Lower bus fares or higher frequencies, what do people want? A microeconomic analysis

Pablo Guarda (Corresponding author)
Department of Transport Engineering and Logistics, Pontificia Universidad Católica de Chile
Vicuña Mackenna 4860, Macúl, Santiago, Chile, 7820436
paguarda@uc.cl

Carlos Bueno
Department of Transport Engineering and Logistics, Pontificia Universidad Católica de Chile
Vicuña Mackenna 4860, Macúl, Santiago, Chile, 7820436
cebueno@uc.cl

Patricia Galilea
Department of Transport Engineering and Logistics, Pontificia Universidad Católica de Chile
Vicuña Mackenna 4860, Macúl, Santiago, Chile, 7820436
pga@ing.puc.cl

Juan Carlos Muñoz
Department of Transport Engineering and Logistics, Pontificia Universidad Católica de Chile
Vicuña Mackenna 4860, Macúl, Santiago, Chile, 7820436
jcm@ing.puc.cl

Juan de Dios Ortúzar
Department of Transport Engineering and Logistics, Pontificia Universidad Católica de Chile
Vicuña Mackenna 4860, Macúl, Santiago, Chile, 7820436
jos@ing.puc.cl

Submitted April 2015

Lower bus fares or higher frequencies, what do people want?

Pablo Guarda, Carlos Bueno, Patricia Galilea, Juan Carlos Muñoz, Juan de Dios Ortúzar

Abstract In the literature, different studies support the implementation of subsidies to ensure a proper operation of the public bus transport systems. However, in the actual state of the practice most of public transportation systems are designed under the scheme of financial self-sustainability. In addition, it is not clearly defined under what conditions these subsidies should be used either to increase the level of service or to reduce bus fares. This paper addresses this issue by developing a mathematical formulation to determine how the government can allocate a fix amount of subsidy (by subsidising the fare or the operation by improving the bus frequencies) in order to increase the modal share in the bus public transport system.

KEYWORDS: *Public Transport, Operator Subsidy, Fare Subsidy*

EXTENDED ABSTRACT

During the last decade, many public transport systems around the world have been restructured through private concessions. In Latin America, the implementation of these new systems has had different approaches, having as a common factor the budget constraint to operate the bus fleets, which requires financial sustainability. Resources come mainly from the revenues obtained from the bus fares and in some cases the use of a subsidy, which is not necessarily optimal in terms of minimizing the overall social cost in the system (CAF, 2010). In the literature, different studies support the implementation of subsidies to ensure a proper operation of the public bus transport systems (Basso and Jara-Díaz, 2010; Basso et al., 2011).

To define how to allocate subsidies Jara-Díaz and Gschwender (2009) formulated an optimization problem to find the optimal frequency at a given bus line that minimizes the bus line cost per hour at a given time horizon, which they called *value of resources consumed* (VRC). As shown in Eq. (1), the objective function weights both the operational cost of the service provider and the users cost; the first term captures both the fixed cost and the variable cost of the operators, the next two terms are the waiting and in-vehicle times of users respectively. Eq. (2) represents the financial constraint of the operators; on the left side is the operational cost of the service per hour (the first term of the objective function), whereas on the right side are the total revenues (A) obtained by the operator during the time horizon analysed.

$$\text{Min VRC}_{\{f\}} = (\text{ft} + \text{tY})\left(\text{c}_\text{o} + \text{c}_1 \frac{\text{Y}}{\text{f}} \cdot \frac{\text{l}}{\text{L}}\right) + \text{P}_\text{w} \frac{1}{2\text{f}} \text{Y} + \text{P}_\text{v} \frac{\text{l}}{\text{L}} \left(\text{T} + \text{t}\frac{\text{Y}}{\text{f}}\right) \text{Y} \tag{1}$$

s.a

$$(\text{fT} + \text{tY})(\text{c}_\text{o} + \text{c}_1 \text{K}) \leq \text{A} \tag{2}$$

Where

VRC	:	Value of the of the resources consumed [US$]
f	:	Frequency [bus/hour]
K	:	Capacity [passengers/bus]
T	:	Time in motion of a vehicle [min]
t	:	Average boarding and alighting time per passenger [min]
L	:	Route distance [km]
l	:	Travel distance [km]
Y	:	Flow of passengers [passengers/hour]
c_o	:	Fixed cost per bus and hour [US$]
c_1	:	Variable cost per unit of bus capacity and hour [US$]
P_w	:	Value of waiting time [US$/hour]
P_v	:	Value of In-vehicle time [US$/hour]
A	:	Total revenues perceived by operator [US$]

Jara-Díaz and Gschwender (2009) computed the total revenues obtained by the service provider as the product of the total flow of passengers (Y) and the sum between the bus fares paid by users and the amount of subsidy (by passenger) provided by the public agency to the operator. Instead, our formulation defines a fix subsidy (S) at the time period analysed. Thus, defining c_f as the cost per bus and hour given a fleet capacity K, under operator subsidization the financial constraint of the service provider is given by the following equation

$$(fT + tY)c_f \leq pY + S \tag{3}$$

Where

p	:	Fare revenue by passenger [US$]
S	:	Total subsidy [US$]

Our model considers user's preferences by assuming that bus demand may vary according to changes of bus fares and/or changes in bus frequency. In order to include bus user preferences, we modelled user choices using a Logit model specification (Ortúzar and Willumsen, 2011), thus, the probability to choose bus mode is given by:

$$P_{iq} = \frac{\exp\,(\beta V_{iq})}{\sum_{A_j \epsilon A_{(q)}} \exp\,(\beta V_{jq})} \tag{4}$$

Where

β	:	Form parameter (usually equals one)
V_{iq}	:	Utility function for individual q and alternative i
$A_{(q)}$	:	Set of available alternatives for individual q

If the public agency is interested to maximize the modal split in the bus public transport system, it should determine the changes in the demand of bus users under two scenarios; i) increasing the bus frequency (and decreasing the waiting times), or ii) using the subsidy to decrease the bus fare paid by each passenger. Based on the logit specification, it is possible to derive a simple equation for the direct elasticity of the bus modal share (p_b) with respect to any explanatory variable with a linear form included in the model (Ortúzar and Willumsen, 2011). Assuming that all individuals has the same utility function and they experience the same level of service in the bus system, computing the aggregated elasticity of bus ridership (weighted average between the probability of choosing bus and the direct elasticity of each individual), the total percentage change in the bus modal share due to either an increase in the bus frequency (ΔD_f) or a reduction of the bus fare price (ΔD_p) are given by the following equations:

$$\Delta D_f = \theta_w (1 - p_b) \Delta w_f \tag{5}$$

$$\Delta D_p = \theta_p (1 - p_b) \Delta p + \theta_w (1 - p_b) \Delta w_p \tag{6}$$

Where

ΔD_f	:	Percentage change in the bus modal share under frequency subsidization
ΔD_p	:	Percentage change in the bus modal share under fare price subsidization
Δw_f	:	Change in waiting time under frequency subsidization
Δw_p	:	Change in waiting time under fare price subsidization
Δp	:	Fare reduction under user subsidization
θ_w	:	Waiting time parameter in the logit utility function
θ_p	:	Cost parameter in the logit utility function
D	:	Total demand for transportation (including all modes)

Let's assume that the public agency provides a total amount of subsidy (S) to the operator in order to improve the bus frequency during the time horizon analysed. In this case, the net increase of the frequency (Δf_o) is equal to the subtraction between the increase of the frequency by the investment S and the reduction of the frequency by the induced demand (Eq. 7). Assuming the financial constraint (Eq. 3) is active, Δf_f is given by the following equation:

$$\Delta f_f = \frac{S}{c_f T} - \frac{\Delta D_o D t}{T} \tag{7}$$

Instead, under fare subsidization there is an increase in demand by decreasing the bus fare price (Δp). Since all passengers must face the same fare, the change of the bus fare can be calculated as the division between the total amount of subsidy S and the sum between the current demand and the induced demand (Eq. 8). In addition, it must be accounted for the decrease of the frequency (Δf_p) due to the increase of demand faced by the bus operator (Eq. 9).

$$\Delta p = -\frac{S}{(p_b D + \Delta D_p D)} \tag{8}$$

$$\Delta f_p = \frac{\Delta D_p D t}{T} \tag{9}$$

Then, defining f_1 as the current bus route frequency, assuming regular headways ($\Delta w \approx \Delta f / 2f^2$) and substituting Eq. (7) and Eq. (9), the average difference in waiting times under frequency subsidization and fare subsidization are given by Eq. (10) and Eq. (11) respectively.

$$\Delta w_f = -\frac{1}{2f_1^2 T}\left[\frac{S}{c_f} - \Delta D_f Dt\right] \tag{10}$$

$$\Delta w_p = \frac{Dt}{2f_1^2 T}\Delta D_p \tag{11}$$

Finally, substituting Eq. 5 and Eq. 10, the increase in the demand by frequency subsidization (ΔD_f) is directly derived (Eq. 12). Instead, the increase in the demand by the fare price reduction (ΔD_p) is obtained solving a quadratic equation (Eq. 13), which is derived using Eq. 6, Eq. 8 and Eq. 11.

$$\Delta D_f(S) = \frac{S}{c_f\left(tD - \frac{2f_1^2 T}{\theta_w(1-p_b)}\right)} \tag{12}$$

$$\Delta D_p^2\left(\frac{2f_1^2 T}{(1-p_b)} - \theta_w tD\right) + \Delta D_p\left(\frac{2f_1^2 T p_b}{(1-p_b)} - \theta_w tDp_b\right) + 2\theta_p SDf_1^2 T = 0 \tag{13}$$

Since the objective is to maximize the modal share in public transport, we must find the region of solutions where is more convenient to give fare subsidies ($\Delta D_p \geq \Delta D_f$) or frequency subsidies ($\Delta D_f \geq \Delta D_p$). Based on data obtained from AMB (2011), we plotted $\Delta D_p(S)$ and $\Delta D_f(S)$ as a function of the total subsidy (S), substituting in Eq. (12) and Eq. (13) for all the other parameters. As shown Figure 1, in the x-axis is the total amount of subsidy divided by both the total demand (including all modes of transportation) and the bus fare, whereas in the y-axis is the percentage change of the demand under any of the two schemes.

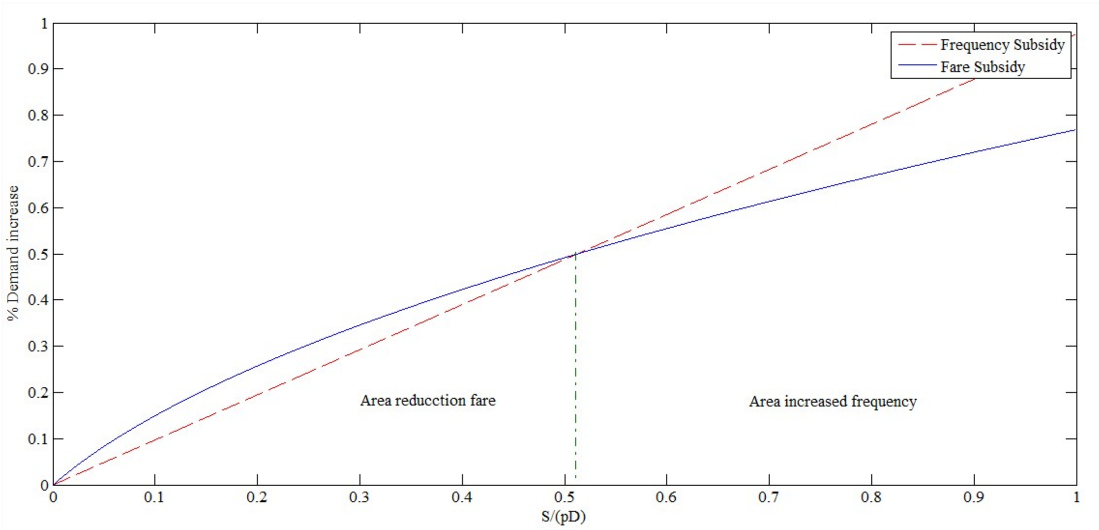

Figure 1. Comparison of the demand increase under a bus fare or frequency subsidization

In conclusion, the mathematical formulation obtained allows us to define the range of subsidy in which it is more convenient to subsidize either the bus fare or the bus frequency. The model includes the effect of losses of frequency due to induced demand, the sensitivity of users both to fare reductions and changes of waiting times, and the financial constraint of bus operators. Our results showed that changes in the parameters related to user preferences directly affect the area of the regions of solutions where is more convenient to apply any of the two subsidization schemes. For example, it was verified that increases of θ_p (a proxy for the sensitivity of bus users to the bus fare cost) increase the range in which the allocation of fare subsidies is preferable to maximize the demand for public transport.

In future research, we will analyse the influence of other factors on the decision of allocating subsidies which were already included in our formulation, such as θ_w (a proxy for the sensitivity of bus users to the waiting times), the current bus modal share (p_b), the current frequency (f_1), the total time in motion of the bus fleet (fT) and the total loading times (tY). Likewise, the model will be evaluated by using real data of other public transportation systems related to operational variables (i.e frequency or loading times) and preferences of different types of user (changes in θ_p and θ_w).

REFERENCES

AMB, 2011. Area Metropolitana Bucamaranga. Diseño y procesamiento de encuestas PD orientadas a la caracterización de los usuarios de transporte de AMB.

Basso, L.J., Guevara, C.A., Gschwender, A., Fuster, M., 2011. Congestion pricing, transit subsidies and dedicated bus lanes: Efficient and practical solutions to congestion. *Transport Policy* **18**(5), 676–684. http://dx.doi.org/10.1016/j.tranpol.2011.01.002.

Basso, L.J., Jara-Díaz, S.R., 2010. The case for subsidisation of urban public transport and the Mohring effect. *Journal of Transport Economics and Policy* **44**(3), 365–372.

CAF, 2010. Observatorio de movilidad urbana para América Latina.

Jara-Díaz, S.R., Gschwender, A., 2009. The effect of financial constraints on the optimal design of public transport services. *Transportation* **36**(1), 65–75. http://dx.doi.org/10.1007/s11116-008-9182-8.

Ortúzar, J. de D., Willumsen, L.G., 2011. Modelling Transport, fourth. ed. John Wiley & Sons, Chichester.