

**“It’s the relativity, stupid!”**

## **Testing Weber’s law in utility-based and regret-based models of travel behavior**

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In this study we extend recent work into the role of Weber’s law in discrete choice theory towards non-regret based models of (travel) choice behavior; and we provide an empirical exploration of the relevance of Weber’s law in the context of utility- and regret-based models of travel behavior.

Being one of the most well-known regularities in the Social Sciences, Weber’s law relates the *actual* difference between physical stimuli (in the context of discrete choice models these are the attributes) to the size of the difference as *perceived* by the decision maker. Reformulated into a discrete choice context, **Weber’s law asserts that the perceived size of a difference in attribute values between alternatives is inversely proportional to the (actual) attribute values themselves.** Take the example of perceived travel time differences. In this context, Weber’s law suggests that a ten minutes actual travel time difference between two routes is perceived to be larger (or more salient) when the routes’ travel times are 5 and 15 minutes respectively, compared to the situation where the two routes’ travel times are 105 and 115 minutes respectively. Empirically, Weber’s law has been firmly empirically established in relation to a wide variety of senses, such as vision, hearing, taste, touch and smell.

**In a recent and very interesting contribution, Jang et al. (in press) introduce Weber’s law to a discrete choice theory context:** in their paper, the authors suggest that when comparing different alternatives (e.g. routes) in terms of their attributes (e.g. travel times), decision makers follow Weber’s law in the sense that differences between attribute values – across alternatives – become more pronounced and influential when the attribute levels themselves are smaller. The authors introduce Weber’s law – including a generalized version thereof which included an estimable parameter that governs the size of the Weber effect – in the specific context of Random Regret Minimization (RRM) models, more specifically two versions thereof (Chorus et al., 2008; Chorus, 2010). This is a natural choice, since RRM models are based on the behavioral premise that comparisons between attributes across alternatives is the key to regret, an emotion which RRM models postulate is being avoided by decision makers. Jang et al.’s results are

promising, in the sense that they find that their (Generalized) Weber’s model empirically outperforms conventional RRM models.

**This paper extends the work of Jang et al. (in press) towards linear Random Utility Maximization (RUM) models, and a range of regret models not considered by the authors.** We do this by combining their generalization with the recently proposed  $\mu$ RRM model (van Cranenburgh et al., 2015), which has linear RUM models and more or less extreme RRM models as limiting cases. It should be noted that the extension of Jang et al.’s approach to linear RUM models is non-trivial: only by rewriting the linear RUM model as a limiting case of the  $\mu$ RRM model, can we estimate to what extent the base values of attribute levels influence the perceived salience of differences between these attribute levels (i.e., this is not achievable in the context of the conventional mathematical formulation of utility as a linear-additive function). The regret function in our  $\mu$ RRM-Weber model is written as follows (where  $x_{im}$  represents the attribute value of a considered alternative,  $x_{jm}$  the attribute value of a competing alternative,  $\beta_m$  represents the estimated attribute weight,  $\mu_m$  an estimable parameter governing the degree of regret aversion, and  $\vartheta_m$  represents an estimable parameter governing Weber’s law, as presented in Jang et al.’s paper):

$$R_i = \sum_{j \neq i} \sum_m \mu_m \cdot \ln \left( 1 + \exp \left\{ \frac{\beta_m}{\mu_m} \cdot \left[ \frac{x_{jm} - x_{im}}{(x_{im})^{\vartheta_m}} \right] \right\} \right)$$

Relevant special or limiting cases arise when  $\mu_m$  approaches zero for all attributes (in that case, the P-RRM model arises which postulates extreme regret aversion); when  $\mu_m$  is large and positive for all attributes (in that case, the linear RUM model is approached); when  $\vartheta_m$  equals zero for all attributes (in that case, there is no Weber effect); and when  $\vartheta_m$  approaches one for all attributes (implying an effect as in the classical version of Weber’s law). As such, the above combined  $\mu$ RRM-Weber model is able to capture a wide range of regret aversion levels in combination with a wide range of Weber-effects.

**Subsequently, we use the above presented  $\mu$ RRM-Weber model to explore the empirical performance of Weber’s law** in the context of various degrees of regret aversion, ranging from the complete absence of regret aversion as in linear RUM models to the presence of extreme regret aversion as in so-called P-RRM models. We do this by estimating choice models based on the above generalized regret function in the context of a variety of previously collected datasets. More specifically, we first systematically explore (the properties of) all 19 special cases of this combined  $\mu$ RRM-Weber model using numerical analyses. Subsequently, we test the performance of the  $\mu$ RRM-Weber model and all its special cases on five empirical data sets.

Preliminary results show that incorporating Weber's law potentially improves model fit, but only in some cases. Furthermore, model fit improvements are generally found to be modest. Interestingly, we find that whereas incorporating Weber's law in models with high levels of regret aversion (such as P-RRM models) may result in improvements in empirical performance, this does not seem to be the case in the context of linear RUM models. We interpret this and other findings in the context of the behavioral differences between regret minimization and utility maximizations.

## References

- Chorus, C.G., 2010. A new model of Random Regret Minimization. *European Journal of Transport and Infrastructure Research*, 10(2), 181-196
- Jang, S., Rasouli, S. & Timmermans, H. (in press). Incorporating psycho-physical mapping into random regret choice models: model specifications and empirical performance assessments. *Transportation*
- Van Cranenburgh, S., Guevara, C. A. & Chorus, C. G. (2015). New insights on random regret minimization models. *Transportation Research Part A: Policy and Practice*, 74(0), 91-109.
- Weber, E. H. (1834). *De Pulsu, resorptione, auditu et tactu: Annotationes anatomicae et physiologicae*: CF Koehler).