Markov assignment for a pedestrian activity-based network design problem

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1 Introduction

Activity path choice problems are often difficult to solve because of the combinatorial complexity and have been dealt with as deterministic and restricted models in low-resolution networks (e.g., Kang et al., 2013). However, pedestrian activities are often probabilistic and should be described in high resolution networks, and it makes the problem much harder to solve using existing approaches. In order to deal with the computational challenges, we propose a network reduction method that is based on Markov processes and the time-space prism (Hägerstrand, 1970). Using the method, we introduce an activity assignment model as an operable framework of evaluating pedestrian behavior including route choices. Moreover, we show that the framework enables one to compute a pedestrian activity-based network design problem.

2 Activity assignment

Consider a spatial network $\mathcal{G}^s = (\mathcal{N}, \mathcal{A})$ and a time-space network $\mathcal{G}^a = (\mathcal{S}_{1:T}, \mathcal{E}_{1:T-1})$ where $t \in \{0, 1, ..., T\}$ is the discretized time at a fixed interval τ, and T is the time-constraint. State $s = (t, i) \in \mathcal{S}_t$ is defined as the combination of node $i \in \mathcal{N}$ and time t, and edge $e = (s_t, s_{t+1}) \in \mathcal{E}_t$ represents link $a \in \mathcal{A}$ at t. An activity path is described as $\psi_{1:T} = [s_0, ..., s_t, ..., s_T]$. We consider that $\mathcal{A} = \mathcal{A}^m \bigcup \mathcal{A}^s$ includes both moving link $a^m \in \mathcal{A}^m$ where pedestrians walk and staying link $a^s \in \mathcal{A}^s$ where the activities are performed (Figure 1).

We assume that the initial state $s_0 = (0, o)$ and the final state $s_T = (T, d)$ are always given and fixed for each individual, then we define $D^o(i)$ and $D^d(i)$ as the minimum

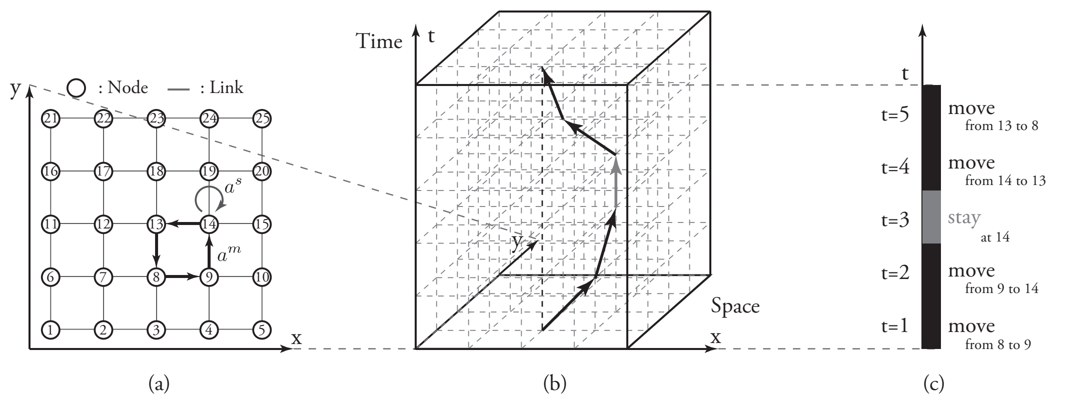

Figure 1. An activity path $\psi_{0:5} = [(8,9),(9,14),(14,14),(14,13),(13,8)]$ in time-space network and its projections. (a) Spatial network graph, (b) time-space network with time constraint $T = 5$ and (c) time use pattern.

number of steps from the origin o to i and from i to the destination d, respectively. $\mathcal{S}_t$ and $\mathcal{E}_t$ are restricted as follows:

$$\mathcal{S}_t = \{s_t = (t,i) | i \in \mathcal{N}, I_t(i) = 1\}, \tag{1}$$

where

$$I_t(i) = \begin{cases} 1, & \text{if } D^o(i) \leq t, D^d(i) \leq T - t \\ 0, & \text{otherwise,} \end{cases} \tag{2}$$

and

$$\mathcal{E}_t = \{e_t = (s_t, s_{t+1}) | s_t = (t,i) \in \mathcal{S}_t, s_{t+1} = (t+1,j) \in \mathcal{S}_{t+1}, \Delta_t(j|i) = 1\}, \tag{3}$$

where

$$\Delta_t(j|i) = I_t(i)\delta(j|i)I_{t+1}(j). \tag{4}$$

$\delta(j|i)$ is the space connection indicator. The right panel of Figure 2 illustrates the state set of $\mathcal{G}^a$, which forms the time-space prism where all possible paths are included. This path set of the time-space prism reflects the behavioral bounds based on the time constraint and is operable and reasonable for pedestrian networks compared to the set of efficient paths (Dial, 1971) or the universal set (Akamatsu, 1996; Fosgerau et al., 2013).

We then formulate an activity path choice model based on the sequential state transitions (e.g., Fosgerau et al., 2013). The probability that an individual at $s_t = i$ chooses $s_{t+1} = j$ is:

$$p_t(j|i) = \frac{\Delta_t(j|i) e^{\mu\{u^t_{ij} + \beta\varphi_{t+1}(j)\}}}{\sum_{j' \in \mathcal{S}_{\sqcup+\infty}} \Delta_t(j|'i) e^{\mu\{u^t_{ij'} + \beta\varphi_{t+1}(j')\}}}. \tag{5}$$

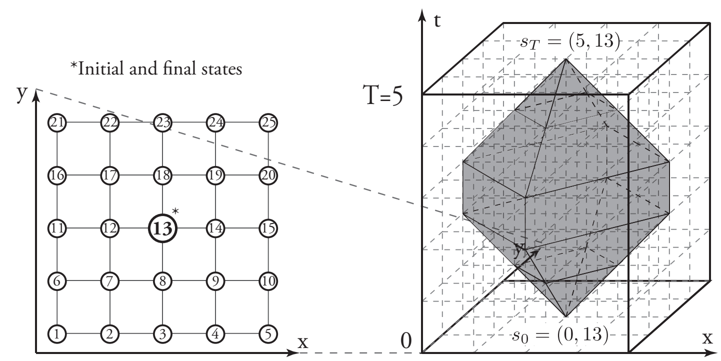

Figure 2. *Time-space prism* constraint and path restriction when $s_0 = 13$ and $s_T = 13$ ($T = 5$) are given.

where $\tilde{u}_{ij}^t = u_{ij}^t + \varepsilon_{t+1}(j)$ is the instantaneous utility of transition. $\varphi_{t+1}^{s_T}(j)$ is the expected maximum utility to the final state $s_T = (T, d)$ and formulated by Bellman equation. It is important that our model includes the time-space prism variable $\Delta_t(j|i)$ for restricting path set and the discount factor $\beta \in [0, 1]$ to describe dynamic behavior. Moreover, we can solve the Bellman equation simply with the backward induction, because $\varphi_t(i)$ is defined for each pair of space and time. Finally, we calculate the flow of each edge f_{ij}^t by the forward calculation, then we obtain the spatial link flow $f_{ij} = \sum_{t=0}^{T-1} f_{ij}^t$.

3 Pedestrian network design: increasing sidewalk area

We look for the configuration of a network that satisfies the maximum duration time for different increases of the sidewalk area [m^2]. The decision variable is the sidewalk width n_{ij} [m] on each moving link $a^m = (i, j) \in \mathcal{A}^m$, which must satisfy $n_{ij}^{\min} \leq n_{ij} \leq n_{ij}^{\max}$, where $n_{ij}^{\min}$ and $n_{ij}^{\max}$ indicate the current width and the maximum road width [m], respectively. It is also incorporated into the utility function u_{ij}^t. We investigate the trade-off curve between two conflicting objectives: maximizing the duration time in the entire network,

$$\max z_1 = \sum_{(i,j)\in\mathcal{A}} \sum_t f_{ij}^t \tau, \tag{6}$$

and minimizing increased sidewalk area,

$$\min z_2 = \sum_{(i,j)\in\mathcal{A}^m} (n_{ij} - n_{ij}^{\min}) l_{ij}. \tag{7}$$

To solve this bi-objective and bi-level programming, we adopt the network update algorithm (Scarinci et al., (to appear)) that has two main steps: activity assignment and network update. The network update modifies the current network, and the new solution is considered as a Pareto front solution if it satisfies the acceptance criterion.

We start with the network equipped with the maximum possible sidewalk area, i.e., $n_{ij}^{(1)} = n_{ij}^{\max}\ \forall (i, j) \in \mathcal{A}^m$, and we iterate the assignment and network update process for

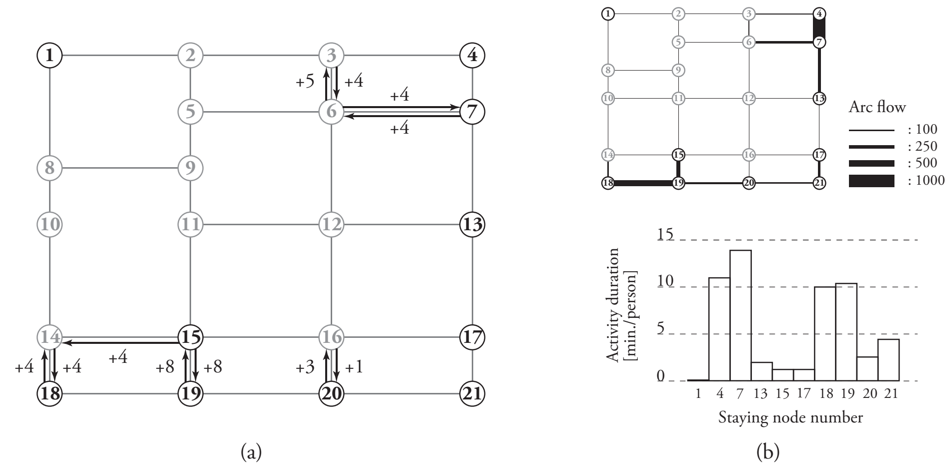

Figure 3. Network design results. (a) Example of a Pareto front solution for the pedestrian network configuration and (b) the activity assignment result.

1000 times, using a simple network. We show an example Pareto front solution in Figure 3(a), where on eleven links the sidewalk width increases. These links are located near the origins/destinations or streets with many staying nodes (indicated in bold). Figure 3(b) shows the activity assignment result in the solution network. Our model can evaluate the use patterns of space and time simultaneously.

4 Conclusion

To deal with complicated pedestrian activity path choice problem, we applied a Markov assignment with the method for restricting path set based on the time-space prism. It enabled us to solve the Bellman equation stably and evaluate the use patterns of space and time simultaneously. We also present a pedestrian activity-based network design problem and investigated the Pareto front solutions.

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