Face validation of a microscopic cycling behaviour model using differential game theory

Alexandra Gavriilidou, Yufei Yuan, Haneen Farah and Serge P. Hoogendoorn

1. INTRODUCTION

Microscopic behavioural models are used in order to study traffic flow operations, evaluate the performance and traffic safety of different infrastructure designs. The increasing interest for cycling in cities necessitates the development of a theory and model pertaining to the movement of cyclists at the operational level. Existing research makes use of modelling paradigms developed for motorised traffic or pedestrians, and as indicated by (Twaddle et al. 2014) in their review, most of the developed models consider the behaviour of cyclists in mixed traffic conditions (i.e. sharing the infrastructure with other road users), thereby lacking characteristics particular to cyclists' behaviour and motion. The studies that focus solely on cyclists tend to look at macroscopic properties instead of aiming to accurately represent the microscopic behaviour. This, along with the general lack of model calibration and validation using empirical data, greatly reduces the applicability of these models.

In this study, we develop a novel modelling paradigm for microscopic cycling behaviour based on effort minimisation and differential game theory, and we describe the framework for its face validation.

2. MODEL DERIVATION

Given the fact that cyclists exert effort for their motion, the theory of effort minimisation can be adopted from the micro-economic theory of subjective utility maximization. Also, due to their size and flexibility, close interactions between cyclists are possible, which can be accounted for using a game theoretical approach. These two principles can be combined with optimal control theory to derive a new microscopic cycling model.

The derivation follows the framework developed by Hoogendoorn and Bovy (2003), while changing the coordinates for the description of the motion from Cartesian to polar, since the steering is better captured by angular movements rather than lateral ones. Additionally, anisotropy is considered with respect to the relative location of others based on the formula for the proximity cost proposed in (Helbing et al. 2002). We assume that a cyclist p minimises the objective J_p over the time interval $[t, t+T)$ by choosing the longitudinal acceleration (i.e. change in speed) a_p and the angular velocity (i.e. change in steering angle) ω_p:

$$J_p = \int_t^{t+T} e^{-s} L \, ds \tag{1}$$

where t and T denote the current and terminal time, respectively. L denotes the running cost equal to:

$$L = \frac{1}{2}\left(v_p{}^0 - v_p\right)^2 + \frac{1}{2}\left(\theta_p{}^0 - \theta_p\right)^2 + \frac{1}{2}a_p{}^2 + \frac{1}{2}\omega_p{}^2 + \sum_q e^{-r_{pq}/R}\,\beta \tag{2}$$

where $v_p{}^0$ and $\theta_p{}^0$ denote the desired speed and direction of cyclist p respectively, v_p and θ_p are the current speed and direction of p, r_{pq} is the distance between cyclists p and q, R is the range of the repulsive interaction, and $\beta = \psi + (1-\psi)\frac{1+\cos\phi_{pq}}{2}$ reflects the anisotropic character of cyclist interaction. The level of anisotropy is determined by the parameter ψ. The variable ϕ_{pq} is the angle between the movement direction of p and the position of q (Figure 1).

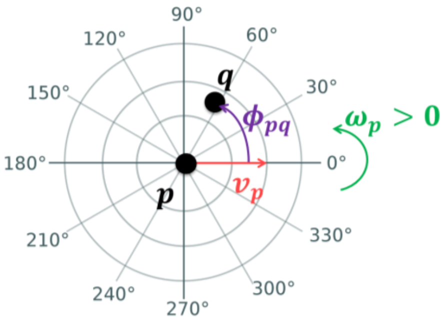

Figure 1: Definition of the angle between cyclists p and q and direction of positive angular velocity.

The differential game approach looks for the control that minimises the objective J_p. This can be achieved via Pontryagin's Minimum Principle (MP), which uses the Hamiltonian function H defined by:

$$H = L + \vec{\lambda}\,\frac{d\vec{\xi}}{dt} \tag{3}$$

where $\vec{\lambda}$ denotes the so-called co-state vector and shows how a small change in the state $\vec{\xi} = (x, y, v, \theta)$ leads to changes in the optimal solution $J_p{}^*$.

Using the MP, the optimal control (i.e. $a_p{}^*$ and $\omega_p{}^*$) is derived and given by the following equations:

$$a_p{}^* = \overbrace{\left(v_p{}^0 - v_p\right)}^{1^{st}\ term} - \overbrace{\sum_q e^{-\frac{r_{pq}}{R}}\left(\frac{\beta\cos\phi_{pq}}{R}\right)}^{2^{nd}\ term} - \overbrace{\sum_q e^{-\frac{r_{pq}}{R}}\left(\frac{(1-\psi)\sin^2\phi_{pq}}{2r_{pq}}\right)}^{3^{rd}\ term} \tag{4}$$

$$\omega_p{}^* = \overbrace{\left(\theta_p{}^0 - \theta_p\right)}^{1^{st}\ term} - \overbrace{\sum_q e^{-\frac{r_{pq}}{R}}\left(\frac{\beta\sin\phi_{pq}}{R}\right)}^{2^{nd}\ term} - \overbrace{\sum_q e^{-\frac{r_{pq}}{R}}\left(\frac{(1-\psi)\sin\phi_{pq}\cos\phi_{pq}}{2r_{pq}}\right)}^{3^{rd}\ term} \tag{5}$$

Both control variables consist of three terms, the first describing the desire to maintain a certain speed and direction, and the others capturing the desire to keep some distance from other cyclists. The 2nd and 3rd term are visualised in Figure 2, where the magnitude of each term is plotted for three r_{pq} values at every possible angle ϕ_{pq} with $R = 2$ and $\psi = 2$. It can be seen that as the distance to other cyclists increases, their effect on the control strategy decreases. Based on these plots, the terms can be physically interpreted. With respect to the longitudinal acceleration, the 2nd term primarily reflects the interactions with cyclists back and front (i.e. at 180 and 0 degrees, respectively), while the 3rd term considers those with cyclists travelling in parallel (i.e. at 90 and 270 degrees). Since no bias based on traffic rules has been specified (e.g. overtaking from the left), cyclists tend to decelerate irrespective of their relative side on the road. The angular velocity indicates the steering direction (Figure 1), which due to the absence of bias fails to react when cyclists approach right in front or behind. The 2nd term urges cyclists to shy away from others especially parallel to them, while the 3rd term focuses on the diagonally surrounding cyclists.

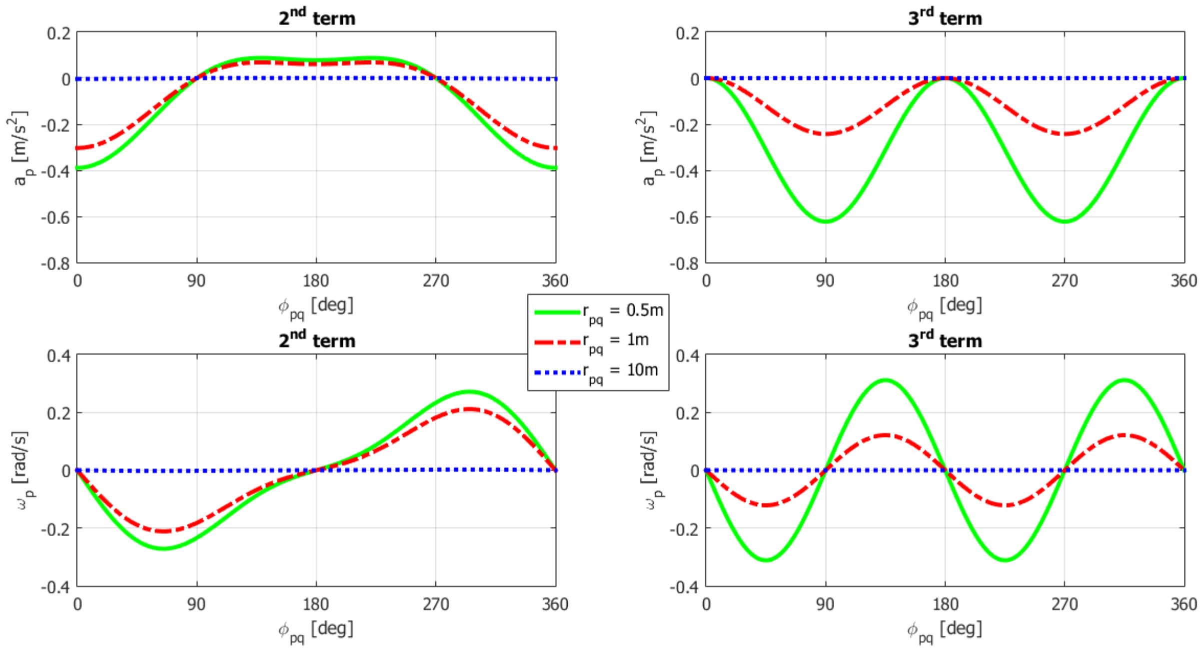

Figure 2: Visual representation of 2nd (left) and 3rd (right) term of longitudinal acceleration (top) and angular velocity (bottom) for three r_{pq} values at every possible angle ϕ_{pq} with $R = 2$ and $\psi = 2$.

The importance of each of these terms differs per person but may also be affected by the situation (e.g. cycling parallel to strangers or company). Therefore, behavioural parameters are introduced as weights (τ, ζ, A, B, C and D) in the components of these functions, leading to the following equations for the derived model:

$$a_p{}^* = \frac{(v_p{}^0 - v_p)}{\tau} - \sum_q e^{-\frac{r_{pq}}{R}} \left(A \frac{\beta \cos \phi_{pq}}{R} + B \frac{(1-\psi) \sin^2 \phi_{pq}}{2 r_{pq}} \right) \tag{6}$$

$$\omega_p{}^* = \frac{(\theta_p{}^0 - \theta_p)}{\zeta} - \sum_q e^{-\frac{r_{pq}}{R}} \left(C \frac{\beta \sin \phi_{pq}}{R} + D \frac{(1-\psi) \sin \phi_{pq} \cos \phi_{pq}}{2 r_{pq}} \right) \tag{7}$$

3. FACE VALIDATION FRAMEWORK

In order to check the plausibility of the model and acquire a reasonable range of values for the model parameters prior to an extensive calibration on empirical data, a face validation framework is followed. It consists of two processes; the first makes use of the macroscopic properties and the second of the cyclist trajectories.

By assuming that cyclists ride behind each other without need for steering, in an infinite loop, equilibrium conditions can be reached. In equilibrium their acceleration is zero and the equation can be solved for the speed as a function of the cyclist density and the parameters A, B, R and τ. Since cyclists are behind each other, $\sin \phi_{pq} = 0$ and the effect of B cannot be studied in this step. By varying the density, a fundamental diagram can be constructed for different combinations of A, R and τ and compared to values for the critical and jam density obtained from literature. The range of R is determined by the critical density, while the others (A and τ) depend on the jam density. An example for $R = 2$ is shown in Figure 3.

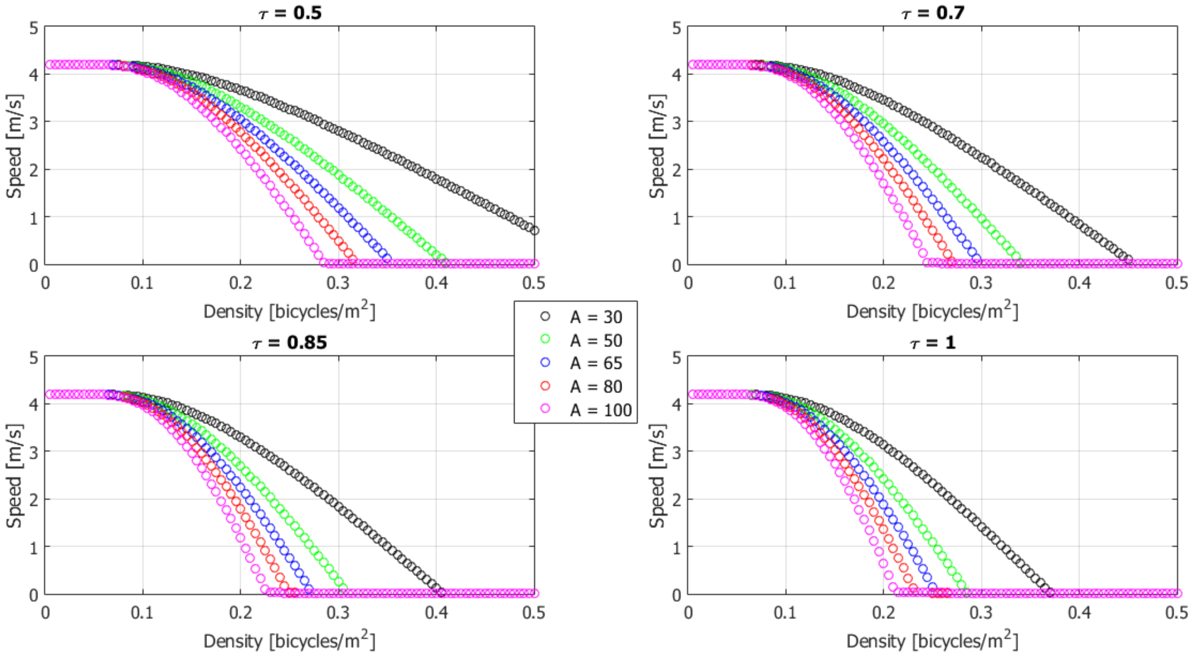

Figure 3: Speed – density plots for $R = 2$ and different parameter settings for A and τ.

From these plots and a range of 0.27 – 0.45 bicycles/m^2 for the jam density according to the literature (Hoogendoorn and Daamen 2016), the acceptable ranges for A and τ can be derived. Following these findings, the assumptions can be lifted and the trajectories of two opposing cyclists can be compared for different combinations of B, C, D and ζ.

The full paper will present the results of both processes in more detail.

REFERENCES

Helbing, D., I. J. Farkas, P. Molnar and T. Vicsek (2002). "Simulation of pedestrian crowds in normal and evacuation situations." Pedestrian and evacuation dynamics **21**(2): 21-58.

Hoogendoorn, S. and P. H. L. Bovy (2003). "Simulation of pedestrian flows by optimal control and differential games." Optimal Control Applications and Methods **24**(3): 153-172.

Hoogendoorn, S. and W. Daamen (2016). "Bicycle Headway Modeling and Its Applications." Transportation Research Record: Journal of the Transportation Research Board(2587): 34-40.

Twaddle, H., T. Schendzielorz and O. Fakler (2014). "Bicycles in urban areas: Review of existing methods for modeling behavior." Transportation Research Record: Journal of the Transportation Research Board(2434): 140-146.