A reaction-diffusion model with region-to-region parameters for large scale traffic networks
Extended Abstract

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1 Introduction

While cascade phenomena have been broadly studied by physicists, understanding and modeling of congestion propagation in large urban city networks still remains a challenge. Most efforts are mainly based on micro-simulations of link-level traffic dynamics without a proper treatment of physical laws. The main purpose of this paper is to reveal the process of congestion formation by exploring empirical and simulated data from large-scale urban networks. Specifically, the authors aim at studying the spatiotemporal relation of congested links, observing congestion propagation from a macroscopic perspective, and develop a dynamic model with a small number of parameters that can properly reproduce the spatiotemporal distribution of congestion and cascade phenomena of traffic. The model is based on two ingredients: a reaction and a diffusion term. The interaction of these two terms brings the model in a self-organized pattern that after appropriate calibration can reproduce realistic traffic scenarios. Vehicles spread through the urban network by diffusion as well as the values of average link speed according to a Fundamental Diagram that relies on density, flow and speed. The reaction term will be the responsible of any exogenous change of concentration of vehicles, e.g. local peaks of demand. The combination of these two terms will reproduce many different scenarios. The final outcome of this work is not the accurate estimation of speeds for every link in a large network, but an elegant physical law that can generate realistic aggregated congestion patterns. The results presented show very good data matching with an available data set of more than 20k taxis GPS in Shenzhen for two different cases: a morning peak hour and a whole working day of urban traffic.

2 The model

The model proposed by the authors is a link-based model. A lot of work has been done in this topic (see as examples [9], [7] and [8]) because link-scaled models are considered a good trade-off between micro-simulation and regional scale model. In fact all this family of models aggregate at link level the macroscopic traffic variables like speed, flow and density.

Our purpose is to simulate the evolution during the day of link speeds of a defined urban network given realistic initial conditions. The peculiarity of this model is that it was inspired by a very general version of the reaction-diffusion phenomenon (in continous [1] and in networks [10]) but it also considers some specific aspects typical of the transportation system and traffic flow theory.

This model is composed of two parts: a diffusion and a reaction term. These two actors operate in a network, represented by a graph $\mathcal{G}(N, E)$ of N nodes and $E \subset N \times N$ links. The diffusion part will be regulated by the combinatorial Laplacian $L = A - kI$ where A is the corresponding adjacency matrix of graph $\mathcal{G}$ and kI the diagonal matrix with the corresponding node degree $k_i, i \in N$ as entries. For the purpose of this work it will be simpler and useful to use the dual representation of the graph where each node $i \in N$ represents link (road) and the elements of the adjacency matrix $A = \{a_{ij}\}$ represent the intersections, that is the connections between links. In particular, the element $a_{ij} = 1$ if and only if link i and link j are adjacent, 0 otherwise. In the matrix L the elements on the diagonal $l_{ii} = -k_i$ where k_i is the number of the adjacent links of i. In literature, one can find another definition of the combinatorial Laplacian, with opposite sign: -1 if there is a link between i and j and k_i in the diagonal. We chose to keep the definition in the seminal paper about reaction-diffusion system in networks (see [10]).

On the other hand, the reaction is regulated by a non linear function $f(\bar{u}, t)$ depending on the vector $\bar{u} = \{u_i\}_{i \in N}$ and the time t. The most general form of the discrete differential equations for every component $i \in N$ of the vector $\bar{u}$ will be:

$$\frac{du_i(t)}{dt} = \rho(i,t) f(\bar{u}(t), t) + \sigma(i,t) \sum_{j=1}^{N} L_{ij} u_j(t). \tag{1}$$

The parameters ρ and σ will be the reaction and diffusion parameter respectively and they could depend on space (link i) and/or time t.

The diffusion term $\sigma(i,t) \sum_{j=1}^{N} L_{ij} u_j(t)$ changes the distribution of u_i values among the links of the network while the reaction term $\rho(i,t) f(\bar{u}(t), t)$ is the responsible for the change of the sum of link speeds ($\sum_i u_i$). The weight for the effects of two terms can be regulated by their respective parameters σ and ρ.

2.1 General settings

Equation (1) could be seen like a very general version of our model. Parameters ρ and σ may be time-dependent or in some case, as in a single peak hour event, they can be consider constant in time. The example shown in the following section reproduce with a simulation of the reaction-diffusion model an emerging congestion in the morning peak hour of a big Chinese city, and in this case ρ

and σ will not depend on time because of the monotonic trend of the decrement in average speed due to congestion.

We are also interested in the space-dependence of these parameters. In particular, we aim to estimate a good partition of the network into homogeneous regions and reaction and diffusion parameter for couple of link belonged to the same region or two different adjacent regions. In this case ρ and σ will be represented by two matrices. The detail of this representation will be discussed in Section 4.

The function f can be chosen from a large set. In general, we want a dependence on the value u_i of the other links i and on time. This because it will describe how congestion "reacts" according the value of speed of the surrounded links, and also according to the increasing or decreasing trend of the traffic at a certain moment t.

We tested our model for 2 different cases come from data in the works [3] and [5]. The first case is a portion of Shenzhen (China) urban network on the 1^{st} of September 2011 from 6am to 8am, that is the morning peak hour. The second case is for a whole day where one can easily distinguish two different congested periods, one in the morning and the other in the afternoon. For the first case, because of the increasing monotonic behavior of the congestion we chose to fix the function f and the parameters ρ and σ. The results, shown in the next section, proof that also with assumptions the model fits very well the speed distribution come from the real data. The second case is more general and it is based on data from the same network of Shenzhen but for a whole day (8th September 2011). For this case we introduce a control parameter that with the comparison between the values issues from the simulation and the real data. In particular, we look at the global average link speed and based on it we modify through a proportional scalar parameter the reaction function. Also the standard distribution is used to regulate the σ diffusion constant and create a continuous simulation that follows the real data with an online calibration.

3 Principles of reaction and diffusion

The two principles of reaction and diffusion that are expressed in the differential equation 1 can be seen as the counterpart of following two empirical facts:

P1) A congested link leads the drivers to prefer one of its neighbor links;

P2) If a link is surrounded by congested links with high probability it will get congested in a brief delay.

$P1$) is simulated by the diffusion term. In fact, diffusion is applied to each link speed values we obtain as result to transfer some quantity from a link to its neighbor that means to get higher u_i in one link and decrease proportionally u_i in its adjacent link. In the other hand $P2$) is made effective by the reaction term depending on the sum of the differences $\Delta u_i = \sum_{j \in N_i} u_i - u_j$ between a link i and its neighbors N_i.

Case 1: Let us consider a peak hour scenario in an urban network. We can observe that congestion propagate with a certain speed through the urban network and also that the global average link speed decreases. To simulate principle $P2$) we decide to use a simple mathematical function, that depends, for each u_i on its neighbors. For this particular case we chose the logarithm function $f(i, u_i) = log(1 - \Delta u_i)$ because of its concavity and simplicity.

Case 2: For the all day simulation we need a symmetric function, limited and easily to adjust with a simply constant ($alpha_t = a * (\bar{\hat{u}} - \bar{u})$) to force the system to increase or decrease the average link speed based on the online measurements. For this aim we chose $f(i, u_i, t) = -tanh(\alpha_t - \Delta u_i)$ that respects all our requirement.

4 Parameters region-to-region

It is well known that the streets that compose an urban transportation system network can be classified according to their types for example as periphery, primary or secondary roads, highway, etc.. So it is natural to imagine that the correlation between two roads belonging to two different types can react in a different way than two links which are one the consecutive of the other and belonging to the same type of road (See Figure 1).

Another differentiation can be done using clustering algorithms that divide the street network into large zones based on the level of congestion and/or well defined Macroscopic Fundamental Diagram ([2]).

For this reason in order to reach more accuracy can be useful to set different reaction (ρ) and diffusion (σ) terms based on the different locations and usage.

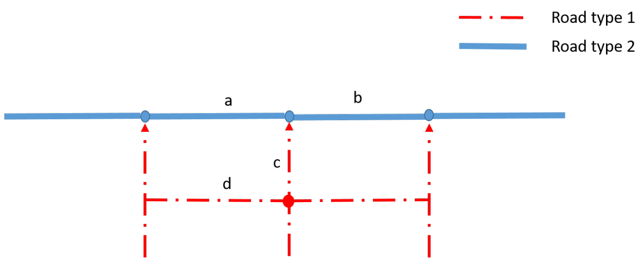

Figure 1: Example of intersection of different types of road in an urban network.

In general, ρ and σ will be represented by two squared matrix of the same dimension than the number of the pointed out regions and set with the values to obtain the best benchmark with the comparison with real data.

5 Results

5.1 Case 1: morning peak hour

In our application, based on a clustering algorithm proposed in a very recent paper, we divided the city of Shenzhen into 3 regions: Periphery (green), Upper part (blue) and Lower part (red). The parameter ρ and σ are two matrices 3×3 used to calibrate the interactions between regions. That is

$$\sigma = \begin{vmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{vmatrix} \qquad \rho = \begin{vmatrix} \rho_{11} & \rho_{12} & \rho_{13} \\ \rho_{21} & \rho_{22} & \rho_{23} \\ \rho_{31} & \rho_{32} & \rho_{33} \end{vmatrix}$$

Where $\sigma_{ij}(\rho_{ij})$ is the weight in the diffusion (reaction) term that a link from zone i has on a link of zone j.

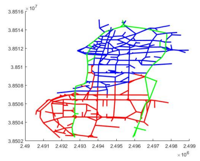

Figure 2: Clustering of Shenzhen (China) downtown street network in 3 zones. This clustering is based on real data about level of congestion during a peak hour

After applying the model of (1) to Shenzhen network and calibrate the matrix-parameters ρ and σ we have been able to reproduce a very accurate prediction of link speeds distribution during a congested phase of the city center, from 6am to 8am. In particular, we reproduce a good spatial correspondence with the real data and the distribution of u_i (See the comparison between contour plot of real data and simulated results in Figure 3 and 4).

5.2 Case 2: Whole day simulation

In this case we tested if our model was flexible and good enough to simulate a whole day with peak and off-peak of the congestion in the city center. As we anticipated in the previous section we consider for this scope the symmetric function $f = -tanh$ and a constant α_t to add on the argument of the reaction function in order to calibrate the model to follow the real data. For what concerns the reaction and diffusion parameters ρ and σ we start from the value founded in Case 1.

With the $\alpha_t = a * (\bar{\hat{u}} - \bar{u})$ we adjust the reaction function in order to follow the global average speed of the network. a is a scalar to weight this control parameter. Moreover, our goal it is also to have a realistic distribution of link speeds and to do this we use another control parameter $\beta_t = std(\hat{u}) - std(u)$ that changes σ in $\sigma + \beta_t$ according with the difference in terms of standard deviation of the distribution of the real speeds $\bar{\hat{u}}$ with the speeds estimated by the model $\bar{u}$ at the corresponding time t.

This control parameter α_t and β_t permit to have very good results with few calibrations. In this sense, as one can see in , the model is able to simulate the on-peak and off-peak of the congestion without loosing the distribution of the link speed (Figure 5 and 6) and the cluster subdivisions (Figure 7).

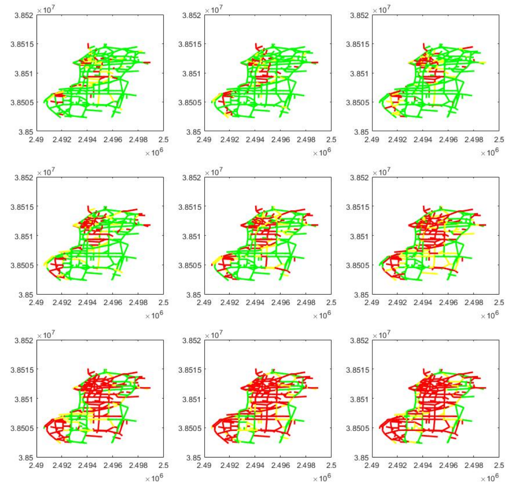

Figure 3: Contour plot of Shenzhen network based on real data of a peak hour of a typical working day from 6am to 8am. In red links high congested, in yellow normal traffic flow and in green good traffic condition.

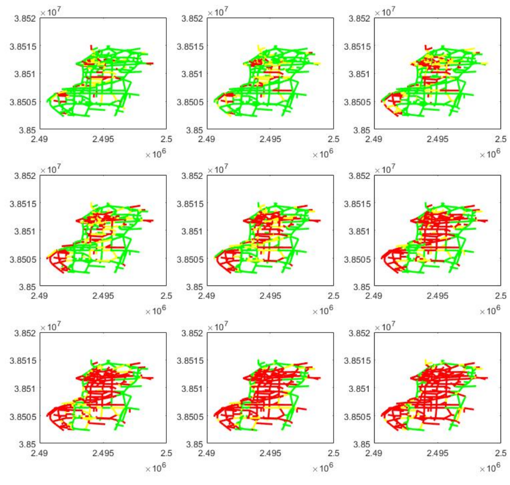

Figure 4: Contour plot of Shenzhen of simulation results starting from the same initial condition than in Figure 3 and using the 3×3 matricial form of parameters ρ and σ.

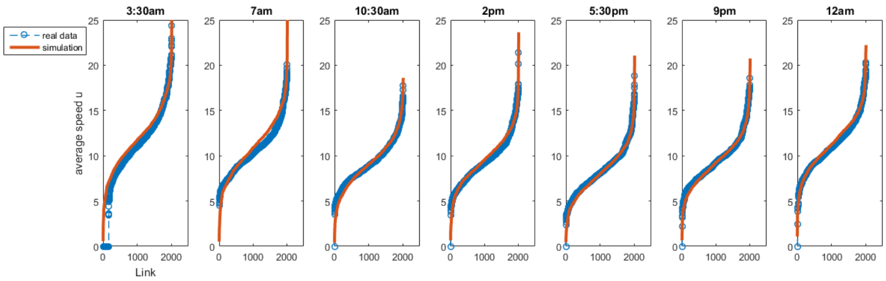

Figure 5: Comparison of the distribution of link speed between Simulation and real data

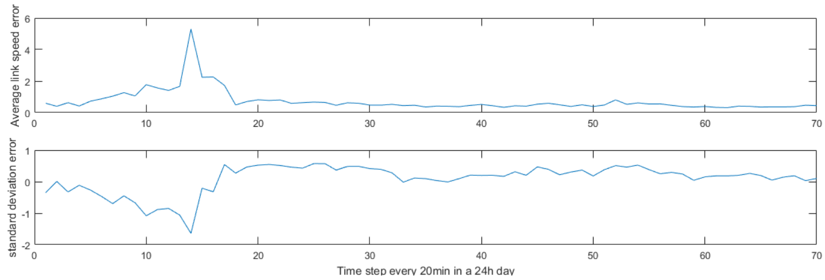

Figure 6: Difference of the value of the average link speeds (top) and standard deviation of the speed distribution (bottom)

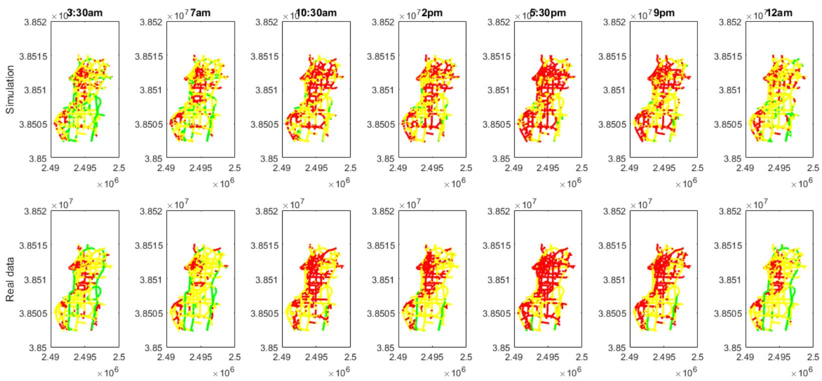

Figure 7: HeatMap comparison with the follow color legend: green for free flow speed, yellow normal traffic and red congested link.

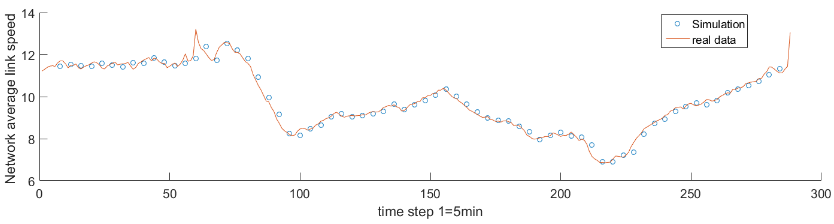

Figure 8: global average link speed for real data (continuous line) and simulation (circled points)

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