A risk averse approach to the optimal capacity problem in the airline cargo industry

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Abstract

The low margins in the airline industry have made them focus in the air cargo area to try to increase their profits. One of the decisions that the airlines face is determining the proportion of the capacity of a flight intended for long-term contracts (known as allotment) and for bookings that are more urgent and come closer to the departure of the flight (known as free). In general, allotment tariffs are lower than free market ones. This suggests that, if possible, one should favour the sale in the free market rather than the allotment contracts. However, demand in the free market has more uncertainty and usually does not arrive with the promised cargo even if the capacity was booked. In this paper we propose a two stage stochastic programming model to determine the total weight that should be assigned to allotment and free reservations, considering as random elements the demand, the tariff, and the show-up rate for the free mode. We also consider risk aversion in the objective function using the CVaR, departing from the classical expected value case. We show that more stable solutions, with low probability of experiencing extreme losses, can be obtained at the expense of reducing the gains. Moreover, we conclude that the use of stochastic programming model provides greater benefits in scenarios with high seasonalities and free demand variability, reaching a 2.28% increase in average income when compared to solving the problem replacing the random parameters by their average values.

Keywords: Air Cargo, Stochastic programming, Conditional Value at Risk.

1 Introduction

The average annual growth of passengers in the airline industry in the last years has been around 5% [3]. Despite this growth, costs such as fuel have increased significantly, which yielded a net profit per passenger transported of only $2.56

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USD in 2012 [4]. Such modest results have forced airlines to search for other sources of revenue, focusing on businesses such as cargo transportation.

One of the main tools that have been used to increase profits in cargo transportation is Revenue Management. However, the implementation of those tools in cargo transportation is still in its infancy given its higher complexity when compared to passenger transportation [17].

One important difference is the possibility that cargo customers can place reservations in two forms: allotment and free. In the allotment mode the customer signs a long term agreement (approximately 6 months long) specifying the weight of the load to be transported, for instance on weekly basis, paying a fixed tariff. The free mode corresponds to the space available in the airplane that was not assigned to allotment contracts, and reservations usually take place closer to the departure date. Free reservations can for example be urgent deliveries, and typically airlines charge more for those types of reservations.

The decision of how much space to assign to each reservation mode is a very difficult one mainly because of *show-up rates*. Show-up rates are defined as the percentage of load that shows-up in the day of the flight to be transported. Unlike passengers, for whom the show-up is a binary event, in the cargo industry the show-up rate has a continuous behavior. The actual load that the customer drops at the airport can be anything between nothing (a complete no-show) until a load greater than what was previously established [5]. Such events happen very often in the free mode, especially because there are no penalties for any of the sides for doing so: it may happen that the airline will not transport 100% of previously agreed load, and it may the case that the customer shows-up a fraction of the load that was negotiated. The system works mainly based on trust, so the incentives for the airline to provide a good service, or for the customer to have a show-up rate of 100%, are mainly dictated by the necessity of being in good terms for future deals.

The impact of uncertainty can be significant to the airline: since free reservations are typically placed a few hours before the flight, the airplane may fly with empty spaces, which in a tight margin industry means losses. Therefore, airlines must aim at maximizing their profits by finding the right balance between the higher profit per kilogram offered by the free reservations, and the safer but less profitable allotment contracts.

There are several papers in the air cargo literature devoted to the determination of optimal overbooking levels. In the works [6] and [24], the authors focus on the prediction of the show-up rate in order to compute the optimal overbooking levels. The publications [12] and [20] solve an optimization problem considering fixed capacity for weight and volume in a single leg flight, while [26] incorporates stochastic capacity, and use fuzzy logic to determine the optimal overbooking level using scenarios to represent uncertainty. Unlike their predecessors, [27] consider multiple leg routes and solve the resulting optimization problem using a newsvendor problem with two locations.

The focus of this work is to efficiently assign load to cargo flights considering three sources of uncertainty: the demand for the free load, the show-up rates, and amount charged for the free load. In the literature some works isolate the free demand problem, and others consider allotment and free jointly. In [1] the authors use Markov decision processes to maximize the expected income of the free reservations, in single-leg flights. The results show a 2.7% increase in the profits in comparison to the first come first booked policy. The work [13] develops a new heuristic using sampling methods and improve the results obtained in [1]. A dynamic programming model is proposed in [14] to solve the problem under uncertain capacity. The goal is to find an optimal capacity control policy that tells when to accept an order.

Their numerical results indicate that the reservations of smaller sizes should be accepted when uncertainty on demand increases.

A very important point that was not considered in previous publications is the management of the risk associated with losses. The expected value is the most common tool to deal with uncertainty, but its use imply that the decision maker is risk neutral. As pointed out by [15], maximizing the expected revenue is not the only concern—it is also necessary to control the impact or the variability of losses. In the context of finance, the seminal work [21] was the first to explicitly incorporate the need to balance risk and return, using the variance as way to control risk. Another risk measure that is widely used in practice is the Value-at-Risk (VaR) ([16], [11]), which is based on the quantile of the loss distribution. Although it is a very intuitive risk measure, the VaR lacks convexity, which complicates significantly its practical use.

The paper [2] proposed the concept of coherent risk measures, which are a set of desirable properties a risk measure should have. The Conditional Value-at-Risk (CVaR), introduced by [25], is a coherent risk measure and therefore is suitable for optimization problems. In this work our objective function consists of a weighted sum of the expected value and the CVaR.

Our contribution is twofold: first we propose a two stage stochastic programming model to determine the total weight that should be assigned to allotment and free reservations, considering as random elements the demand, the tariff, and the show-up rate for the free mode. Second, we depart from the classical expected value case and consider risk aversion in the objective function using the CVaR. We show that more stable solutions, with low probability of experiencing extreme losses, can be obtained at the expense of reducing the gains. We construct realistic experiments, based on real data from a major Chilean cargo company, considering 9 different combinations of demand for the free mode. Each experiment reflects the presence of possible seasonalities experienced by the company, as well as situations with higher than usual variability.

The rest of the paper is organized as follows. In Section 2 we describe the problem we are modelling, discussing the assumptions we make, and a description of the data we have from the industrial partner. In Section 3 we formulate the problem, discuss how it can be solved and present two metrics to evaluate solution quality. Section 4 contains extensive numerical experiments based on real data. Section 5 concludes the paper and presents futures avenues of research.

2 Model description

In this section we will describe the cargo problem we want to model and the practical aspects related to the formulation. In particular, we described the variables of the model, the data provided by the cargo company, and how uncertainty was modelled.

2.1 Allotment and free demand

Our model assumes the cargo company is planning its cargo policy for a fixed time period in the future, say 6 months, and needs to define *before* the period starts how much kilograms will be assigned for contracts, the so called allotment. The remaining capacity can be filled with the free demand, that is, demand that is random and is not attached to any type of contract. The price paid for the allotment is usually lower than that of the free demand, but it offers a safer alternative because the latter exhibits great variability in terms of prices and of show-up rates.

Allotment contracts have a fixed duration, specified at the moment the contract is signed, and they are valid for all flights within the contract's time window. The cargo company has a maximum demand for allotment contracts, which is a known number D_A. In addition, there is a fixed tariff T_A that is charged to the allotment customer, in dollars per kilogram. Finally, there is show-up rate SUR_A, which represents the percentage of the assigned allotment cargo that actually showed up for the flight. The cargo company has to decide how much of the allotment demand it will sell via contracts. In our model we will assume there is only one contract during the time horizon under study. One can think this contract is a proxy for all contracts that were signed for this period.

Those three parameters are assumed to be deterministic while the free demand are specific for each flight i, and are characterized by a random demand D^i_F, a tariff T^i_F and a show-up rate SUR^i_F.

2.2 Uncertainty modelling

The uncertain parameters of the model are all related to the free customers: the demand at each flight, the tariff that will be charged to those customers, and their show-up rate. We assume that the demand, tariff and show up rate of the free customers to be independent. Furthermore, demand and tariff are modelled as a lognormal distributions [23]. The use of a lognormal distribution to model individual booking requests has been widely used in the literature ([9], [18], [23]). Following [10] we assume that the free demand of a flight, which is the sum of the individual booking requests, follows this same distribution with the same first and second moments. Aside from being positive and allowing values that are significantly greater than the mean, the lognormal distribution generates values that resemble the data that we encountered in practice. The same applies to the tariff charged to free customers.

Finally, to model the *show up rate*, we use a discrete distribution similar to the one proposed by [24]. To generate this distribution we create different bins (show up rate ranges) with different probabilities of occurrence to each one. To set the optimal number of bins, we first follow the methodology developed by [8] that uses bins of equal size. However, given the small amount of data, some bins contains only one observation of the data. For this reason, we create 5 bins of different size that better reflect the data behaviour.

2.3 Data description

The test data for this model is based on real data provided by a major cargo airline for a single market between December 2013 and March 2014. The data base include information of each flight, with specific detail of each shipment order: the client name, number of the order, flight date, shipment origin and destination, load reserved and the load flown, tariff and reservation mode. However, the data base does not record the date of each booking and bookings that were not accepted.

Flight capacity is fixed an equal to 100,000 kg representing the capacity of a CAO Boeing 777

Table 1 summarizes the value of the parameters used for the free and allotment contracts

Table 1: Parameters for the free and allotment contracts

Parameter	Value
D_A	51,847 kg.
T_A	2,5$[USD/kg.]$
SUR_A	1
C	100,000 $[kg.]$
V	3
D_F	$\mu = 11.32[kg.]$; $\sigma = 0.365$
T_F	$\mu = 1.525[USD/kg.]$; $\sigma = 0.044$

3 Formulation

In this section we show the explicit optimization formulation of the problem. We also discuss how the Sample Average Approximation will be used in order to solve the problem.

3.1 Risk neutral formulation

The risk neutral formulation aims at maximizing the average income obtained with contracts plus free demand. The problem fits into the framework of two-stage stochastic programming: the airline has to decide how much should be assigned for allotment *before* uncertainty is revealed, and when the values of the random elements are known, the assignment for the free demand must be chosen. We formulate the problem as follows:

$$\begin{aligned} \underset{X_A}{\text{Max}} \quad & T_A X_A \text{SUR}_A + \frac{1}{V}\sum_{i=1}^{V} \mathbb{E}\left[Q(X_A, \xi(\omega_i))\right] \\ \text{s.t.} \quad & 0 \leq X_A \leq D_A. \end{aligned} \tag{1}$$

The constraint means that the capacity X_A destined for allotment contracts must be smaller or equal than the total demand D_A for such contracts. The objective function of (1) has two terms: the first represents the profit obtained through contracts in any flight, which is the tariff T_A multiplied by the assigned capacity times the show-up rate for allotment SUR_A, and the second represents the average return per flight obtained through free contracts. Thus, the objective function value can be interpreted as the average income per flight when using a contract with demand D_A and tariff T_A. Function $Q(X_A, \xi(\omega_i))$ represents the profit obtained with one flight *given* amount reserved for the contract X_A, and realizations of demand, tariff and show-up rate for the free customers for each flight $i(\omega_i)$ The second stage can be explicitly written as the optimal value of a minimization problem:

$$Q(X_A, \xi(\omega_i)) = \underset{X_F^{ij}}{\text{Max}} \quad T_F^{ij} X_F^{ij} \text{SUR}_F^{ij} \tag{2}$$

$$\text{s.t.} \quad 0 \leq X_A \leq D_A, \tag{3}$$

$$X_A \text{SUR}_A + X_F^{ij} \text{SUR}_F^{ij} \leq C, \tag{4}$$

$$0 \leq X_F^{ij} \leq D_F^{ij}, \tag{5}$$

Constraint (4) states that the sum of effective cargo, that is, the assigned cargo multiplied by the show-up rate, for allotment and free customers cannot exceed the airplane's capacity C. Constraint (5) is the analogous of constraint (3) for the free demand.

The random variables are continuous, and the expectation of the function $Q(X_A, \cdot)$ is impossible to compute explicitly. Following [19], we can approximate the problem using the Sample Average Approximation (SAA) method. Each sample j represents a vector of three components: $(D_F^{ij}, \text{SUR}_{ij}, T_F^{ij})$. For N samples the problem becomes

$$\begin{aligned} \underset{X_A, X_F^{ij}}{\text{Max}} \quad T_A X_A \text{SUR}_A + \frac{1}{V}\frac{1}{N}\sum_{i=1}^{V}\sum_{j=1}^{N} T_F^{ij} X_F^{ij} \text{SUR}_F^{ij} \\ \text{s.t.} \quad 0 \leq X_A \leq D_A, \\ X_A \text{SUR}_A + X_F^{ij} \text{SUR}_F^{ij} \leq \text{C}, \quad i = 1, \ldots, V, j = 1, \ldots, N, \\ 0 \leq X_F^{ij} \leq D_F^{ij}, \quad i = 1, \ldots, V, j = 1, \ldots, N. \end{aligned}$$

The expected value model is a completely valid formulation, but as discussed before it does not take into account risk. We now turn our attention to the risk averse model with CVaR.

3.2 Risk averse formulation: the λ-CVaR model

The λ-CVaR model incorporates risk in the objective function, using the weight λ to balance risk and return. For the sake of completeness, let us define the CVaR rigorously. The Value-at-Risk of a random variable X that represents losses at confidence level α is defined as

$$\mathrm{VaR}_\alpha[X] := \min\{t \mid F(t) \geq \alpha\} = \min\{t \mid P(X \leq t) \geq \alpha\}. \tag{6}$$

The Conditional Value-at-Risk (CVaR) of a continuous random variable X with cdf $F(\cdot)$ with risk level $\alpha \in [0,1]$ is defined as the average of losses given that such losses were higher than the VaR:

$$\mathrm{CVaR}_\alpha[X] = \mathbb{E}[X | X > \mathrm{VaR}_\alpha[X]]. \tag{7}$$

Albeit simple, definition (7) is of little use in an optimization context. The key result in [25] is the proof that CVaR can be expressed as the optimal value of the following optimization problem:

$$\mathrm{CVaR}_\alpha[X] = \min_{\eta \in \mathbb{R}} \left\{ \eta + \frac{1}{1-\alpha} \mathbb{E}[(X-\eta)_+] \right\}, \tag{8}$$

where $(a)_+ := \max(a, 0)$. Moreover, they show that $\mathrm{VaR}_\alpha[X]$ is an optimal solution of the optimization problem in (8). The λ-CVaR model is

$$\underset{X_A}{\mathrm{Max}} \quad T_A X_A \mathrm{SUR}_A + \frac{1}{V} \sum_{i=1}^{V} \lambda \mathbb{E}\left[Q(X_A, \xi(\omega_i))\right] + (1-\lambda)\mathrm{CVaR}_\alpha[Q(X_A, \xi(\omega_i))] \tag{9}$$

$$\text{s.t.} \quad 0 \leq X_A \leq D_A.$$

Using expression (8) and defining auxiliary variable u_{ij} to linearize the CVaR, problem (9) can be approximated by

$$\begin{aligned} \underset{X_A, X_F^{ij}}{\mathrm{Max}} \quad & T_A X_A \mathrm{SUR}_A + \frac{1}{V} \sum_{i=1}^{V} \left\{ \lambda \left[\frac{1}{N} \sum_{j=1}^{N} T_F^{ij} X_F^{ij} \mathrm{SUR}_F^{ij} \right] + \right. \\ & \left. (1-\lambda) \left[\theta + \frac{1}{N(1-\alpha)} \sum_{j=1}^{N} u_{ij} \right] \right\} \\ & \text{s.t.} \quad 0 \leq X_A \leq D_A, \\ & u^{ij} \geq 0, \quad i = 1, \ldots, V, j = 1, \ldots, N, \\ & u^{ij} + \theta + T_A X_A \mathrm{SUR}_A + T_F^{ij} X_F^{ij} \mathrm{SUR}_F^{ij} \geq 0, \quad i = 1, \ldots, V, j = 1, \ldots, N, \end{aligned}$$

adding constraints (4),(5) from the expected value problem (2).

3.3 Benchmark policies

In the literature of stochastic programming it is common to use two metrics to obtain additional information about the usefulness of a model. The first metric is the Value of the Stochastic Solution (VSS), which is a measure of the gain in considering randomness in the formulation. If the VSS is zero then replacing the random parameters by their averages yields the same results as the two stage stochastic programming model.

The other metric is the Expected Value of Perfect Information, or EVPI. The EVPI measures how much a decision maker would be willing to pay for perfect information, that is, for accessing the outcomes of the random variables before they are revealed. An EVPI of zero means that the solution of the two stage problem, which is obtained *without* knowing the outcomes of the random variable, is as good as the solution with complete information. More details can be found in [7]. In the next section we will present our numerical results, including the value of both metric for each of our experiments.

4 Numerical experiments

4.1 Experiment design

To test how does the model perform under different demand patterns we create 9 demand categories. Each demand category reflects the presence of possible seasonality and is represented by its mean and standard deviation. Three levels of demand mean and standard deviation are considered: High, Medium and Low. Table 2 summarizes these 9 categories.

Table 2: Free demand categories

	Standard Deviation		
Demand Mean	High (H)	Medium (M)	Low(L)
High (H)	HH	HM	HL
Medium (M)	MH	MM	ML
Low (L)	LH	LM	LL

In the base category MM we assume that the demand mean of $D_{MM} = 88,560$ *kg.* and a standard deviation of $33,503$ *kg.* To generate the High (Low) mean demand level, we took the D_{MM} and amplify(reduce) in a 25%. We did something analogous to create the High(Low) levels of standard deviation by modifying the coefficient of variation in a $\pm 15\%$ in relation to the Medium level.

Based on these demand categories we built 9 experiments to test the two different models presented in section 3. An experiment is composed by 3 different demand category each of them representing a season. Table 3 shows a summary of these experiments.

The results of these experiments are presented in the next section.

Table 3: Description of Free demand experiments

Experiment	Characteristic	Flight 1	Flight 2	Flight 3
1	Base case	MM	MM	MM
2	Variability increase	MH	MH	MH
3	Demand increase	HM	HM	HM
4	Demand decrease	LM	LM	LM
5	Three different seasons	MM	LM	HM
6	One high demand season	MM	HM	MM
7	One low demand season	MM	LM	MM
8	Three seasons with high variability	MH	LH	HH
9	Decrease variability	ML	ML	ML

4.2 Results

As is usually the case with stochastic programming problems, we cannot solve the resulting problem directly since the expected value cannot be computed explicitly. As mentioned in Section 3.1, we will approximate the problem using SAA in order to obtain good candidate solutions.

4.2.1 Risk neutral results

We now present the main results obtained for each of the nine experiments. Using SAA we obtained upper and lower bound for the unknown optimal value v^*. First we ran $M = 100$ experiments with $N = 500$, which generated 100 candidate solutions and 100 objective function values[1]. According to [19], the average of those values is an (statistical) upper bound for v^*, and confidence intervals can be constructed. The best candidate $\hat{x}_A^*$ obtained among the initial 100 runs will be used to estimate a lower bound. By fixing the allotment capacity equals to $\hat{x}_A^*$, we chose a much higher value of $N' > N$ to construct accurate statistical estimates of the objective function value associated with $\hat{x}_A^*$, including confidence intervals.

By constructing lower and upper bounds we have not only a candidate solution that can be implemented in practice, but we also can infer about the quality of such candidate by comparing its objective function value with the upper bound. Results for each of the nine experiments considering a value of $N' = 1 \cdot 10^6$ are shown in Table 4.

Table 5 shows the results compared with our benchmark policies explained in section 3.3. The *EVPI* in this case represents the maximum willingness to pay for having perfect information of the demand that a particular flight will face. According to the results of Table 5 this willingness to pay is on average of 37,690

[1] The values of M and N were chosen for computational tractability. Additional experiments with combinations of M and N are reported in the appendix

Experiment	Best $f(\hat{x}_N^j)$ (95% conf. int.)	$E[\hat{v}_N]$ (95% conf. int.)
1 MM–MM–MM	$353,779 \pm 57$	$354,360 \pm 443$
2 MH–MH–MH	$339,820 \pm 62$	$340,490 \pm 496$
3 HM–HM–HM	$379,334 \pm 75$	$380,491 \pm 608$
4 LM–LM–LM	$328,087 \pm 41$	$328,160 \pm 348$
5 MM–LM–HM	$347,937 \pm 51$	$348,392 \pm 397$
6 MM–HM–MM	$360,925 \pm 63$	$361,395 \pm 530$
7 MM–LM–MM	$343,086 \pm 56$	$343,286 \pm 415$
8 MH–LH–HH	$335,351 \pm 72$	$335,538 \pm 494$
9 ML–ML–ML	$368,109 \pm 53$	$368,773 \pm 408$

Table 4: Lower and Upper Bound Estimates for v^*

[USD] per flight. In contrast, the value of the stochastic solution (VSS) is on average only 1 % better than solving the problem considering the expectation of the expected value (EEV). However, the VSS increases in scenarios 2 and 8 where the free demand has higher variability, reaching a 1.64% and 1.35% improvement compared with the EEV.

Table 5: Results for the Risk neutral formulation

Experiment	$\widehat{\text{EVPI}}$	$\widehat{VSS}$
1 MM–MM–MM	39,137	2,434
2 MH–MH–MH	41,323	4,608
3 HM–HM–HM	38,580	3,062
4 LM–LM–LM	26,052	1,828
5 MM–LM–HM	40,613	3,191
6 MM–HM–MM	40,734	2,905
7 MM–LM–MM	37,259	2,533
8 MH–LH–HH	41,545	5,489
9 ML–ML–ML	33,968	733
Average	37,690	2,975

4.2.2 Risk averse results

In this section we discuss the influence of considering risk in the objective function using the CVaR. First, we study the influence of the weight λ to balance risk and return. Second, we explore the impact of the confidence interval α. For both analysis we use experiment 1 which is the base case experiment.

Influence of the balance weight λ : Figure 1 shows how the optimal allotment capacity change for different values of λ, considering a confidence level

$\alpha = 0.95$. Remember that a value of $\lambda = 1$ represents the risk neutral approach where only returns mater, whereas $\lambda = 0$ only gives importance to risk.

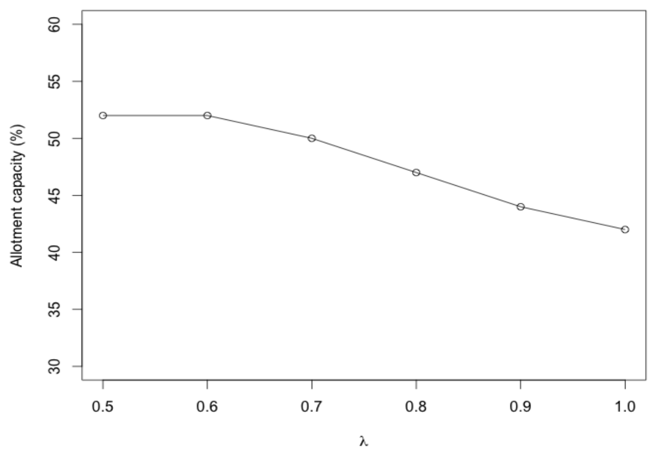

Figure 1: Allotment capacity as a function of λ for $\alpha = 0.95$

Figure 1 shows a tendency to reduce the allotment capacity reserved as the value of λ increase. In this case, when $\lambda = 0.6$ the optimal percentage of allotment capacity is 52% which represents accepting all the potential demand (remember that $D_A = 51,847[kg.]$). These results are coherent to what is observed in practice since allotment reservations gives stability to gains, reducing the probability of extreme losses.

Influence in the value of the confidence level α: The value of α indicates the confidence level with which the VaR of a function is determined. This value is then used to work with the CVaR of the same function. Figure 2 shows the allotment capacity that should be reserved for different values of α, considering $\lambda = 0.6$.

Figure 2 shows that the capacity intended for allotment contracts increase with the value of α. In other words, this result suggest that in order to achieve higher expected profits in $(1 - \alpha) \cdot 100\%$ of the worst cases, which corresponds to the cases where the revenue per flight are lower than the VaR of the revenue function of the airline, capacity reserved for allotment must increase when α grows. The result above shows again that allotment contracts provides security to airlines revenue.

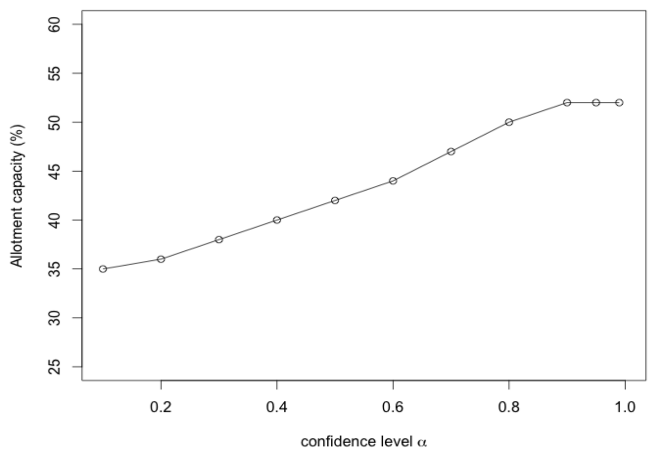

Figure 2: Allotment capacity as a function of α for $\lambda = 0.6$

4.2.3 Comparison between Risk neutral and Risk averse results

So far, we have not analysed the optimal value of the λ-CVaR model because, except in the case of $\lambda = 1$, where the problem is identical to the risk neutral model, the objective function of the λ-CVaR model has no physical interpretation. In order to compare the risk neutral model and the λ-CVaR model, we took the values of the capacity X_A destined for allotment contracts obtained for each of these two models as known parameters. We then generate a new batch of 500 scenarios and compute for each scenario and for each model, the expected income obtained fixing the value of X_A. Finally we calculated the average income and standard deviation for all scenarios and compared it against the EEV solution.

Table 6 shows for each experiment the percentage difference for both the expected income per flight and the standard deviation of this income compared against the EEV for the Risk Neutral (RN) model and for the CVaR model with $\lambda = 0.7$. A negative percentage value represents greater profits or a reduction in the standard deviation value compared to the EEV.

In terms of income, Table 6 shows that the risk neutral model shows on average for the nine experiments a 1 % higher income than the EEV, while the risk averse model attain incomes a 0.8% lower than the EEV. For the RN approach, the greater benefits are produced in experiments 2 and 8 which presents high seasonality and situations with higher than usual variability, whereas the lower benefits are obtained in experiments 9 and 1 where there is only one season and a medium or low variability. Regarding the risk averse approach, the greater benefits are obtained in experiment 8, while in experiments 1,3, 5,6,7 and 9 the

Table 6: Comparison results between risk neutral and risk averse model

Experiment		Dif RN (%)	Dif $\lambda = 0.7$ (%)
1 MM–MM–MM	Income	-0.57	2.01
	Std.Dv.	-20.86	-57.33
2 MH–MH–MH	Income	-1.60	-0.02
	Std.Dv.	-27.74	-56.95
3 HM–HM–HM	Income	-0.65	2.37
	Std.Dv.	-20.66	-58.78
4 LM–LM–LM	Income	-0.24	-0.12
	Std.Dv.	-22.64	-29.14
5 MM–LM–HM	Income	-1.03	1.25
	Std.Dv.	-23.10	-59.14
6 MM–HM–MM	Income	-1.21	0.83
	Std.Dv.	-22.97	-62.34
7 MM–LM–MM	Income	-0.91	0.18
	Std.Dv.	-23.55	-55.78
8 MH–LH–HH	Income	-2.28	-1.41
	Std.Dv.	-29.68	-53.87
9 ML–ML–ML	Income	-0.14	1.95
	Std.Dv.	-12.45	-51.09

average incomes are lower than the EEV. The above results suggest that both models achieve the best results on scenarios with high demand variability and seasonalities.

Regarding standard deviation, both models present significant reductions when compared against the EEV. While, the RN model shows on average a 22.7% reduction in standard deviation, the risk averse model attain average reductions of 53.8% . This result suggest that both models give more stable solutions than the EEV reducing the probability of experiencing extreme losses. Moreover, the risk averse model shows to reduce this variability in a more efficient way than the RN.

5 Conclusions

In this paper we proposed a two stage stochastic programming model to determine the total weight that should be assigned to allotment and free reservations. In our formulation, we consider the risk aversion in the objective function using the CVaR and compare it against the risk neutral formulation.

Our work suggest that using the risk neutral model can achieve on average a 1% increase in the incomes per flight than solving the model using the average values.

The use of the risk neutral model also gives more stable solutions, reducing the probability of experience extreme losses, which is reflected by a 23% decrease in the standard deviation of the incomes when compared to solving the problem replacing the random parameters by their average values.

Regarding the risk averse approach, this model tend to give more stable solutions than the risk neutral model, at the expense of reducing average incomes. Thus, the risk averse model is more suitable in scenarios with higher than usual variability and in presence of seasonalities where expected incomes can vary significantly.

We also conclude that the use of stochastic programming model provides greater benefits in scenarios with high seasonalities and demand variability, reaching a 2.28% increase in average income compared to using average values.

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A Appendix A

A.0.4 Influence of the number of batches M and scenarios N

We use the Risk neutral formulation and experiment 1 to test the influence of N and M in the stability of the objective function. To solve the optimization problem we use Gurobi 5.6.2 [22] .

Influence of the number of batches M : First, we test the influence in the number of batches M while mantaining fixed the number of scenarios $N = 50$ ([19]). We consider different values of $M = 50, 100, 500, 1000, 2000$ and build for each of them a 95 % confidence interval of the expected income of each flight, and the relative error of these incomes as shown in Table 7

Table 7: Confidence interval and relative error of the expected income of each flight for different values of M

M	$E[\hat{v}_N][USD]$	Relative Error
50	$352,414 \pm 2,252$	0.64%
100	$354,522 \pm 1,365$	0.39%
500	$353,967 \pm 651$	0.18 %
1000	$355,217 \pm 492$	0.14%
2000	$354,967 \pm 337$	0.09%

Table 7 shows that the relative error is less than 1% for the different values of M, showing that the optimal solution of the risk neutral formulation is stable in the number of batches.

Influence of the number of scenarios N : Once we have fixed $M = 100$ we now study the influence of the number of scenarios N in the stability of the results ([19]). Table 8 shows confidence interval and the relative error of the income for $N = 50, 100, 500, 1000$.

Table 8 shows that the relative error is very small for all cases and the expected income value is stable around 354,500 *USD*.

Table 8: Confidence interval and relative error of the expected income of each flight for different values of N

N	$E[\hat{v}_N][USD]$	Relative Error
50	$354,522 \pm 1,365$	0.39%
100	$355,140 \pm 1,241$	0.34%
500	$354,360 \pm 443$	0.12 %
1000	$354,114 \pm 331$	0.09%