Incitements for transportation collaboration by cost allocation

Joen Dahlberg[a,b,*], Stefan Engevall[a], Maud Göthe-Lundgren[a,b], Kurt Jörnsten[c], Mikael Rönnqvist[d]

[a] *Department of Science and Technology, Linköping University, Sweden*
[b] *The Swedish National Road and Transport Research Institute, Linköping, Sweden*
[c] *Norwegian School of Economics and Business Administration, Bergen, Norway*
[d] *Dpartement de gnie mcanique, Universit Laval, Qubec, Canada*

Abstract

In this paper, we focus on how cost allocation can be used as a means to create incentives for collaboration between companies, with the aim of reducing the overall operational cost. Collaboration is assumed to be preceded by a simultaneous invitation of the companies to collaborate with an initiating company. We make use of concepts from cooperative game theory and develop specific cost allocation mechanisms in order to introduce a solid base, a part of a business model, with potential to establish large collaborations among the companies. The cost allocation mechanisms are tested on a case that involves transportation planning activities at forestry companies. Although the case study is from a specific transportation sector, the findings in this paper can be adapted to collaborations in any type of transportation planning activity. Two of the cost allocation mechanisms ensure that any sequence of companies joining the collaboration represent a monotonic path, that is, any sequence of collaborating companies is such that the sequences of allocated costs are non-increasing for all companies.

Keywords: Dahlberg, Collaboration, Forest industry, Monotonic Path, Cost Allocation

*Corresponding author
Email address: joen.dahlberg@liu.se (Joen Dahlberg)
URL: http://www.liu.se/personal/itn/kts/joeda33?l=en (Joen Dahlberg)

1. Introduction

Collaboration in transportation has become a growing area of research, resulting in a need for development of new transportation planning models. Some areas where collaborative transportation solutions are desirable or necessary are country side freight transportation, supply chain operations, and sector specific collaborations. Here the objective is, mainly, to increase vehicle load factors and to reduce the total transportation costs. Another area of growing interest is city logistics (Thompson and Taniguchi, 1999), alternatively urban freight transportation, where the main objectives, besides efficiency of the transportation system, are to reduce congestion, pollution and noise.

In this paper we study collaborative transportation planning in a specific sector , where shipments and companies are considered as components of an integrated logistics system, which is to be optimized. Examples of such logistics systems are, e.g., collaboration between forest companies (Frisk et al., 2010) and alliances of cargo carriers (Houghtalen, 2007). When the integrated logistics system is optimized, the total transportation cost normally decreases, compared to the sum of the costs of the non-collaborative transportation plans. Then, the question arises of how the overall costs should be allocated among the companies. This is the cost allocation problem. This is a rather crucial question, since it implies incitement to the companies to take part in the collaboration. This question has been treated in different ways in the present literature. In Frisk et al. (2010), a cost allocation method, *the Equal Profit Method* (EPM), is proposed, such that relative cost savings are as equal as possible for all participating companies. Houghtalen et al. (2007) propose models for capacity exchange prices as incentives for collaboration. Agarwal and Ergun (2010) study cost allocation with side payments in maritime transport, namely liner shipping. Dai and Chen (2012) propose three profit allocation mechanisms based on the Shapley value. The profit allocation mechanisms ensure stable allocations and are tested on a carrier collaboration problem with pick and delivery requests and time window constraints. Vanovermeire and Sörensen (2014) argue that companies with large flexibilities should be rewarded, however considering two-partner coalitions, flexibility is not usually rewarded. Thus, they propose an alternative approach.

The size of a collaboration may be crucial, larger collaboration are generally more difficult to manage. Guajardo and Rönnqvist (2015) study the optimal size of a collaboration with respect to this matter. In Frisk et al. (2010) it is assumed that the collaboration of the companies is decided upon a priori, although, it is possible that one company or a coalition of companies can do better, in terms of savings, on their own. In the considered application, however, it shows that it is possible to allocate costs in such a way that no company, or coalition of companies, can do better on their own. Cruijssen et al. (2005) study another approach, where the a priori decision is replaced by a negotiation, where each company is invited to the collaboration by means of a cost savings offer. A logistics service providers proactively select a group of shippers with a strong synergy potential. It is crucial to decide in what sequence the companies are to be invited, since different sequences allow for different savings offers to the companies.

In this paper we consider a third situation, when companies simultaneously are invited to collaborate, but it is not possible to predetermine which of them will accept, and in which order. This situation puts a focus on establishing the best conditions for offering savings to the companies, and, thereby, achieving as large collaboration as possible. This is done by choosing a suitable cost allocation mechanism. A cost allocation mechanism consist of a cost allocation method from cooperative game theory in combination with some alteration, see Section 4.1. We examine and analyze how collaboration can be achieved in a situation where companies of the forest industry sector simultaneously are invited collaborate. The aim is to achieve full collaboration. We assume that a company accepts to participate if the collaboration has the effect of reducing (or at least not increasing) its total transportation cost. We study which effects different cost allocations mechanisms, based on game theoretical concepts, have on the offered savings and, thereby, on the incentive to collaborate. This is done by adapting notions and ideas presented in Cruijssen et al. (2005).

While considering a sequence of companies joining a collaboration, Audy et al. (2012) study the effect on the full collaborations final profit allocation depending on the leading companies. When a new company join the collaboration the marginal profit is allocated, contrary to the case in this paper, where the total cost is reallocated. A result from allocating marginal profit is a diversion in the final allocation, that is, the sum of all profit allocated to each company, much depended on the leading company.

With the aim of generating as good conditions as possible to establish a full collaboration we study the number of monotonic paths (MPs) generated by the use of different cost allocation mechanisms. A MP represents a sequence of companies, whom all agree to collaborate. A more formal definition of MP is presented in Section 4. In order to offer savings to each company, the cost allocation problem is solved each time a new company is considered. We assume that the new company accepts, and are accepted by the others, if no one is allocated a higher cost according to the new cost allocation, compared to the previous cost allocation. Further, we introduce a relaxation of MP, with the aim of increasing the likelihood of a full collaboration in a final collaboration.

We report on the effects of generating paths based on the Shapley value (Shapley, 1953), the Nucleolus (Schmeidler, 1969) and two version of the EPM (Frisk et al., 2010). A complete enumeration of all possible combinations of accepting companies and for all cost allocation mechanisms is applied. For some of the approaches it is possible to only consider cost allocations such that the path is monotonic. However, it may occur that there are no such cost allocations for a specific path, and the interpretation is that the path is not a MP.

We are mainly interested in full collaborations, but we also consider what we define as large (not necessarily full) collaborations. An indication of the size of a final collaboration, given a specific path, is the size of the collaboration at the first point when a company declines to collaborate. We refer to this as the length of an path, see Section 6.1, and such a path is referred to an Incomplete Monotonic Path, IMP. If a path is not an IMP, it is a complete path, equivalent to a full collaboration.

The outline of this paper is the following. In Section 2 we describe some basics from cooperative game theory. In Section 3 we elaborate on different ways to achieve collaboration. We relate to the work of Cruissen et al. (2005). In Section 4 we describe MPs, a mathematical definition is presented as well as the cost allocation mechanisms we study. Section 5 consists of a description of a case study (Frisk et al., 2010) and, for this paper, relevant data from that study. In Section 6 we present our results based on full path enumeration for different cost allocation mechanisms. The number of MPs is presented as well as details on the number of MPs, starting with certain companies. We also report on to what extent the inclusion of certain companies result in an IMP. In Section 7, we discuss the computational results and draw some conclusions.

2. Cost allocation

When allocating the total cost among the companies we make use of cost allocation methods from cooperative game theory. In this context, each company is considered as a player of a *cooperative cost game*, (N, c), where N is the set of players, known as the *grand coalition*, and c is the *characteristic cost function* of the game. The value $c(S)$ expresses the total cost of a collaboration of the players in *coalition* $S \subseteq N$. For simplicity we define $c(j) := c(\{j\})$.

Each *cost allocation method*, that provides us with a cost allocation, can fulfill a number of properties, or fairness criteria. There is, however, no cost allocation method that satisfies all properties listed in the literature. Below we list some of the basic properties. It is assumed that all players have the opportunity to form and collaborate in coalitions.

A cost allocation method fulfills *efficiency* if it allocates the total cost, $c(N)$, among the players $j \in N$, that is $\sum\limits_{j \in N} y_j = c(N)$ where y_j is the cost allocated to player j.

A cost allocation method fulfills *symmetry* if two arbitrary players, i and j, that have the same marginal cost with respect to all coalitions not containing i and j, have equal costs allocated to them; $y_i = y_j$, that is,
if $c(S \cup \{i\}) - c(S) = c(S \cup \{j\}) - c(S), \forall S \subseteq N, i, j \notin S$ then $y_i = y_j$.

A cost allocation method fulfills the *dummy player property* if an arbitrary player, whom is a dummy in the sense that he neither helps nor harms any coalition he may join, receives a cost saving of zero, that is,
if $c(S \cup \{j\}) - c(S) = c(j), \forall S \subseteq N j \notin S$ then $y_j = c(j)$.

A cost allocation method fulfills *additivity* if given two arbitrary characteristic cost functions c_1 and c_2, for each player the allocated cost based on $c_1 + c_2$ must be equal to the sum of the allocated costs based on c_1 and c_2, respectively, that is $y^1 + y^2 = y^{1+2}$ where $y^i = (y^i_1, \dots, y^i_n)$ is the cost allocation based on c_i.

A cost allocation method fulfills *individual rationality* if no player pays more than its stand alone cost, that is
$y_j \leqslant c(j), \forall j \in N$.

A cost allocation method fulfills *group rationality* if no coalition pays more than its characteristic function cost, $\sum\limits_{j \in S} y_j \leqslant c(S), \forall S \subseteq N$.

The core of the game is defined as those cost allocations, y, that satisfy the constraints

$$\sum_{i \in S} y_i \leqslant c(S), \qquad \forall S \subset N. \tag{1a}$$

$$\sum_{i \in N} y_i = c(N), \tag{1b}$$

That is, no single player or coalition of players should together be allocated a cost that is higher than the cost of the individual or of the coalition, if they acted without the others. A cost allocation in the core is said to be stable. The constraints (1a) ensure individual and group rationality, and equation (1b) ensures efficiency.

For each coalition, S, and a given cost allocation, y, we can compute the *excess*, $e(S, y) = c(S) - \sum_{j \in S} y_j$ which expresses the difference between the total cost of a coalition and the sum of the costs allocated to its players. If a cost allocation is not in the core, at least one excess is negative. Observe that the cost savings of player j, $\bar{c}_j$, is equal to his individual cost minus his allocated cost, that is, $\bar{c}_j = e(\{j\}, y) = c(j) - y_j$.

A game is said to be *proper* if $c(S)+c(T) \geqslant c(S \cup T), S \cap T = \emptyset, S, T \subseteq N$, that is, the cost function is sub-additive. In such a game it is always profitable (or at least not unprofitable) to form larger coalitions.

2.1. The Shapley value

A well-known cost allocation method from cooperative game theory is *the Shapley value* (Shapley, 1953), which provides us with a unique solution to the cost allocation problem. The Shapley value for a player, i, is the average marginal cost of the player while considering all paths, assuming the players are added sequentially.

Thus, the Shapley value allocates to player j the value

$$y_j = \sum_{S \subseteq N} \frac{(|S|-1)!(|N|-|S|)!}{N!} [c(S) - c(S \backslash \{j\}]$$

where $|S|$ denotes the number of players in the coalition S. The summation in this formula is the summation over all coalitions S. The quantity $c(S) - c(S\backslash\{j\})$ is the amount by which the cost of coalition $S\backslash\{j\}$ increases when player j joins it, here denoted by the marginal cost of player j with respect to the coalition $S\backslash\{j\}$.

The Shapley value is based on three axioms (properties) formulated by Shapley (1953). These properties express that the Shapley value satisfies the properties of efficiency, symmetry and *law of aggregation* (additivity). For an exact formulation of these properties we refer to Shapley (1953).

The Shapley value provides us with a cost allocation that is unique, however there is no general guarantee that it is stable, that is, it does not necessarily fulfill group rationality. The Shapley value fulfills the dummy player property.

2.2. The Equal Profit Method

In Frisk et al. (2010) the EPM was suggested, as a cost allocation method for computing a cost allocation for a negotiation situation. The EPM provides us with a stable allocation, such that the maximum difference in pairwise relative savings, $|\frac{c(i)-y_i}{c(i)} - \frac{c(j)-y_j}{c(j)}| = |\frac{y_i}{c(i)} - \frac{y_j}{c(j)}|$, is minimized, where $\frac{c(i)-y_i}{c(i)}$ is the relative savings of player i.

To find such an allocation we need to solve the linear programming problem

$$
\begin{aligned}
&[P_{EPM}] && \min f_0, && && \text{(2a)}\\
&s.t. && \frac{y_i}{c(i)} - \frac{y_j}{c(j)} \leqslant f_0, && \forall (i,j) \in N \times N, && \text{(2b)}\\
& && \sum_{i \in S} y_i \leqslant c(S), && \forall S \subset N, && \text{(2c)}\\
& && \sum_{i \in N} y_i = c(N). && && \text{(2d)}
\end{aligned}
$$

The first constraint set, (2b), measures the pairwise difference between all players relative savings. The variable f_0 is used in the objective to minimize the largest difference. The two other constraint sets, (2c) and (2d), define all stable allocations, that is, the core. If the core is empty, then there are no cost allocation according to the EPM.

2.3. The Nucleolus

To define *the Nucleolus* we need the following. Let x and z denote two arbitrary vectors in $\mathbb{R}^n$ and let $\hat{x}$ and $\hat{z}$ denote sorted lists of each vector respectively. If there exist a $k \in N$ such that $\hat{x}_i = \hat{z}_i, \forall i < k$ and $\hat{x}_k > \hat{z}_k$, then we say that $\hat{x}$ is *lexicographically greater* than $\hat{z}$.

Let the associated excess vector of the cost allocation y be a non-decreasing list of excess values. *The Nucleolus* is the cost allocation y that has the lexicographically greatest associated excess vector and such that individual rationality holds.

Since excess can be seen as a measure of profit for a coalition we aim at maximizing the minimum profit of any coalition. For a more formal definition of this concept, we refer to Schmeidler (1969). The Nucleolus satisfies both the symmetry property and the dummy player property. If the core is non-empty, the Nucleolus is in the core, that is, it represents a stable cost allocation.

Procedures to calculate the Nucleolus are presented by Kopelowitz (1967) and Dragan (1981). The procedure we use solves a sequence of optimization problems. In each iteration some inequalities are changed to equality constraints, based on the values of the dual variables, corresponding to the constraints, in an optimal solution. The k:th optimization problem in the sequence is formulated as P^k_{NUC}:

$$[P^k_{NUC}] \qquad \max w_k \tag{3a}$$

$$s.t. \qquad y_i \leqslant c(i), \qquad \forall i \in N \tag{3b}$$

$$w_k + \sum_{i \in S} y_i \leqslant c(S) \qquad \forall S \in \mathcal{P}(N) \setminus \Pi_k \tag{3c}$$

$$w^*_{\kappa(S)-1} + \sum_{i \in S} y_i = c(S) \qquad \forall S \in \Pi_k \tag{3d}$$

$$\sum_{i \in N} y_i = c(N) \tag{3e}$$

Where $\mathcal{P}(N) = \{S \subseteq N | S \neq \emptyset\}$, $\Pi_0 = \emptyset$, $\Pi_\lambda = \Pi_{\lambda-1} \cup \{S \in \mathcal{P}(N) \setminus \Pi_{\lambda-1} | \pi^*_{\lambda-1}(S) > 0\}$, $\forall \lambda > 0$, $\kappa(S) = \min_{S \in \Pi_k} k$. Further, $\pi^*_\lambda(S)$ is an optimal value of the dual variable corresponding to the constraint S in (3c) and w^*_λ is the optimal objective function value of problem P^λ_{NUC}. The problems P^k_{NUC} is solved for $k := k + 1$ until a unique solution is found.

3. Collaboration

There are several ways to form collaborations for transportation activities; shippers may outsource their transportation activities to a Logistics Service Provider (LSP) of their choice. Cruijssen et al. (2005) suggests a reverse process, insinking, where the LSP invite the shippers instead. In this way the LSP can be more efficient by proactively select shippers with similar distribution networks, that is, shippers with strong synergy potential. The selected shippers are offered cost savings. It is further suggested that the selection of shippers is a sequential process where the next shipper is selected based on the current collaboration and game theoretical concepts, in such a way all previous cost saving offers remain or improve. In this sense the cost saving offers are monotone and this sequence or selection path is said to be a MP, see Section 4 for a mathematical definition.

In this paper, we use the concepts of insinking and MP, but instead of a sequential invitation we study a simultaneous invitation. The idea is that companies are invited to sign up for the collaboration, and when they do, they are presented with a cost saving that may be further improved each time a new company join the collaboration. Observe that this implies that the simultaneous invitation leads to a sequential order of joining companies, that is, they are not signing up all at once. When collaboration in transportation planning is achieved by simultaneous invitation, an underlying aim is to achieve collaborations that are, from a system perspective, good and large collaborations usually are. If all companies accept we denote this by a full collaboration.

Each time a new company (*player*), j, is offered a cost saving, $\bar{c}_j$, the player accepts to participate if the cost saving has a non-negative value. And the player is accepted by the other players if their new allocated cost is less than or equal to the prior allocation. However, since we do not know which players will finally participate, it is not possible to, during this process, compute a cost allocation, y, based on full collaboration. Instead, a cost allocation is computed, based on a collaboration of all players already collaborating (*committed players*), together with player j. We refer a cost allocation based on a full collaboration as a baseline cost allocation, one for each cost allocation method.

4. Monotonic Path

Although the invitation is simultaneous, the players receive cost saving offers sequentially. They receive a cost saving when they announce their transportation activities. We call a sequence of those players, who receive a cost saving offer, a *path*. Each time a new player is offered a cost saving we refer to this as a new step in the path. Let $y_{i,k}$ denote the cost allocated to player i in step k and N_k the set of committed players including the player currently offered a cost saving. Let step 0 represent an initial state, where $N_0 = \emptyset$ and $y_{i,0} := c(i), \forall i \in N$.

A path is said to be a MP if $y_{i,k-1} \geqslant y_{i,k}, \forall i \in N, k \in \{1, 2, \dots, n\}$ where $y_{i,k-1}$ is the cost allocated to player i in the previous step and $y_{i,k} := y_{i,k-1}, \forall i \in N \backslash N_k$. When at least one of the inequalities is violated the path is an IMP.

4.1. Cost allocation mechanisms

In order to increase the likelihood for large collaborations, e.g. increase the number of MPs, we have added two modifications of the straight forward implementation of the enumeration of MPs. The first one prevents cost allocations that would result in an IMP. However, it may occur that all cost allocations are excluded. In that case the path is IMP by default. For those cost allocation methods that are solved as an optimization problem, we prevent undesirable cost allocations by using additional (side) constraints, see Section 4.1.2. For the second modification the monotonicity of an MP is relaxed, instead of require a constant improvement we allow regression to some extent. We call the relaxed version of MP, semi monotonic path (SMP), see Section 4.1.1.

Further, when the cost allocation problem is solved by the EPM, we face the problem of non-uniqueness of the solution. We resolve this problem with two different approaches. One is by using lexicography, and we call this the Lexicographic Equal Profit Method (EPML), see Section 4.1.3. The second approach is by using an upper bound on the allocation, and we call this the Bounded Equal Profit Method (EPMB), see Section 4.1.4. The EPMB does not fulfill efficiency. In order to fulfill efficiency, we use the EPML for the last step, the step corresponding to a full collaboration.

4.1.1. Semi Monotonic Paths

When simultaneous invitation is applied, it could be sufficient to inform players about their savings twice; once when they initially join the collaboration and once when the final collaboration is achieved. Therefore, we consider SMPs; each time a new player joins, all players must have an improved (or equal) cost saving compared to the cost saving obtained when they initially join the collaboration, instead of requiring a continuous improvement (or equal). Let k^i denote the step in which player i was offered a cost saving, and accepted to join the collaboration. A path is said to be a SMP if $y_{i,k^i} \geqslant y_{i,k}, \forall i \in N, k \geqslant k^i$.

4.1.2. Side constrained cost allocation

For the Nucleolus and for the EPM the cost allocation problem can be formulated and solved as optimization problems. Then it is possible to avoid IMPs by modifying the cost allocation method by adding individual constraints to the optimization problem. The constraints are identical to the mathematical definition of MP and SMP, that is $y_{i,k-1} \geqslant y_{i,k}, \forall i \in N$ and $y_{i,k^i} \geqslant y_{i,k}, \forall i \in N$ respectively.

However, it is plausible that the baseline cost allocation is excluded in the process. In that case, the cost allocation deviates from the standard methods and baseline allocations. Therefore we have performed analyses for the different cost allocation mechanisms, concerning how often such exclusions occur and on the magnitude of deviation from the baseline cost allocations. In our case study, for all MPs and SMPs, the cost allocations according to both the EPML and the EPMB converge to the EPM baseline cost allocation. However, this is not the case for the Nucleolus. When applied to the Nucleolus, these side constraints make no paths IMP, that is, all paths are MP or SMP. We refer to Section 6.3 for further reports on the computational results.

Proposition
Given a proper cooperative cost game (N, c), applying the cost allocation mechanisms, where the cost allocation method is the Nucleolus and side constraints for either MP or SMP are used, results in all paths being MP and SMP respectively.

Proof
Let P_{NUC+} denote the iterative process P_{NUC} but with the added side constraints. Assume that player k receives a cost saving in step k. There exists a feasible solution to P_{NUC+} for N_1; $y_{1,1} = c(1)$. Assume there exists a feasible solution to P_{NUC+} for N_k, thus, $\sum_{i \in N_k} y_{i,k} = c(N_k)$ holds. In step $k+1$ the side constraints are $y_{i,k} \geqslant y_{i,k+1}, \forall i \in N_k$ and $c(k+1) \geqslant y_{k+1,k+1}$. Let us set $y_{i,k} := y_{i,k+1}, \forall i \in N_k$.
$c(k+1) \geqslant$[proper game]$\geqslant c(N_{k+1}) - c(N_k) = c(N_{k+1}) - \sum_{i \in N_k} y_{i,k+1} =$[efficiency]$=$ $y_{k+1,k+1}$. That is, we found a feasible solution satisfying all side constraints, efficiency and individual rationality.

Example
The difference between using side constraints and not is illustrated in the following example, including five players. The values of the characteristic function for all coalitions are given in Table 1. We considered a specific path where the players join the collaboration in the order 3, 2, 5, 1 and 4. In Table 2, we show the computed cost allocations according to the Nucleolus, as the players successively join. Each column in Table 2 corresponds to a player and each row to a new step, where a new player joins. The individual costs of the players are given below the players' numbers. The values of Table 2 are the allocated costs to the players in the different steps. A blank value indicates that the player is not currently participating in the collaboration.

Coalition S	$c(S)$	Coalition S	$c(S)$
{1}	208	{1,2,3}	1096
{2}	944	{1,2,4}	1144
{3}	288	{1,2,5}	2056
{4}	496	{1,3,4}	568
{5}	1200	{1,3,5}	1568
{1,2}	1096	{1,4,5}	1744
{1,3}	392	{2,3,4}	1056
{1,4}	576	{2,3,5}	1952
{1,5}	1400	{2,4,5}	2024
{2,3}	992	{3,4,5}	1656
{2,4}	1064	{1,2,3,4}	1160
{2,5}	1904	{1,2,3,5}	2056
{3,4}	488	{1,2,4,5}	2104
{3,5}	1464	{1,3,4,5}	1744
{4,5}	1664	{2,3,4,5}	2016
		{1,2,3,4,5}	2120

Table 1: The characteristic function values of the small example

As seen in Table 2, when cost allocations are computed according to the Nucleolus, the path becomes IMP at step 5, as player 3 is allocated a higher cost compared to the one in the previous step (marked as red). If instead side constraints for MP are added to the Nucleolus computations, the last step results in a cost allocation $y = (156, 587, 118, 179, 1080)$. Thus, the path is MP. This is because in step 5 there is one added constraint stating that

$y_3 \leqslant 118$. This implies a slight increase of costs for player 2 and 4, compared to the costs in Table 2.

player i	1	2	3	4	5
join\\$c(i)$	208	944	288	496	1200
3			288		
2		824	168		
5		704	168		1080
1	156	702	118		1080
4	156	583	126	175	1080

Table 2: The cost allocations, for each step in the path, according to the Nucleolus for a small example.

4.1.3. The EPML

If there is more than one solution to P_{EPM}, at least two players have non-unique feasible allocations with the same optimal objective function value. Since the objective of the EPM is to reduce the difference in relative savings, it is arguably fair in that sense to further reduce the difference in relative savings between these players.

Consider P_{EPM} and a sorted list, $\bar{s}$, of the relative saving differences from the constraint set 2b, in descending order. Then the cost allocation according to the EPML is the solution to P_{EPM} with the lexicographically smallest $\bar{s}$.

Dahlberg et al. (2015) present and discuss the EPML in more detail.

4.1.4. The EPMB

This approach takes advantage of the potential degree of freedom of solutions to P_{EPM}; instead of choosing an arbitrary solution, the cost allocated to each player is set to an upper bound. The cost allocated to player i is calculated as follows:

$$y_{i,k} = \min y_i, \tag{4a}$$

$$s.t. \quad \frac{y_{j'}}{c(j')} - \frac{y_j}{c(j)} \leqslant f^*, \qquad \forall (j', j) \in N_k \times N_k, \tag{4b}$$

$$\sum_{j \in S} y_j \leqslant c(S), \qquad \forall S \subset N_k, \tag{4c}$$

$$\sum_{j \in N} y_j = c(N_k), \tag{4d}$$

where f^* is the optimal objective function value of P_{EPM} for N_k. It is noteworthy to point out that $\sum_{i \in N_k} y_{i,k} \geqslant c(N_k)$. In practice, the cost of a collaboration should be allocated amongst the players, that is, $\sum_{i \in N} y_i = c(N)$. Therefore, in the last step, we use the EPML instead.

5. A case study

In this paper we apply our cost allocation mechanisms on a transportation planning case from the forest industry, as presented in Frisk et al. (2010), where shipments and forest companies are considered as components of an integrated logistics system.

We consider a number of forest companies, who collaborate in order to minimize their total transportation cost. We focus on a tactical problem, which often ranges from one to several weeks and deals with transportation of logs from harvest areas (supply points) to industries such as paper mills, pulp mills, saw mills and heating plants (demand points).

In this paper, the collaboration costs, $c(S)$, are those used in Frisk et al. (2010), and are computed by the use of FlowOpt (Forsberg et al., 2005), developed at Skogforsk.

5.1. Baseline cost allocations

There is a relatively large difference in size between the eight companies of this case study. In the second column of Table 3 we present the individual costs (without a collaboration). In the subsequent columns we show cost allocation results, computed according to the Shapley value, the EPM and the Nucleolus, as presented in Frisk et al. (2010). In the columns we give the cost allocations based on the three cost allocation methods, together with savings as compared to the individual costs. Full collaboration provides an overall saving of 8.3%.

By definition, the cost allocation corresponding to the EPM is stable and the relative savings for the companies are as equal as possible. Further, since the core of the game is non-empty, the cost allocation that represents the Nucleolus is stable. Finally, we observe that also the Shapley value produces a cost allocation that is stable . These three cost allocations represent the baseline cost allocations, for the three cost allocation methods, respectively.

Company	Ind. cost	Shapley	Savings	EPM	Savings	Nucleolus	Savings
Company A	3.778	3.586	5.1	3.523	6.7	3.650	3.4
Company B	14.859	13.528	9.0	13.549	8.8	13.207	11.1
Company C	4.742	4.102	13.5	4.324	8.8	4.081	14.0
Company D	2.067	1.889	8.6	1.884	8.8	1.935	6.4
Company E	10.340	9.747	5.7	9.428	8.8	9.848	4.8
Company F	4.959	4.503	9.2	4.522	8.8	4.546	8.3
Company G	1.884	1.587	15.8	1.718	8.8	1.667	11.5
Company H	0.333	0.310	6.9	0.304	8.8	0.318	4.6
Sum	**42.963**	**39.253**		**39.253**		**39.253**	

Table 3: Individual cost [$MSEK$] and allocation of costs according to the Shapley value, the EPM and the Nucleolus, and the corresponding savings [%], see Frisk et al. (2010).

6. Computational study

In this section we present computational results given by an enumeration of all paths and of the cost allocation mechanisms presented in Section 4.

We apply four types of cost allocation methods (EPML, EPMB, Nucleolus, Shapley value), two types of monotonicity (MP, SMP) and for all cost allocation methods but for the Shapley value we have two cases, whether we add side constraints or not. This means we have fourteen different cost allocation mechanisms. When referring to these cost allocation mechanisms we first specify the cost allocation method, then if SMP or MP is considered. We indicate if we use side constraints with +, e.g. Nucleolus SMP+, indicates that the Nucleolus with added side constraints is used and that SMPs are considered.

The script program Python has been used mainly for the enumerations of all paths and for all cost allocation mechanisms, as well as for calculating the Shapley value and save the results. For all optimization problems, the optimization solver Gurobi has been used, which has a Python API.

6.1. Path length

Here, we present the number of paths of a certain length, or at what step the paths become IMP. The length of a path determine the first time a player decline their offer (or being rejected by the committed players) and indicate the success of establishing large collaborations. With an underlying aim to establish large collaborations, it is relevant to see how well the different cost allocation mechanisms perform in that regard.

We define the length of a path as the number of steps the path is monotonic, e.g. the length of each MP and SMP in this case study is eight. If a path becomes IMP at step k, the length is $k-1$. In Table 4 we report on the number of paths with certain lengths for each cost allocation mechanisms. For eight players there are a total of 40320 possible paths. The values in columns 3-5 are the number of paths and column 6 indicate the average length of the paths. No paths are of length 1.

Cost allocation mechanism		Length 2-6	7	8	average
EPML	MP	16920	12336	11064	6.68
	MP+	15936	12408	11976	6.73
	SMP	4590	3222	32508	7.65
	SMP+	3702	3222	33396	7.7
EPMB	MP	14904	12600	12816	6.79
	MP+	14424	12480	13416	6.81
	SMP	3738	3222	33360	7.69
	SMP+	3474	3222	33624	7.71
Nucleolus	MP	40320	0	0	3.31
	MP+	0	0	40320	8.00
	SMP	32047	4250	4023	4.73
	SMP+	0	0	40320	8.00
Shapley value	MP	39540	728	52	3.93
	SMP	23599	4759	11962	5.74

Table 4: Number of paths of certain lengths and the average length, for each cost allocation mechanism.

When SMP is considered, the number of complete paths is naturally larger than when MP is considered, and in this case study quite much larger. However, the increase in average length is about 1.

There is a slight increase of complete paths for EPML SMP+, EPML MP+, EPMB SMP+, EPMB MP+ compared to EPML SMP, EPML MP, EPMB SMP, EPMB MP respectively. However, for Nucleolus MP+ and Nucleolus SMP+ the amount of complete paths are 100%, compared to 0% and 7.6% for Nucleolus MP and Nucleolus SMP respectively. That is, we can ensure that all paths are MPs, see the proof in Section 4.1.

With the underlying aim of achieving a large collaboration, Nucleolus MP+ and SMP+ seem promising. But, as mentioned in Section 4.1, these two mechanisms do not always converge to the Nucleolus baseline allocation.

6.2. The Nucleolus side constraint deviation

Here, we present the magnitude of the deviation from the Nucleolus baseline allocation when using side constraints. Since all other cost allocation mechanisms converge to their baseline allocations there is no deviation and therefore moot to include them. The deviation from baseline allocation creates an uncertainty regarding fairness (from a cooperative game theory point of view). But the concern may be reduced if the deviation is relatively small and the overall consequences for using side constraints are positive.

In Table 5 we report, for each player and for both cases of monotonicity, SMP and MP, on the smallest and largest cost (in percentages) relative to the cost associated with the Nucleolus baseline allocation. For Nucleolus MP+, zero paths converge to the baseline and for Nucleolus SMP+, 8813 paths do converge. However, all allocations are stable, that is, they are in the core.

Cost allocation mechanism:		Nucleolus MP+		Nucleolus SMP+	
Company	Baseline cost	Min cost	Max cost	Min cost	Max cost
Company A	3.650	−1.1%	0.0%	−1.1%	0.0%
Company B	13.207	0.0%	0.6%	0.0%	1.0%
Company C	4.081	−0.9%	0.9%	−0.9%	0.9%
Company D	1.935	−0.4%	0.1%	−0.4%	0.1%
Company E	9.848	−0.4%	0.0%	−0.4%	0.0%
Company F	4.546	−0.3%	0.4%	−0.3%	0.4%
Company G	1.667	−4.4%	0.0%	−4.4%	0.0%
Company H	0.318	−3.0%	0.0%	−3.0%	0.0%

Table 5: Comparing the difference between the baseline Nucleolus cost allocation and the cost allocation according to the Nucleolus with added side constraints.

Out of all eight players, company B is the only one that never got a lower cost for any path. This might be explained by it being the largest company and having strong synergy with the others. Those coalitions including company B gain quite large reduced costs in terms of absolute (not relative) values. These coalitions excess values are therefore relatively large. Their excess values are in the end of the associated excess vector and are therefore seldom in risk of being cut by the side constraints. Similar (but opposite) can be argued for the small companies. It is also evident that medium sized companies have both reduced cost for some paths and increased costs for other paths, except for company E. A possible explanation is that the relative cost saving of company E, according to the Nucleolus baseline cost allocation, is small. It is likely that the cost allocated to company E, according to the Nucleolus, in certain coalitions are less than the baseline cost allocation.

At least for this cooperative game, the deviations are small. In the worst case scenario, one player receives a 1% increase in costs compared to the baseline, and since the baseline cost is 8.8% less than the individual cost, the player still save quite a lot.

6.3. Leading player

Here, we have separated the number of complete paths depending on the *leading player*, that is, the first player in the path. Audy et al. (2012) study collaborative planning in logistics operations. Their case study is based on the same data as the case in this paper. One of their research questions is about the formation of collaborative groups. The question is answered by studying leading coalition (or player) and different profit allocation methods. Contrary to what we do, they allocate marginal profit/cost whenever a new player joins the collaboration. In this manner, the final profit allocation depends on the path. Although the different approaches, it is interesting to highlight similarities and differences in our result analysis.

In Table 6 we report more details about the complete paths for 11 cost allocation mechanisms, depending on the leading player, e.g. 816 paths (top left entry), or 7%, out of 11064 started with company A for EPML MP.

Since no paths for the cost allocation mechanism Nucleolus MP are complete and 100% of the paths for Nucleolus MP+ and Nucleolus SMP+, those values do not add anything to the discussion and are therefore excluded.

Audy et al. (2012) note that if company A is not a leading player, they wont end up in the final collaboration. The allocation models, suggested by Audy et al., allocate profit proportional to the size (monetary value) which indicate a stronger correlation with the EPM than the Nucleolus and the Shapley value.

Company A is a better leading player for cost allocation mechanisms EPML SMP+, EPML SMP, EPMB SMP+, EPMB SMP rather than EPML MP+, EPML MP, EPMB MP+, EPMB MP, while comparing percentages. In general company B is a little below average as a leading player (around 10%), but when Nucleolus MP or Shapley value MP are considered, no paths starting with company B is complete and only a few are complete when Shapley value SMP is considered.

Cost allocation mechanism		# of paths complete paths starting with company: A	B	C	D	E	F	G	H	Total (A-H)
EPML	MP	816	1188	1440	1608	1776	1260	1518	1458	11064
		7%	11%	13%	15%	16%	11%	14%	13%	100%
	MP+	882	1188	1572	1734	1848	1452	1644	1656	11976
		7%	10%	13%	14%	15%	12%	14%	14%	100%
	SMP	4890	3736	3608	4054	4206	3868	3964	4182	32508
		15%	11%	11%	12%	13%	12%	12%	13%	100%
	SMP+	4974	3736	3800	4174	4290	4012	4156	4254	33396
		15%	11%	11%	12%	13%	12%	12%	13%	100%
EPMB	MP	1014	1188	1704	1866	1998	1452	1776	1818	12816
		8%	9%	13%	15%	16%	11%	14%	14%	100%
	MP+	1110	1188	1800	1962	2118	1548	1872	1818	13416
		8%	9%	13%	15%	16%	12%	14%	14%	100%
	SMP	4986	3736	3788	4162	4320	3970	4144	4254	33360
		15%	11%	11%	12%	13%	12%	12%	13%	100%
	SMP+	5016	3736	3848	4222	4332	4012	4204	4254	33624
		15%	11%	11%	13%	13%	12%	13%	13%	100%
Nucleolus	SMP	962	0	237	349	418	404	888	765	4023
		24%	0%	6%	9%	10%	10%	22%	19%	100%
Shapley value	MP	13	0	0	0	0	8	16	15	52
		25%	0%	0%	0%	0%	15%	31%	29%	100%
	SMP	2358	164	1257	928	1047	1748	2172	2288	11962
		20%	1%	11%	8%	9%	15%	18%	19%	100%

Table 6: Number of complete paths starting with player i for 11 cost allocation mechanisms

6.4. Terminators

Here, we present how often each player declined their cost saving offer (or being rejected by the committed players). That is, how many paths that are IMP due to player i. It is a complement to studying the leading player in Section 6.3. It is arguably interesting to study both constructive behaviour and destructive behaviour.

In Table 7 we report on the number of paths that become IMP when a player i join in step k, the path is terminated by player i. Observe that since Nucleolus SMP+ and Nucleolus MP+ have 100% complete paths, there are no such players. Hence no info is given for those cost allocation mechanisms. The first column gives the step in which the path becomes IMP. In the appendix there are accumulated bar graphs for Table 7 as a visual aid.

Cost alloc. mech.	(join in) step k	# of paths becoming IMP in step k, for company: A	B	C	D	E	F	G	H	Total (A-H)
EPML MP	5^{th}	0	0	0	144	432	144	0	576	1296
	6^{th}	384	0	0	624	1584	384	432	1056	4464
	7^{th}	1752	0	720	1512	2832	672	912	2760	11160
	8^{th}	0	0	4992	2016	2808	0	2520	0	12336
	Total	2136	0	5712	4296	7656	1200	3864	4392	29256
EPML MP+	5^{th}	0	0	0	144	432	144	0	576	1296
	6^{th}	240	0	0	624	1440	384	432	1056	4176
	7^{th}	1392	0	720	1512	2496	672	912	2760	10464
	8^{th}	0	0	4992	2016	2880	0	2520	0	12408
	Total	1632	0	5712	4296	7248	1200	3864	4392	28344
EPML SMP	5^{th}	0	0	0	36	108	36	0	144	324
	6^{th}	108	0	0	168	420	96	108	336	1236
	7^{th}	384	0	120	420	834	168	240	864	3030
	8^{th}	0	0	840	600	1062	0	720	0	3222
	Total	492	0	960	1224	2424	300	1068	1344	7812
EPML SMP+	5^{th}	0	0	0	36	108	36	0	144	324
	6^{th}	60	0	0	168	372	96	108	144	948
	7^{th}	264	0	120	420	714	168	240	504	2430
	8^{th}	0	0	840	600	1062	0	720	0	3222
	Total	324	0	960	1224	2256	300	1068	792	6924
EPMB MP	5^{th}	0	0	0	144	432	144	0	576	2196
	6^{th}	240	0	0	624	1440	96	432	816	3648
	7^{th}	1392	0	720	1512	2616	48	912	2760	9960
	8^{th}	0	0	4992	2016	3072	0	2520	0	12600
	Total	1632	0	5712	4296	7560	288	3864	4152	27504
EPMB MP+	5^{th}	0	0	0	144	432	144	0	576	1296
	6^{th}	240	0	0	624	1440	96	432	1056	3888
	7^{th}	1392	0	720	1512	2496	48	912	2160	9240
	8^{th}	0	0	4992	2016	2952	0	2520	0	12480
	Total	1632	0	5712	4296	7320	288	3864	3792	26904
EPMB SMP	5^{th}	0	0	0	36	108	36	0	144	324
	6^{th}	60	0	0	168	372	24	108	240	972
	7^{th}	264	0	120	420	762	12	240	624	2442
	8^{th}	0	0	840	600	1062	0	720	0	3222
	Total	324	0	960	1224	2304	72	1068	1008	6960

EPMB SMP+	5^{th}	0	0	0	36	108	36	0	144	324
	6^{th}	60	0	0	168	372	24	108	144	876
	7^{th}	264	0	120	420	714	12	240	504	2274
	8^{th}	0	0	840	600	1062	0	720	0	3222
	Total	324	0	960	1224	2256	72	1068	792	6696
Nucleolus MP	3^{rd}	1920	240	480	960	960	1680	1200	1680	9120
	4^{th}	3312	528	1728	1152	1200	2256	2112	2448	14736
	5^{th}	2376	744	1596	1284	1056	1848	1392	1452	11748
	6^{th}	780	324	632	472	336	740	392	492	4168
	7^{th}	70	40	130	32	32	100	50	94	548
	8^{th}	0	0	0	0	0	0	0	0	0
	Total	8458	1876	4566	3900	3584	6624	5146	6166	40320
Nucleolus SMP	3^{rd}	960	120	240	480	480	840	600	840	4560
	4^{th}	1704	264	840	528	600	1224	960	1224	7344
	5^{th}	1770	480	1008	714	720	1428	936	1062	8118
	6^{th}	1484	598	732	632	712	1294	532	860	6844
	7^{th}	1146	636	570	395	543	918	378	595	5181
	8^{th}	823	658	659	398	516	616	209	371	4250
	Total	7887	2756	4049	3147	3571	6320	3615	4952	36297
Shapley value MP	3^{rd}	1200	0	480	0	240	720	960	1200	4800
	4^{th}	2640	0	1152	0	480	1440	2160	2448	10320
	5^{th}	3120	144	1608	0	516	2052	2568	2940	12948
	6^{th}	1868	240	1136	0	500	1296	1512	1828	8380
	7^{th}	562	236	434	0	316	454	522	568	3092
	8^{th}	82	134	110	0	120	98	160	24	728
	Total	9472	754	4920	0	2172	6060	7882	9008	40268
Shapley value SMP	3^{rd}	600	0	240	0	120	360	480	600	2400
	4^{th}	1056	0	504	0	192	648	864	1032	4296
	5^{th}	1422	36	666	0	216	954	942	1272	5508
	6^{th}	1584	48	762	0	280	1076	802	1286	5838
	7^{th}	1491	156	727	0	271	1027	808	1077	5557
	8^{th}	1163	320	693	0	218	829	963	573	4759
	Total	7316	560	3592	0	1297	4894	4859	5840	28358

Table 7: Number of paths player i ended when joining as the j:th player

When the EPML or the EPMB is applied for cost allocation, company A and company F terminate relatively (compared to other companies, except company B) few paths and when the Nucleolus or the Shapley value is applied for cost allocation, company A and company F terminate relatively many paths. Company E has an opposite behaviour.

When EPML MP, EPML MP+, EPMB MP or EPMB MP+ is considered, company C terminates relatively (compared to other companies) more paths than when EPML SMP, EPML SMP+, EPMB SMP or EPMB SMP+ is considered.

When the EPML or the EPMB is the cost allocation method, company A, company F and company H terminates paths relatively early, that is, in step $5-7$ and company C terminates paths later, that is, step $7-8$. Most paths that are terminated by company D, company E or company G are terminated in the later steps, that is, step $7-8$, similar to company C.

Overall it is evident that the choice of a cost allocation mechanism affects the results for individual players. Different players are favoured by different mechanisms.

7. Conclusions and future research

In this paper we examine and analyze how collaboration can be achieved in a situation where companies of the forest industry sector (Frisk et al., 2010) are invited to register for collaboration. The collaboration is established by companies joining sequentially. The sequence is represented by a path in a network. We study which effects different cost allocations mechanisms, based on game theoretical concepts, have on the offered savings and, thereby, on the possible size of the collaboration. The length of a path is an indication of the possible size of the collaboration and a full collaboration is equivalent to a complete path.

It is obvious that the number of complete SMPs is larger (or equal) than the number of complete MPs. Our computational results show indeed that this difference is very large. If the EPML or the EPMB is applied, roughly 30% of the MPs are complete whereas, roughly 80% of all SMPs are complete. This implies that, when simultaneous invitation is applied based on SMPs, there are good possibilities for obtaining large collaborations. Further, when side constraints are added to the EPM there is a slight increase in the number of complete MPs/SMPs. For the Nucleolus without added side constraints, all (with few exceptions for SMPs) paths are IMPs, however using the added constraints all paths are complete. The fact that the Nucleolus with added side constraints results in all paths to be complete is not exclusive for our case. A proof that it holds for a general case is provided. However, when the Nucleolus with added side constraints is applied, the cost allocation seldom converge to the baseline cost allocation. The deviations between those cost allocations and the baseline cost allocation are small, around $\pm 1\%$, for our case.

Further research could involve different rules for the process of joining companies. In this paper we end a path when a company reject their cost allocation offer (or being rejected by any of the committed companies). It is reasonable to assume that some other company could join the collaboration still. It is also reasonable to assume that a company who reject their cost allocation offer might reconsider at a later step of the path, if said company could register more then once and thus are offered a different cost saving.

Another possible continuation could be to apply these ideas to other applications and thus check the generality of the results. There are no methodological assumptions that are case specific. However, the results depends on the characteristic function values which depends on the collaboration's potential for cost savings. Collaborations in some applications might be more beneficial then for other applications.

Acknowledgements

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Appendix

The following plots are representing Table 7. Each figure corresponding to one cost allocation mechanism.

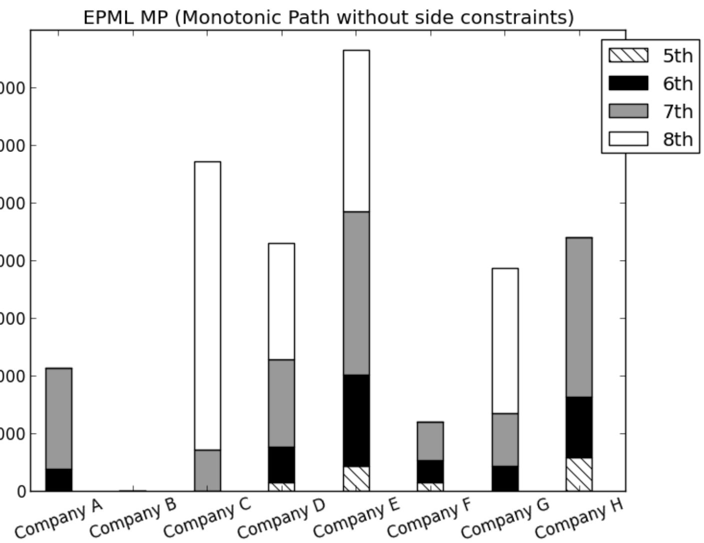

Figure 1: A bar graph representation of Table 7. The cost allocation mechanism is EPML MP

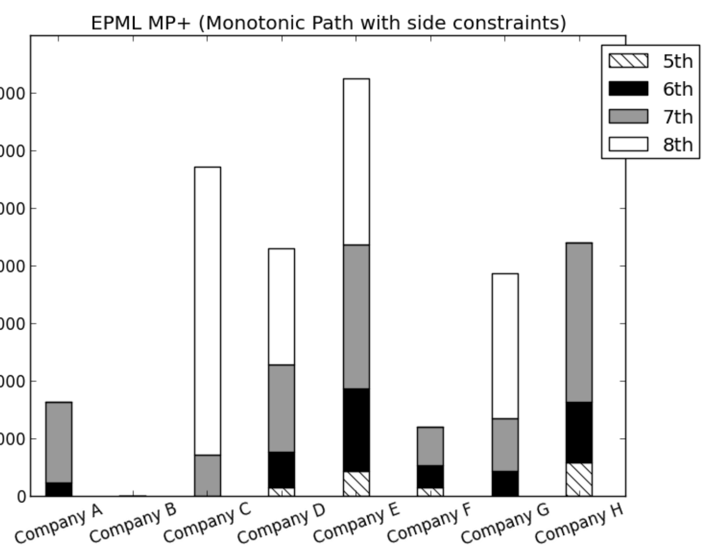

Figure 2: A bar graph representation of Table 7. The cost allocation mechanism is EPML MP+

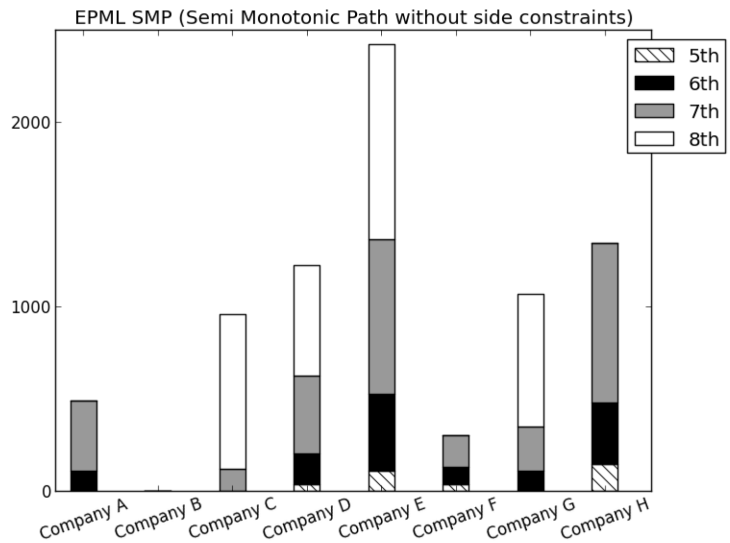

Figure 3: A bar graph representation of Table 7. The cost allocation mechanism is EPML SMP

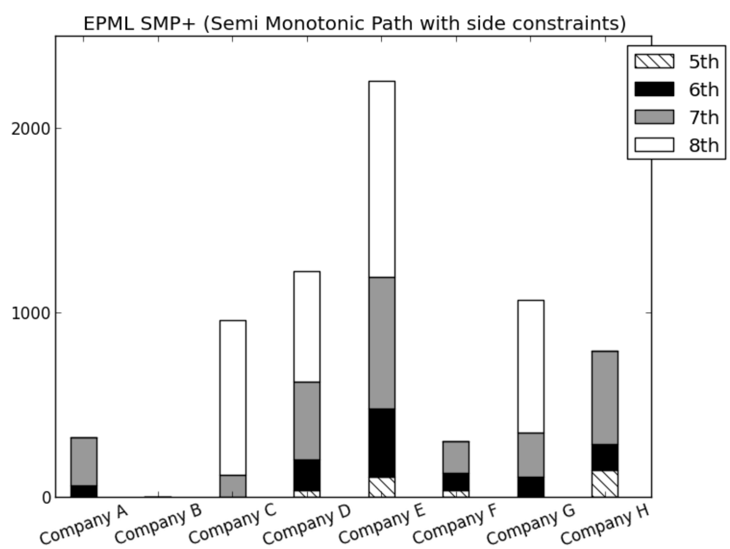

Figure 4: A bar graph representation of Table 7. The cost allocation mechanism is EPML SMP+

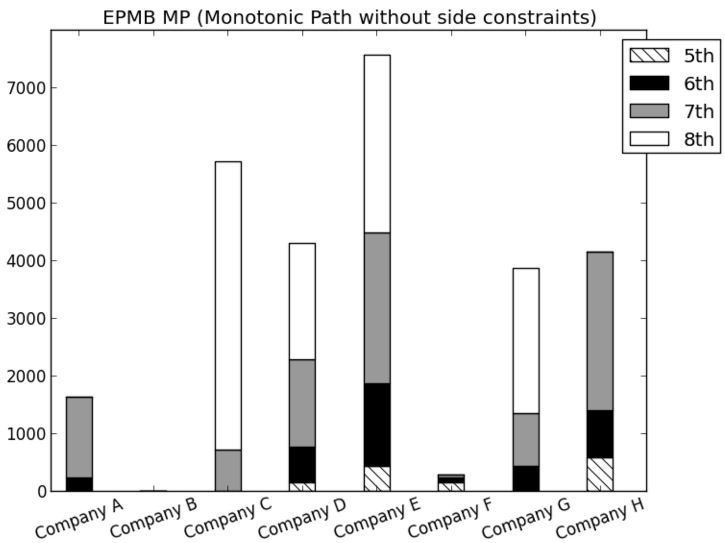

Figure 5: A bar graph representation of Table 7. The cost allocation mechanism is EPMB MP

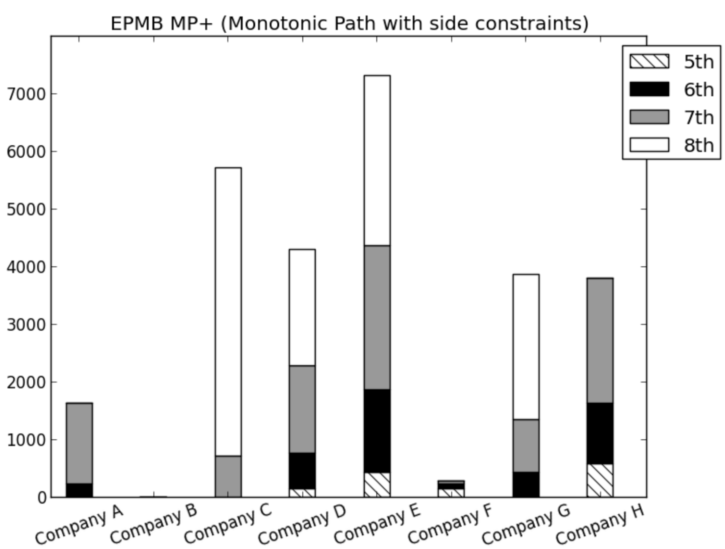

Figure 6: A bar graph representation of Table 7. The cost allocation mechanism is EPMB MP+

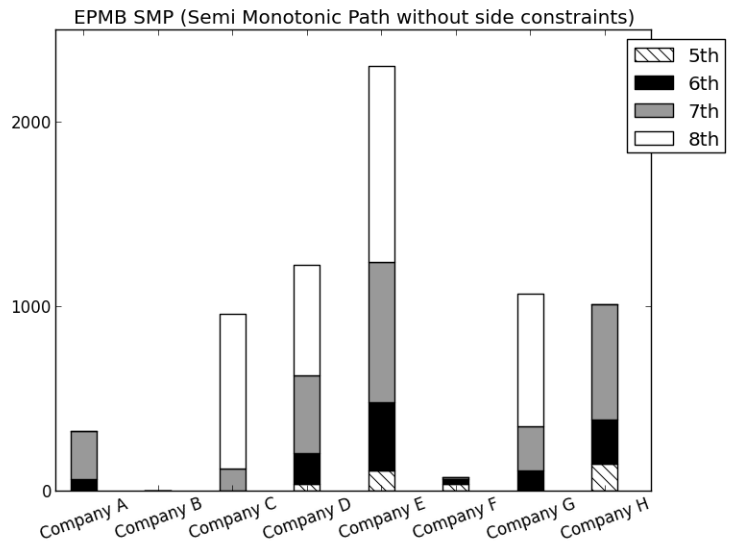

Figure 7: A bar graph representation of Table 7. The cost allocation mechanism is EPMB SMP

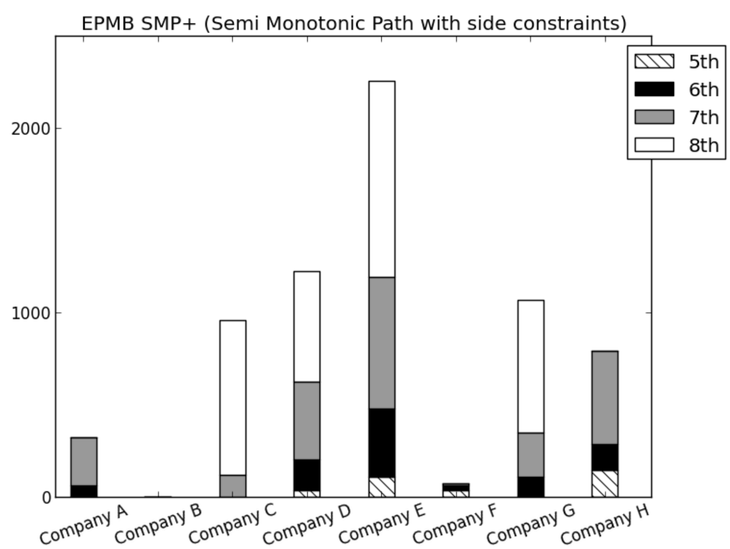

Figure 8: A bar graph representation of Table 7. The cost allocation mechanism is EPMB SMP+

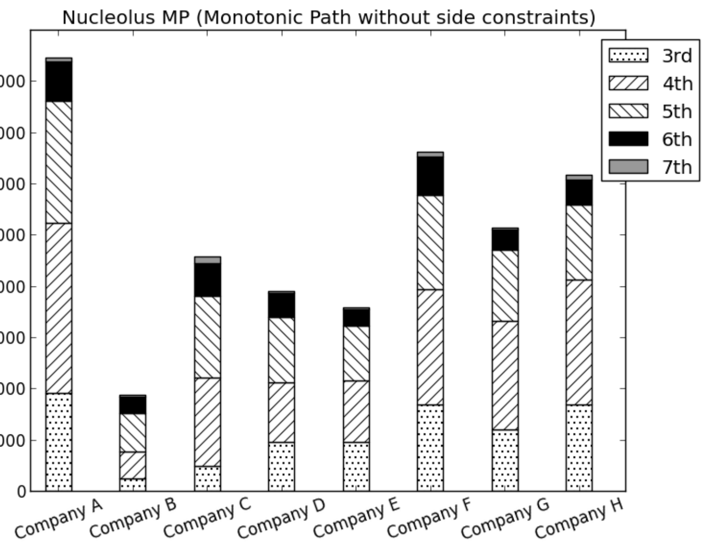

Figure 9: A bar graph representation of Table 7. The cost allocation mechanism is Nucleolus MP

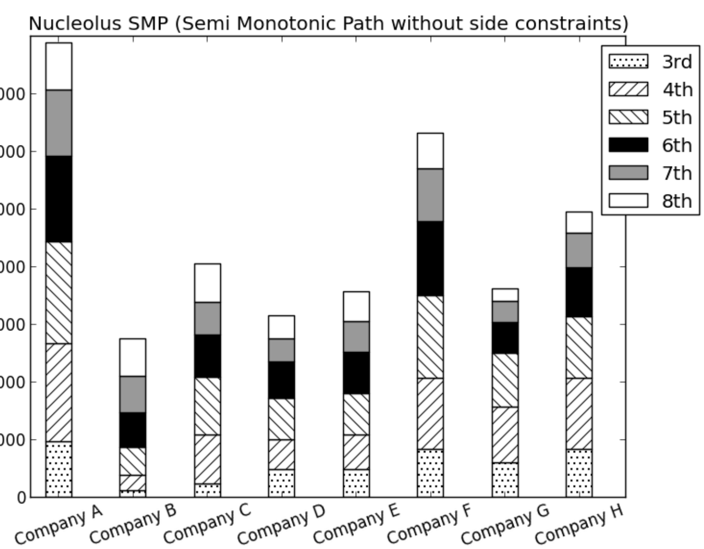

Figure 10: A bar graph representation of Table 7. The cost allocation mechanism is Nucleolus SMP

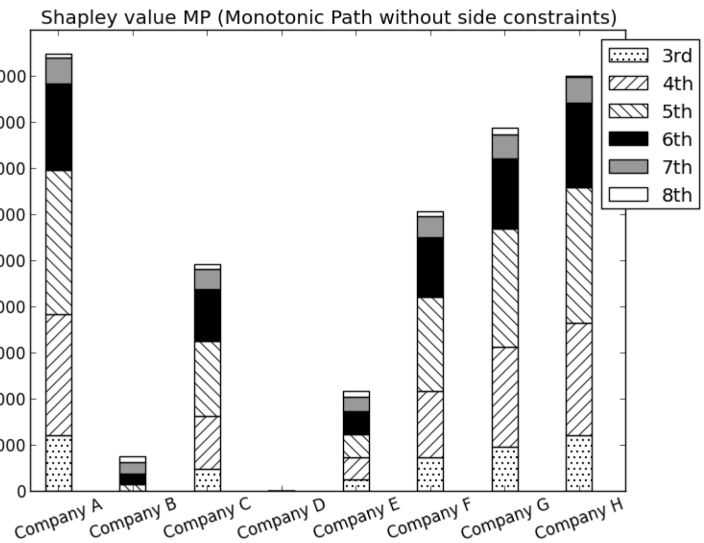

Figure 11: A bar graph representation of Table 7. The cost allocation mechanism is Shapley value MP

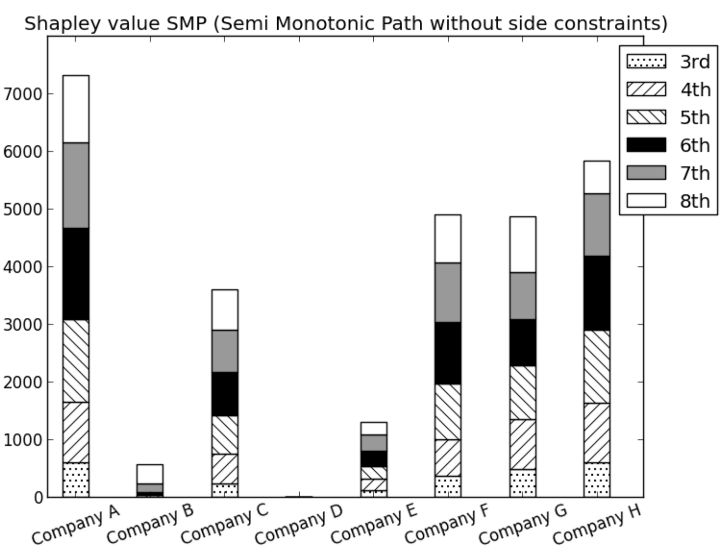

Figure 12: A bar graph representation of Table 7. The cost allocation mechanism is Shapley value SMP