice model based on the random utility principle with a linear set of constraints that can be embedded in any MILP formulation. The probabilistic nature of the choice model is addressed with simulation. For each error term in the utility function, we generate draws based on its distributional assumption. Each draw corresponds to a behavioral scenario. Once the draws have been generated, the utility associated with alternative $i \in \mathcal{C}$ by customer n in scenario r becomes deterministic and is a linear function of the decision variables.

The behavioral assumption is that customer n chooses alternative i if its utility is the largest within the choice set of alternatives available to her. In this case, the binary variable w_{inr} modeling the choice takes value 1. With this representation, the total expected demand of alternative i is obtained by simply averaging the sum of the choice variables over the number of considered scenarios.

2 Demand-based benefit maximization problem

The approach described in Section 1 can be used to model numerous applications, such as the design of a train timetable in transportation or the shelf space allocation problem in retail. For the sake of illustration, we characterize a demand-based benefit maximization problem. Briefly, an operator that sells services to a market, each of them at a certain price and with a certain capacity (both to be decided), aims at maximizing its benefit, understood as the difference between the generated revenues and the operating costs.

We consider the case study of a parking services operator in order to test the resulting formulation, which is motivated by an advanced disaggregate choice model from the recent literature (Ibeas et al., 2014). We assume a population of $N = 50$ customers and a choice set consisting of $J = 3$ services: paid on-street parking (PSP), paid parking in an underground car park (PUP) and free on-street parking (FSP). Since the latter does not generate any revenue, it is used to model the customers leaving the market. We perform different experiments, such as price calibration or price differentiation by population segmentation, and the results exhibit that this formulation is a promising powerful tool to configure systems based on the heterogeneous behavior of customers.

Notwithstanding the scope of this framework, the size of the resulting problem is high, which makes it computationally expensive. Table 1 shows the solution time and the obtained results for $R = 50$ draws and two different assumptions on the operator: (i) it can decide if a service is offered or not, and (ii) it is forced to offer all services. In this case study, $R = 50$ provides a good compromise between computational time and accuracy of the results. In practice, however, a higher number of draws is desirable to be as close as possible to the true value, and populations are larger than the sample considered here. In any case, as the number of draws and/or the number of customers increases, so does the complexity of the model.

	Solution	**Capacity**		**Demand**			**Prices**		
Assumption	time (h)	PSP	PUP	PSP	PUP	FSP	PSP	PUP	Benefit
(i)	18.7	20	-	19.4	-	30.6	0.76	-	6.27
(ii)	33.7	15	5	14.8	4.56	30.7	0.76	1.32	4.99

Table 1: Capacity and price characterization for assumptions (i) and (ii) and $R = 50$

3 Lagrangian relaxation

The underlying structure of the proposed formulation is particularly well suited for decomposition methods. Furthermore, they represent an alternative to valid inequalities since these techniques can be applied in a more general way. In this case, we rely on Lagrangian relaxation (Fisher, 2004). The general idea is to relax the set of complicating constraints by transferring them to the objective function with an associated Lagrangian multiplier, which imposes a cost on violations. In practice, the relaxed problem can be solved more easily than the original problem.

For the demand-benefit maximization problem, we can come up with two separate subproblems, each of them composed of an exclusive set of variables and an objective function consisting of the terms depending on the subproblem's variables. More precisely, one subproblem concerns the decisions of the operator and the other the choices of the customers. Since both subproblems share the choice variable, they cannot be separated by relaxing any of the constraints present in the MILP model. To this end, we define a duplicate of the choice variable w_{inr} (w'_{inr}), and we relax the associated constraint $w_{inr} = w'_{inr}$. In this way, we have a variable representing the choice in each subproblem.

The operator subproblem includes the choice variable w'_{inr} and a binary variable associated with the choice of capacity y_{iq} (1 if service i is offered with capacity c_{iq}, 0 otherwise). It can be seen as a Capacitated Facility Location Problem (CFLP), where $\mathcal{C}$ represents the set of potential facility locations, N the index for the set of customers, y_{iq} the location decision variables and w'_{inr} the allocation decision variables. The objective function can be interpreted as the maximization of the net profit of the operator, which is defined as the difference between the revenue generated from the serviced customers and the cost of the location of the selected facilities. The constraints of the operator subproblem are equivalent to the ones in the CFLP. Although the CFLP is NP-hard, well-known solution methods, or even Lagrangian relaxation, can be considered to solve large instances.

The customer subproblem comprises the remaining variables and constraints. We characterize an algorithm to solve it by assuming that the utility is decreasing as a function of the price. This is actually a reasonable assumption, as utility usually decreases when the price increases. The general idea is to iterate over n and r (i.e., a subproblem is solved for each customer and scenario), and to set the choice of customer n in scenario r to the available service with the highest utility. The price that the customer has to pay to access the service is set to the highest price such that the choice does not change to the service with the second highest utility.

Some preliminary tests on both subproblems manifest the strengths of this approach. Indeed, the computational time of both subproblems for $N = 50$, $R = 50$ and a given value for the Lagrangian multiplier is shorter than 3 seconds. We are currently developing an iterative method called subgradient method in order to optimize the Lagrangian dual, which is the problem on the Lagrangian multipliers.

References

Marshall L. Fisher. The lagrangian relaxation method for solving integer programming problems. *Management Science*, 50(12_supplement):1861–1871, 2004.

A. Ibeas, L. dell'Olio, M. Bordagaray, and J. de D. Ortúzar. Modelling parking choices considering user heterogeneity. *Transportation Research Part A: Policy and Practice*, 70:41 – 49, 2014. ISSN 0965-8564. doi: http://dx.doi.org/10.1016/j.tra.2014.10.001.

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