The Gini index of demand imbalances in public transport networks

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Extended abstract submitted for
the 7th Symposium of hEART
28 February 2018

1 Introduction

Along a public transport line operators serve multiple spatio-temporally differentiated markets with the same second-best capacity. Hörcher and Graham (2018) show in the simplest back-haul setting that the asymmetry in demand between jointly served markets may have crucial impact on (1) the optimal capacity, (2) the equilibrium occupancy rate of vehicles and thus the crowding experience of passengers, (3) optimal pricing decisions, and (4) the financial and economic performance of public transport provision.

In this research the authors extend the analysis of the back-haul problem to a more realistic urban public transport setting: a transit line along which capacity is still indivisible due to operational constraints, but more than two origin-destination pairs have to be served. We investigate what may be a suitable measure of demand imbalances in this setting that could replace the share of main haul demand in total ridership in the back-haul problem (Hörcher and Graham, 2018). We show that what matters in a network is not only the *spread of demand* between line section matters, but also the *spread of operational costs* between them. For example, it may be relatively more expensive to handle excess demand on long line section. To characterise the joint distribution of demand and operational costs, we propose the Gini coefficient of demand imbalances, a statistical index frequently used in macroeconomics as a measure of income inequality.

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2 The Gini index in public transport context

The Gini coefficient measures statistical dispersion between two frequency distributions. In a public transport context we can adopt this concept by plotting the cumulative share of operational costs on line sections against the cumulative share of demand, after sorting both variables in increasing order. The resulting Lorenz curve is the diagonal of the graph in case of perfect equality, while if all demand is concentrated on a negligably short line section, then the Lorenz curve moves along the two sides of the graph (see Figure 1). The Gini index is the share of the area between the actual Lorenz curve and the one belonging to perfect equality (area A), and the area between the two extrema (area A+B). Thus the Gini index ranges between zero and unity.

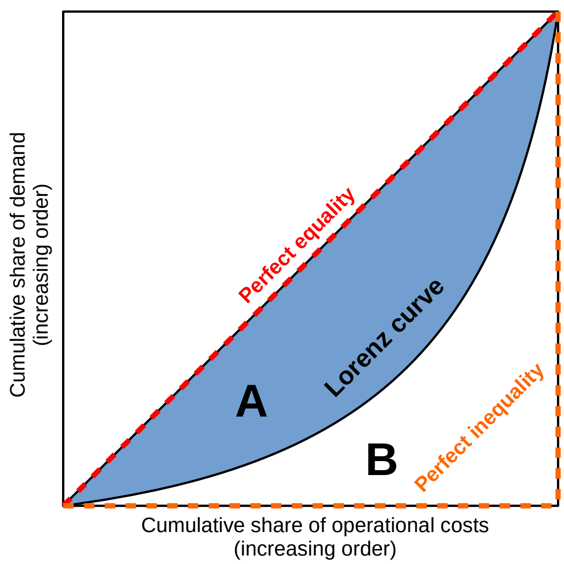

Figure 1: The interpretation of the Gini coefficient in a public transport context

In this research we explore the Gini indices of real world metro demand patterns derived from smart card data. In the main body of the analysis we investigate in a simulation experiment whether the Gini index can serve as a proxy for the numerical optimum of capacity provision and pricing. In other words, we pose the following research question: controlling for scale effects, what is the impact of demand imbalanced, measured by the Gini coefficient, on supply decisions, user costs and eventually on social welfare?

3 Quantitative analysis

Our research approach is partly based on empirical demand patterns recovered from smart card data. Section 3.1 presents preliminary insights into demand imbalances along four metro lines. As data collection for more metro demand patterns is currently ongoing, and we can hardly acquire line-level operational costs due to joint fixed costs between lines, the rest of this analysis is now complemented with synthetic demand patterns in Section 3.2.

3.1 Demand imbalances in reality

In order to get an empirical insight into what demand patterns transport operators face in reality, let us look at data gathered in a large Asian metro network. Let us focus on separate metro lines and time periods when capacity (i.e. the length and frequency of trains) is kept constant. In the metro network under investigation this is the case between 7.30 and 10.30, later on referred to as morning peak, and between 11.00 and 16.00 in the afternoon. Figure 2 shows frequency distributions of ridership in eight operational regimes, where each observation corresponds to the passenger throughput of an interstation section in 15-minute intervals.

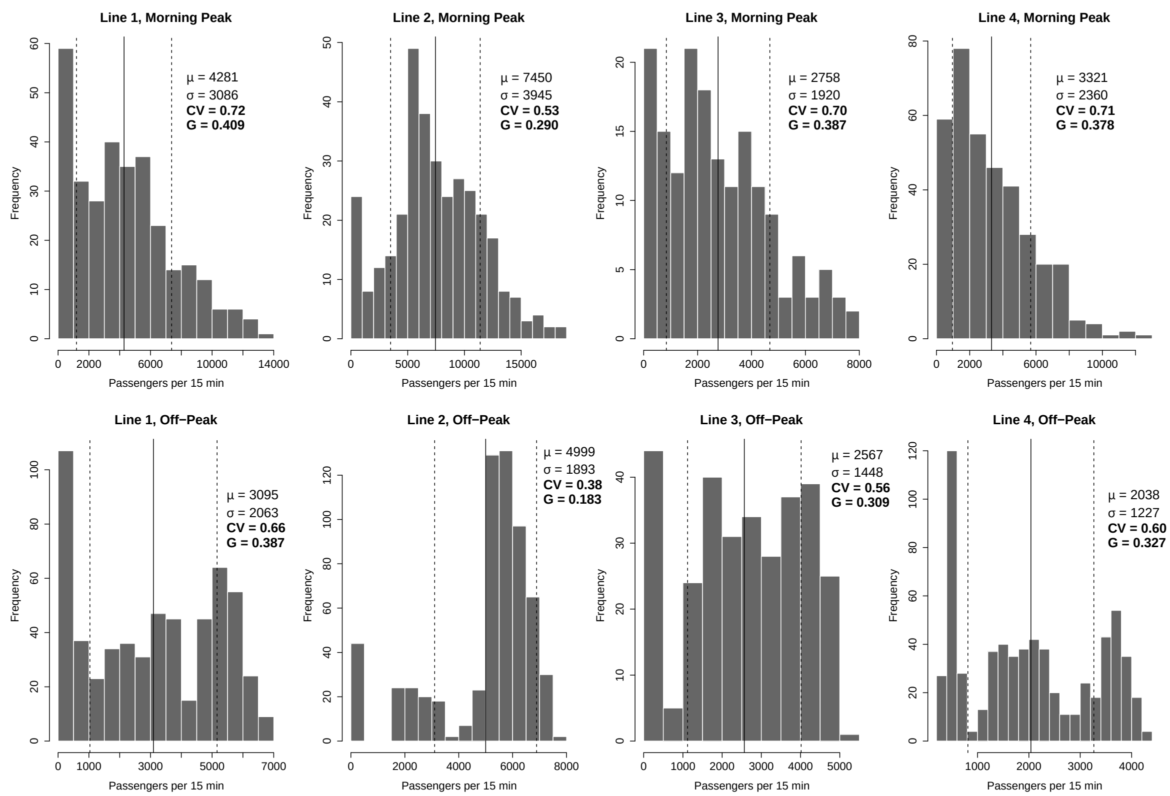

Figure 2: Peak and off-peak demand patterns of four urban metro lines, derived from smart card data. Each observation corresponds to the passenger throughput of an interstation section in 15-minute intervals.

First of all, note that the histograms are surprisingly diverse; none of the standard probability distirbution functions can be identified as the universal distribution of metro demand patterns. Morning peak distributions show some similarity in case of Lines 1, 2 and 4. These may be associated with a gamma or log-normal distribution, as there is a decreasing pattern towards high demand levels. Line 2 is an outlier not only in terms of the shape of the histogram, but also in the sense that mean ridership (μ) is higher and the standardised measure of spread (coefficient of variation, CV) is lower than for the three other lines. The distribution of off-peak demand shows even more randomness. Lines 1 and 4 have a disproportionately high number of line sections where demand is under 1000 passengers per 15 minutes, Line 3 has almost a homogeneous distribution, while in case of Line 2 the demand pattern is heavily skewed towards higher ridership levels.

The lack of uniformity in demand distributions suggests that the standard measures of spread may not be appropriate for characterising demand imbalances. Also, travel times on line sections range between less than 2 minutes to more than 5 minutes, which implies that the share of inter-station markets in operational costs is not similar either. This is the main reason why we decided to turn towards a more compact measure of the joint distribution of demand and operational costs. Among the selected metro demand patterns the Gini coefficient ranges between 0.18 and 0.41.

3.2 Numerical simulation

In order to investigate the relationship between supply variables, passenger well-being and the Gini index, we generate a random sample of demand patterns for the simple network depicted in Figure 3. Controlling for scale effects is a crucial feature of the analysis, as otherwise it would be difficult to disentangle the consequences of scale economies and demand asymmetries. In order to avoid this threat we generate the demand patterns such that the total number of passengers as well as passenger miles are kept constant at 4000 passengers and 2500 passenger hours, respectively. This ensures that scale effect cannot arise in either waiting time or in-vehicle travel time costs. The Gini indices vary between 0.1 and 0.5 in the randomly generated sample, which is almost the same range as what we found for real metro lines.

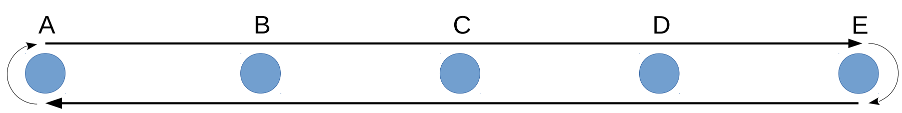

Figure 3: Network layout for simulation experiment

The randomly generated demand patterns are then numerically optimised with respect to frequency (f) and vehicle size (s), according to a social cost minimisation objective.

$$\min_{f,s} TC = C_{op} + C_{user}, \tag{1}$$

where operational costs are a function of cycle time (t_c) and three parameters,

$$C_{op} = t_c\, f \cdot (a + b\, s^c), \tag{2}$$

and total user costs are

$$C_{user} = \sum_i Q_i \left[\alpha_w\, 0.5 f^{-1} + \sum_j \delta_{ij} t_j\, \alpha_v \big(1 + \varphi\, q_j (fs)^{-1}\big)\right]. \tag{3}$$

In the user cost expression i is the index of OD pairs, and $j \in L$ represents the links in the network. Q_i is the inelastic demand on OD pair i, and q_j is ridership on link j, such that

$q_j = \sum_i \delta_{ij} Q_i$. We consider two user cost components: the cost of waiting and in-vehicle travelling, where the latter is applied with a crowding dependent multiplier. In the formula α_w and α_v are the value of waiting and in-vehicle time, respectively, and φ is the parameter of the linear crowding multiplier. Note that $q_j(fs)^{-1}$ is the occupancy rate on link j and δ_{ij} is a dummy set to unity if link j is used by OD i passengers.

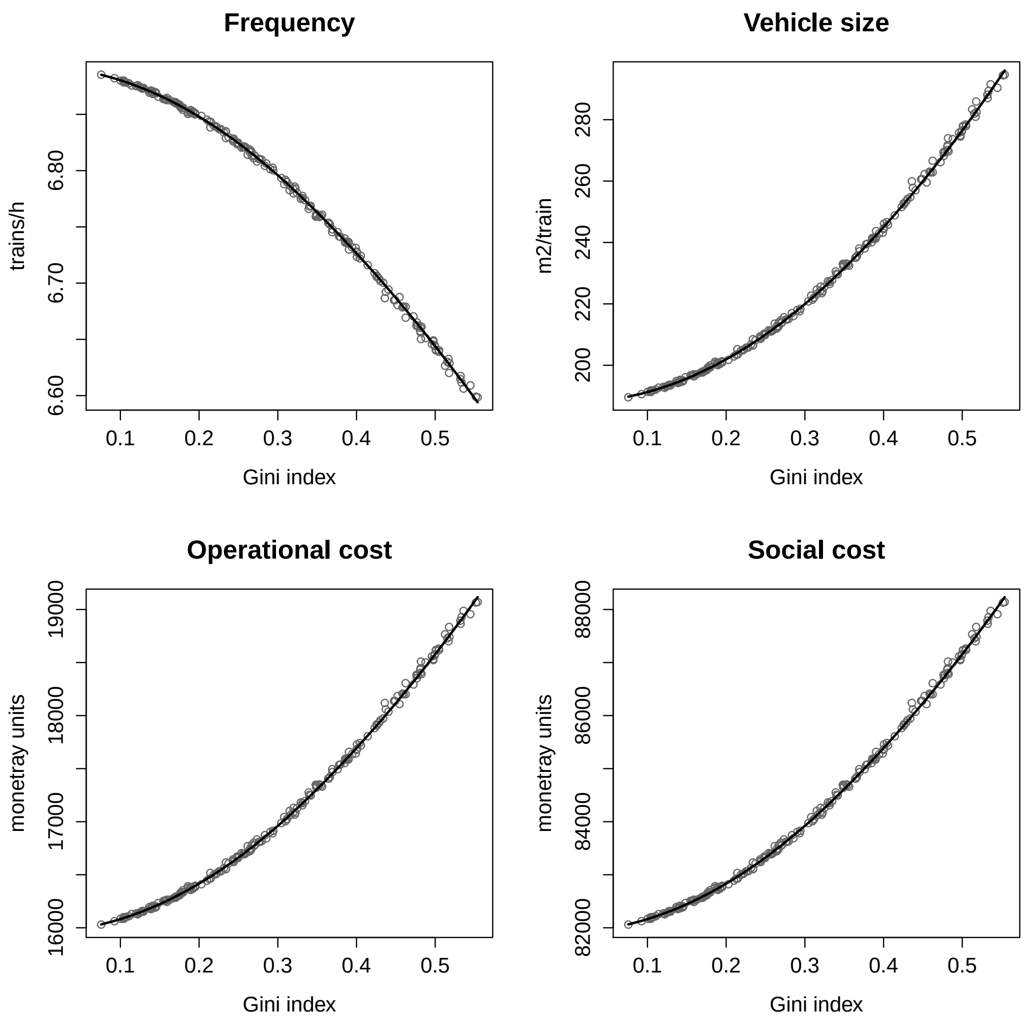

Figure 4: Simulation results – Optimal capacity and the resulting operational costs and social welfare

We calibrated the model with parameters borrowed from earlier capacity optimisation studies, i.e. Rietveld et al. (2002), Jara-Díaz and Gschwender (2003) and Rietveld and van Woudenberg (2007), and the crowding multiplier comes from Hörcher et al. (2017).

The main outcome of this preliminary analysis is that *ceteris paribus* the Gini coefficient is a surprisingly good predictor of the optimal capacity (frequency as well as vehicle size) and the resulting operational and aggregate social costs, as Figure 4 depicts. The shape of the relationship between the Gini index and supply variables is very similar to what we found in the back-haul problem (Hörcher and Graham, 2018). As the concentration of demand increases, frequency is gradually replaced with higher vehicle size, because the disutility of crowding becomes more important than to waiting time costs. The optimal cost of operations increases with the magnitude of demand imbalances, just like the aggregate cost for society.

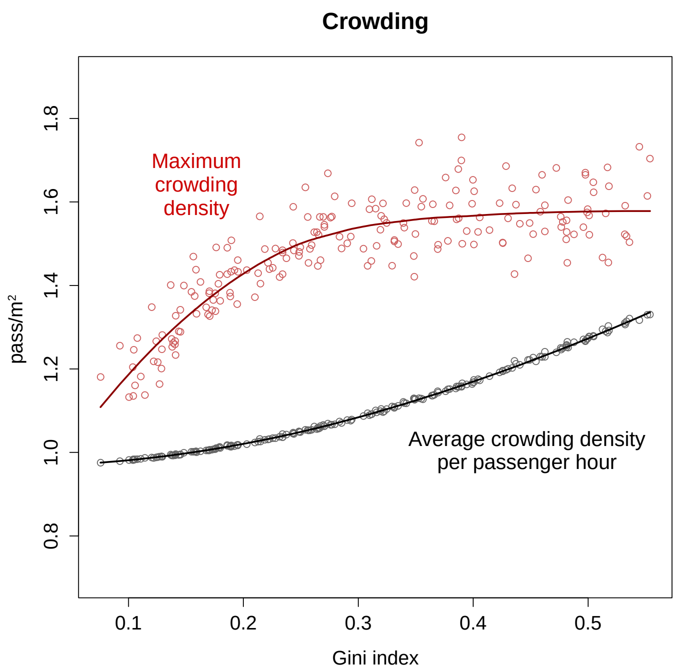

Figure 5: Simulation results – Average and maximum crowding density under optimal capacity provision, in function of the Gini index

More insights can be gained by plotting crowding related simulation variables against the Gini index. Figure 5 shows that the new measure of line-level demand imbalances explains very well the increase in crowding disutility experienced by the average passenger (weighted by the duration of their trips). As intuition suggests, the greater the asymmetry in demand between markets served by the same capacity, the higher the average crowding disutility, even at constant passenger mile performance.

Maximum crowding density values, however, have a much wider spread around the best fitting nonlinear curve, using the Gini index as predictor variable. In other words, above around $G = 0.3$, the possibility of extreme crowding conditions cannot be explained by this measure of demand imbalances.

4 Future research and relevance

The analysis introduced above will be extended in several ways. First of all we collect a larger set of metro smart card data, in order to validate the relationship between supply variables and the Gini index, simulated in Figure 4, with real observations on individual line level.

In addition, we intend to verify the simulation results with analytical derivations based on functional representations of the Gini coefficent (Dorfman, 1979). Although our preliminary experiments suggest that the Gini coefficient is a suitable measure of demand imbalances, future research may consider more advanced inequality measures to be adopted for travel demand applications (Atkinson, 1983).

Future plans include the investigation of the inequality measure under *elastic demand* on each OD pair of the network, in which case ridership levels as well as the Gini index itself may depend on supply side variables. Elastic demand would allow for an investigation of pricing strategies in the presence of network-level demand imbalances. The core research goal in this case will be to identify the impact of the Gini index on the efficiency of differentiated vs. flat fare policies.

Why is the analysis of demand imbalances relevant for research and policy? Transport services make connections between geographically separated areas of a heterogenous urban space, and are therefore affected by the spatial and temporal concentration of economic activity. Said differently, travel patterns are strongly linked to city structure, which is almost exogenous for public transport operators. Therefore studying demand asymmetries is essentially about how urban spatial structure affects the key operational and economic features of public transport provision. Policies that affect the spatial and temporal pattern of activities in the urban economy will influence the effectiveness of public transport provision as well, and therefore optimal public transport interventions should reflect the spatial environment of operations.

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