A study of the effect of social network topology on information propagation speed

Anna TAKAYASU[1], Yusuke HARA[2] and Masao KUWAHARA[3]

Keywords: Information Propagation, Social Network, Consensus Problem

1. Introduction

In this paper, we consider the effect of social network topology on the information propagation speed of an updated subjective traffic state.

At present, with an increase of information from individuals on social media, it has become possible to obtain not only official authorized information, such as news, from the internet, television and car navigation systems, but also various information through SNSs, such as Twitter and chat, more rapidly and easily. Generally, people update their subjective traffic state for roads using such information and road information based on their experience, and choose their routes. However, at the time of a disaster, such as an earthquake, there are cases in which radio and navigation systems cannot report the state of an entire region, and understand the recovery state and state of collapse because of the great change in road conditions. Therefore, it is known that determine their routes using a subjective traffic state that is updated using information from their acquaintances at the time of a disaster. Thus, the information propagation from individuals may have a large influence on the traffic state.

Regarding the study of the information propagation mechanism from individuals, Furuta et al. [1] extended Kermack and McKendrick[2]'s SIR model and presented the information update flow of the subjective probability regarding the safe or dangerous state of the region to a random choice of residents. This model considered information about region-level safety as an entire road network instead of a human relationship. To discuss the propagation of detailed information, such as the travel time of road links, we need to consider the connection between observers of road links.

Therefore, our final purpose is to clarify the effect of the propagation of detailed traffic-state information as a subjective link travel time of individuals on the traffic state of an entire road network by considering the connection between each individual. First, we understand the information propagation from individuals and consider the effect on the information propagation speed.

The aim of this study is to

・determine the relationship between the information propagation speed and algebraic

[1] Graduate School of Information Sciences, Tohoku University, 6-6-06 Aoba, Aramaki, Aoba-ku, Sendai, Japan
[2] Department of Civil Engineering, the University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo, Japan
[3] Graduate School of Information Sciences, Tohoku University, 6-6-06 Aoba, Aramaki, Aoba-ku, Sendai, Japan

connectivity that expresses the network topology using numerical simulation; and
・derive that relationship analytically.

2. Problem Statement

2.1. Social Network and Road Network

In this study, each individual exchanges information about the traffic state of their neighbor roads with their acquaintances, and complements the information for an entire area. We consider two networks; a social network (SN) and road network (RN). The SN is the network such that node $i \in \mathbf{N}$ denotes an individual and links represent the relationship between individuals. If the SN is a directed graph, it is possible to consider the relation between individuals on twitter and mass media. However, at present, we have decided that the SN should be an indirect connection graph for simplicity. The RN is the network for which link $l \in \mathbf{E}$ denotes a road link and each link has the traffic state, such as traveling time and closed state by blocked. Individuals are on the nodes of the RN. They observe the traffic state of neighbor links of that node, and have information about the subjective traffic state of each road link based on whether it is observed or unobserved. Information in this study regards the traffic state of RN links without misinformation.

2.2. Assumption of Information Propagation

Each individual propagates its information about each road based on the following three assumptions:

(1) Each individual exchanges its information with all its acquaintances that are connected in the SN at the same time.

(2) Each individual updates its information according to the information from others connected in the SN at a constant rate θ.

(3) Each individual does not update its information of the road observed links.

Under these assumptions, each individual updates its information as $\mathbf{x}_i = \left(x_i^1, x_i^2, \ldots, x_i^l\right) \in [0,1]$ based on the information from its acquaintances and the information that they observe, and propagates the updated information to its acquaintances. The initial information about each road link is expressed as two cases: $x_i^l = 1$ if the node i observes road link l, and $x_i^l = 0$ otherwise.

3. Modeling of the Information Propagation

We propose an information propagation model by considering the certain information that is observed and the uncertain information from others separately. If road link l is unobserved by node i, then the information road link l at time period k + 1 is

$$x_i^l(k+1) = (1 - |N_i|\theta) \cdot x_i^l(k) + \theta \cdot \sum_{j \in N} a_{ji} \cdot x_j^l(k) \tag{1}$$

otherwise,

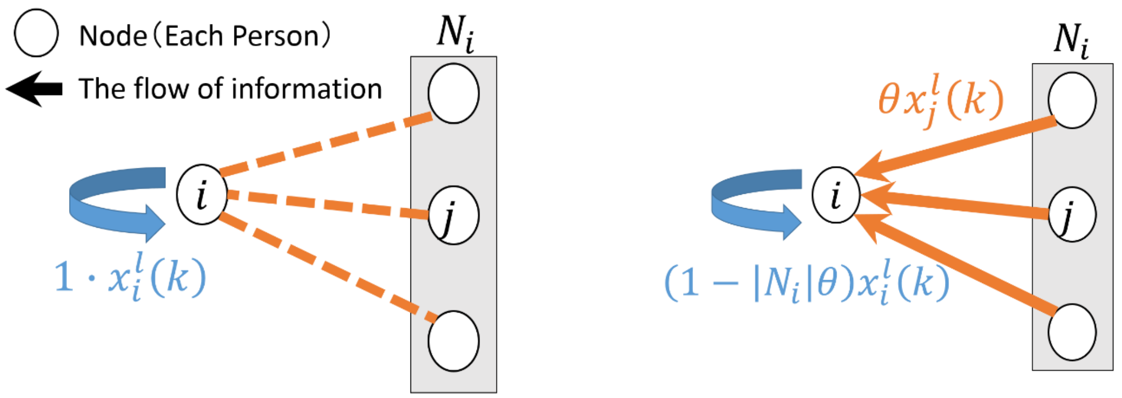
$$x_i^l(k+1) = x_i^l(k) = 1 \tag{2}$$

where a_{ji} is the element of the adjacent matrix that is one if the nodes j and i are connected and zero otherwise; $N_i \in \mathbf{N}$ is the set of individuals connected with individual i; and θ is the information adoption rate. The larger θ, the more information the node obtains from other nodes connected in the SN. Although weighting θ according to the reliability of acquaintances is possible using the idea of PageRank [3], we do not consider the weight at present.

From Equations (1) and (2), the information propagation speed is expressed using the difference equation of the amount of information:

$$\frac{d\boldsymbol{x}(k)}{dk} = \frac{\boldsymbol{x}(k+1) - \boldsymbol{x}(k)}{1} = -\theta(\mathbf{1} - \boldsymbol{x}^{\boldsymbol{l}}(0))\boldsymbol{L}\boldsymbol{x}^{\boldsymbol{l}}(k) \tag{3}$$

where $\boldsymbol{x}^{\boldsymbol{l}}(0)$ is the row vector the element is the initial amount of information $x_i^l(0)$ of each individual and $\boldsymbol{L} \in \mathbf{R}^{|\boldsymbol{N}| \times |\boldsymbol{N}|}$ is the Laplacian matrix proposed by Spielman[4], which is represented as $\boldsymbol{L} = \boldsymbol{D} - \boldsymbol{A}$ for the adjacent matrix $\boldsymbol{A}$ and degree matrix $\boldsymbol{D}$.



(a) Unobserved road link (b) Observed road link

Figure 1. Information update method of this study

4. Numerical Simulation

4.1. Index

We used algebraic connectivity as the parameter for the SN topology. Algebraic connectivity is the minimum eigenvalue of the Laplacian matrix without zero and a very important parameter for expressing network connectivity. We used the step number, called the convergence step number, as the parameter of the information propagation speed at the time that the average amount of information from the entire network exceeds some criterion because the convergence behavior is asymptotic.

4.2. Settings of the Numerical Simulation

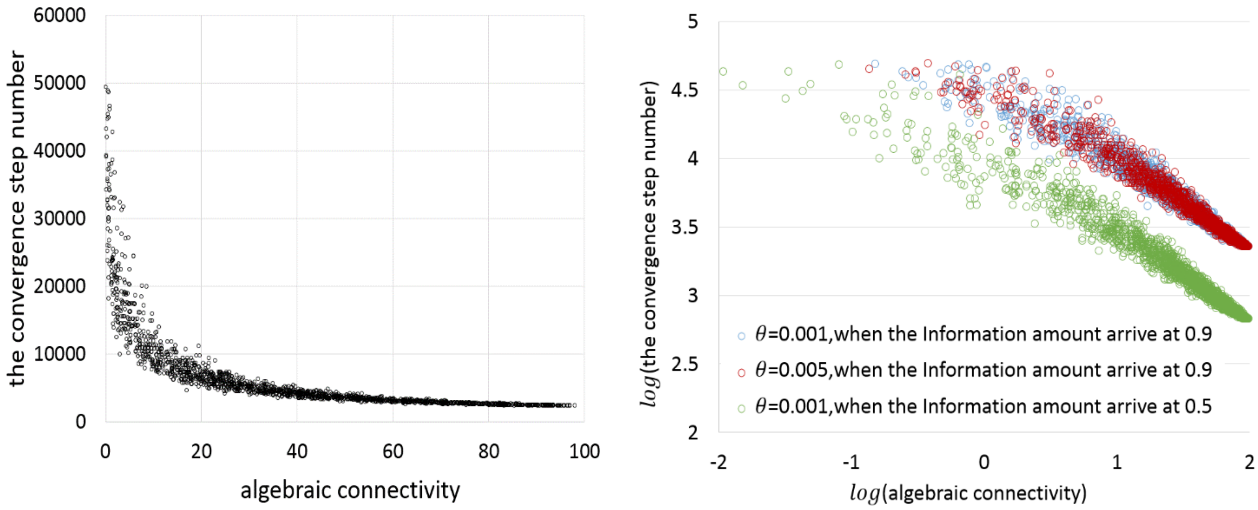
The settings of the numerical simulation are expressed in Table 1. At present, the larger the information adoption rate and rate of the observer, the faster the information propagation speed. However, we decided these value should be constant because they do not have a large effect on propagation behavior.

Table 1. Setting of the numerical simulation

Type of network	Number of nodes	Number of links	Information adoption rate $\boldsymbol{\theta}$	Rate of the observer
SN	100	Random (200-2000)	0.001	1/100
RN	N/A	1	0.001	1/100

4.3. Relationship between the Information Propagation Speed and Algebraic Connectivity

For approximately 5000 random SNs, we determined the power law between algebraic connectivity and the convergence step number because the plots of their logarithm values form a line with a small amount of scattering as shown in Figure 2. Therefore, the information propagation speed of our model is expressed sufficiently by algebraic connectivity.



(a) For the case of $\boldsymbol{\theta} = \mathbf{0.001}$ and the time is the amount of information over 0.9

(b) Logarithm values in some cases

Figure 2. Relationship betwee algebraic connectivity and the convergence step number

4.4. Theoretical Analysis

Obtaining the analytical equation that expresses the information propagation speed using algebraic connectivity is important for analyzing the difference in the traffic state, considering information propagation using the SN topology. First, we derived the solution of the differential equation of Olfati-Saber and Murray[5]'s consensus model, which is the basis for our model.

The consensus model expresses the propagation of one unit of information such that all individuals eventually have an average amount of information follows:

$$\frac{d\boldsymbol{x}(k)}{dk} = \frac{\boldsymbol{x}(k+1) - \boldsymbol{x}(k)}{1} = -\theta \boldsymbol{L}\boldsymbol{x}(k) \tag{4}$$

where $\boldsymbol{L}$ is Lapracian and the second smallest eigenvalue λ_2 of Laplacian (the smallest one λ_1 is always 0) is algebraic connectivity that express the SN topology. We determined that the solution of Equation (4) is expressed by the eigenvalues $\lambda_1, \ldots, \lambda_n$ of $\boldsymbol{L}$ and the corresponding eigenvectors $\boldsymbol{v}_1, \ldots, \boldsymbol{v}_n$ as

$$\boldsymbol{x}(k) = a_1\boldsymbol{v}_1 + a_2 e^{-\theta\lambda_2 k}\boldsymbol{v}_2 + \cdots + a_n e^{-\theta\lambda_n k}\boldsymbol{v}_n \tag{5}$$

When $\lambda_3, \ldots, \lambda_n$ are same values with λ_2 or obviously larger than λ_2, $e^{-\theta\lambda_n k} \approx e^{-\theta\lambda_2 k}$ is executed. Then the answer of Equation (4) can be approximated and reformed and be expressed the relation between SN topology and the information propagation speed of one individual on SN as

$$logK = -log\lambda_2 + (-log\theta + \log\left(-\,log\left(\frac{\alpha - a_1 v_{11}}{a_2 v_{21}}\right)\right)) \tag{5}$$

where $a_n \in \mathbf{R}$ is a coefficient, K is the convergence step number and α is the convergence value. By substituting $\theta' = \theta(\mathbf{1} - \boldsymbol{x}^{\boldsymbol{l}}(0))$ and $\alpha = 1$ into Equation (5), the convergence step number of this study can also be expressed using algebraic connectivity.

5. Conclusion

We determined that algebraic connectivity had a large influence on the information propagation speed of the SN in our model, which expressed whether road links were observed or unobserved. By proposing the route choice model on the RN based on updated information, it was possible to estimate the traffic state using the information propagation speed of each SN topology, and to create a benchmark of the correct timing of information provision. Additionally, we can extend the model of this study to make it realistic by considering the change of SN topology and weighting of the information adoption rate.

Acknowledgement

This work was supported by JSPS KAKENHI Grant Number 26220906. And we thank Maxine Garcia, PhD, from Edanz Group(www.edanzediting.com/ac) for editing a draft of this manuscript.

References

[1] Furuta, M., Iryo, T., Hara, Y. and Kuwahara, M.: Dynamic model of decision of evacuation timing based on the information propagation behavior of individuals, *The Committee of Infrastructure Planning and Management*, Vol. 51, CD-ROM, 2015 (in Japanese).

[2] Kermack, W. O. and McKendrick, A. G.: A contribution to the mathematical theory of epidemics, *Proc. R. Soc. Lond. A,* Vol. 115, pp.700 - 721, 1927.

[3] Brin, S. and Page, L.: The anatomy of a large-scale hypertextual Web search engine, *Proc. of 7th International World Wide Web Conference*, pp. 107 - 117, 1998.

[4] Spielman, D.: Spectral graph theory, Combinatorial Scientific Computing, Chapman and Hall, CRC, pp.495 - 524,2012.

[5] Olfati-Saber, R. and Murray, R. M.: Consensus problems in networks of agents with switching topology and time-delays, *IEEE Trans. Autom. Control, Vol. 49, No. 9, pp.1520 - 1533*, 2004.