Restricted Stochastic User equilibrium with Threshold (RSUET)

- Model formulation, solution methods and large-scale test

Thomas Kjær RASMUSSEN [a], David Paul WATLING [b], Carlo Giacomo PRATO [a], Otto Anker NIELSEN [a]

[a] Department of Transport, Technical University of Denmark
Bygningstorvet 116B, 2800 Kgs. Lyngby, Denmark

[b] Institute for Transport Studies, University of Leeds
36-40 University Road, Leeds, LS2 9JT, United Kingdom

The Deterministic and Stochastic User Equilibrium (DUE and SUE) have limitations by allowing only minimum cost routes to be used in the DUE and requiring all routes to be used in SUE. The Restricted SUE (RSUE) was proposed to remove these limitations and facilitate large-scale application. The RSUE use random utility maximisation models for flow allocation among equilibrated non-universal choice sets. However, unattractive paths may be used at equilibrium which poses behavioural and computational challenges for large-scale applications. We address this issue by proposing a generalisation of the RSUE, which adds a behaviourally realistic 'threshold' condition that the cost on used paths needs to fulfil. A corresponding generic solution algorithm is proposed and several variants of this tested on the large-scale Zealand network. These showed very attractive computation times to equilibrated solutions and that the modification supports an improvement in behavioural realism, especially for high-congestion cases. We validated the choice set composition on aggregate as well as disaggregate level by comparisons to 16,618 observed route choices.

Introduction

The Deterministic User Equilibrium (DUE, Wardrop, 1952) and the Stochastic User Equilibrium (SUE, Daganzo and Sheffi, 1977) have limitations by allowing only minimum cost routes to be used in the DUE and requiring all routes to be used in SUE. To overcome these limitations and facilitate large-scale application, Watling et al. (2015) and Rasmussen et al. (2015) introduced the Restricted Stochastic User Equilibrium (RSUE) model framework and proposed a solution method. The RSUE model uses well-known random utility maximisation (RUM) models for flow allocation among equilibrated non-universal choice sets, and it was formulated to allow for a non-universal choice set by implicitly posing a condition on the costs on unused paths.

While ensuring that no attractive paths are left unused, the RSUE model does not hinder the use of *unattractive* paths at equilibrium. This is exemplified by Figure 1 and Figure 2, which illustrate the relative generalised costs, at equilibrium, between each used route and the cheapest used route for the corresponding choice set for the application of the MNL RSUE(min) and RSUE(max) on the Sioux Falls network (downloaded from Bar-Gera, 2013).

Some used routes, though not very much used, have a considerably higher generalised cost (e.g., 100-200% higher) than the cheapest route for the corresponding OD movement. This does not seem behaviourally justifiable, and we believe that this arises from the fact that the conditions only pose a cost restriction on unused routes and not on the used routes. In other words, there are no conditions ensuring reasonability of the used routes, and there is no possibility to specify whether a small or large set of used routes is generally preferred. It is thus not an algorithmic problem only, but rather seems to stem from the lack of a mechanism in the underlying equilibrium conditions.

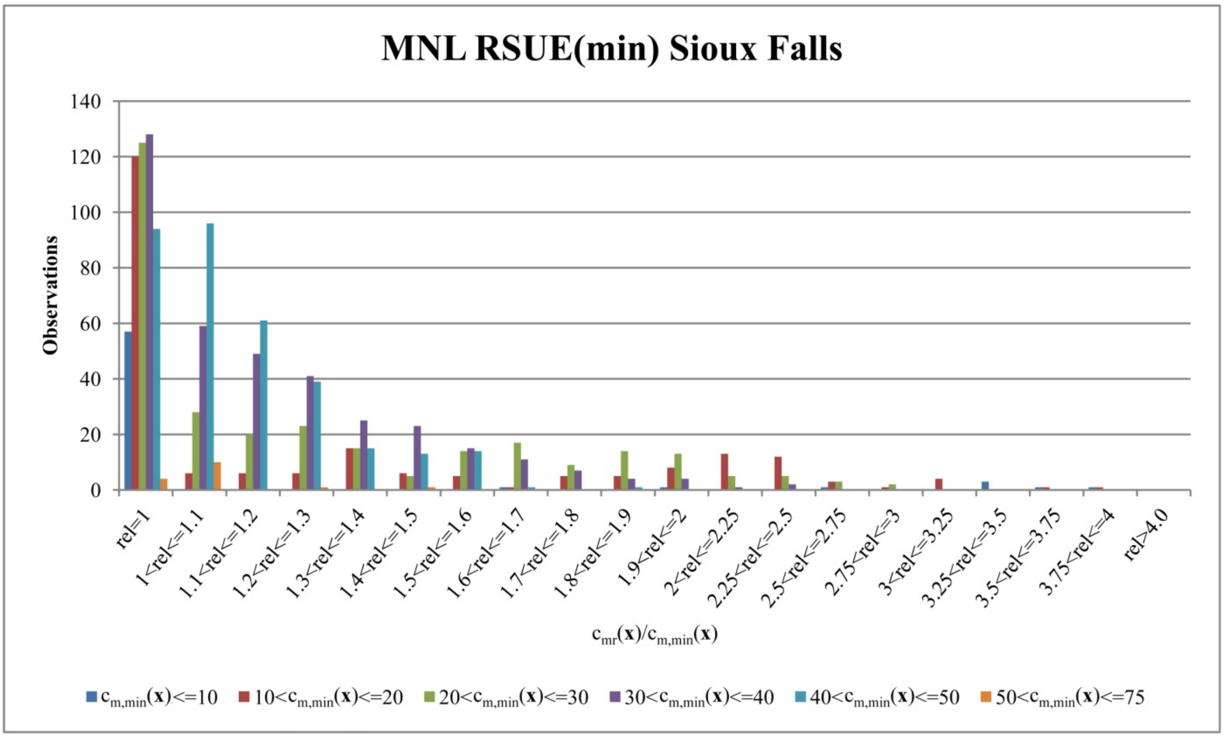

Figure 1 – Number of observations as a function of the ratio between the cost of a used route and the cost of the cheapest used route in the choice set for the MNL RSUE(min) application, Sioux Falls network. Grouped by the cost of the cheapest used route in the choice set

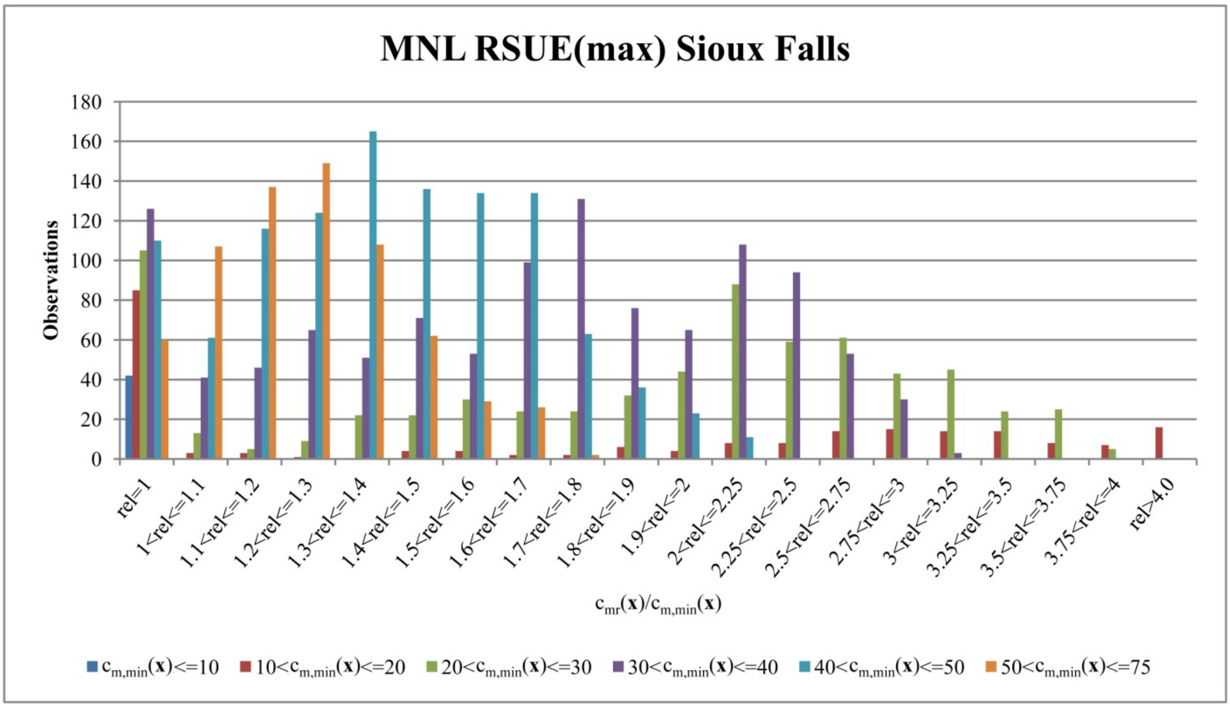

Figure 2 – Number of observations as a function of the ratio between the cost of a used route and the cost of the cheapest used route in the choice set for the MNL RSUE(max) application, Sioux Falls network. Grouped by the cost of the cheapest used route in the choice set

In this paper we aim to address this issue and allow only attractive paths being used through the formulation of a generalisation of the RSUE model, namely the Restricted Stochastic User Equilibrium with Threshold (RSUET) model. The RSUET has built-in rules providing aid for the solution algorithms to utilise in the exclusion of paths, ensuring that a RSUET flow solution is an equilibrated solution on equilibrated choice sets consisting only of the attractive paths. We propose a corresponding consistent solution algorithm to the RSUET model. We apply several of its variants s to the large-scale Zealand area network, and validate the results using observed link flows and GPS data.

Model formulation

Consider a network as a directed graph consisting of A links, indexed a=1, 2, …, A with flow f_a and generalised cost function $t_a(\mathbf{f})$ on link a. $\mathbf{f}$ denotes the A dimensional vector of link flows. Let there be M origin-destination (OD) pairs, indexed m=1, 2, …, M. Let d_m refer to the demand for OD-pair m. Let R_m denote the index set of all simple paths (without cycles) between the origin and destination of OD-pair m, and let x_{mr} and $c_{mr}(\mathbf{x})$ be the flow and cost on route r for OD-pair m. $\mathbf{x}$ and $\mathbf{c(x)}$ denotes the vector of route flows and generalised route travel cost functions, respectively. Let the random utility U_{mr} for each route:

$$U_{mr} = -\theta \cdot c_{mr}(\mathbf{x}) + \xi_{mr} \qquad \left(r \in R_m; m = 1,2,\ldots,M\right) \tag{1}$$

where $\xi = \left\{\xi_{mr} : r \in R_m, m = 1,2,\ldots,M\right\}$ are random variables following some given joint probability distribution, and θ > 0 is a given parameter. Let $\tilde{R}_m$ be any non-empty subset of R_m and we define the 'conditional' probability (assuming continuous, unbounded error distributions $\left\{\xi_{mr} : r \in R_m, m = 1,2,\ldots,M\right\}$):

$$P_{mr}(\mathbf{c}(\mathbf{x}) | \tilde{R}_m) = \Pr\left(-\theta \cdot c_{mr}(\mathbf{x}) + \xi_{mr} \geq -\theta \cdot c_{ms}(\mathbf{x}) + \xi_{ms}, \forall s \in \tilde{R}_m\right) \quad \left(r \in \tilde{R}_m \subseteq R_m; m = 1,2,\ldots,M\right) \tag{2}$$

Definition: Restricted Stochastic User Equilibrium with Threshold (RSUET(Φ,Ω))

The route flow $\mathbf{x} \in G$ *is a RSUET(Φ,Ω) if and only if for all* $r \in R_m$ *and* m = 1,2,…,M:

$$x_{mr} > 0 \quad \Rightarrow \quad r \in \tilde{R}_m \quad \wedge \quad x_{mr} = d_m \cdot P_{mr}\left(\mathbf{c}(\mathbf{x}) | \tilde{R}_m\right) \quad \wedge \quad c_{mr}(\mathbf{x}) \leq \Omega\left(\left\{c_{ms}(\mathbf{x}) : s \in \tilde{R}_m\right\}; \varsigma_m\right) \tag{3}$$

$$x_{mr} = 0 \quad \Rightarrow \quad r \notin \tilde{R}_m \quad \wedge \quad c_{mr}(\mathbf{x}) \geq \Phi\left(\{c_{ms}(\mathbf{x}) : s \in \tilde{R}_m\}; \mathbf{\upsilon}_m\right) \tag{4}$$

where $\Phi\left(\mathbf{c}_{R_m}, R_m\right)$ is formulated as in the RSUE and specifies, dependent on the cost on used paths, a 'reference cost' to be fulfilled by unused paths.

The function $\Omega\left(\left\{c_{ms}(\mathbf{x}) : s \in \tilde{R}_m\right\}; \varsigma_m\right)$ is exogenously defined and specifies one threshold value (internal reference cost) per OD movement. This value is a maximum cost that any used path can have for the OD movement, and ensures that no unattractive paths are used at equilibrium. In the definition above, the Ω-function is specified in a way that enables it to be formulated in numerous different ways. This could e.g. be an absolute non-negative threshold, a relative threshold relative to the minimum cost used route, or a combination of the above. In the application to the large-scale network we consider the following threshold function:

$$\Omega\left(\left\{c_{ms}(\mathbf{x}) : s \in \tilde{R}_m\right\}; \tau_m\right) = \tau_m \cdot \min\left\{c_{ms}(\mathbf{x}) : s \in \tilde{R}_m\right\}$$

where $\tau_m \geq 1$. The choice of the thresholds in Ω causes these to have more or less influence on the solutions. We can either choose to have relatively low computational costs with relatively few used routes (and therefore a strong effect of the threshold), or to enable the inclusion of more used routes (and less effect of the threshold) at a higher computational cost. On the one hand, when choosing a higher threshold the parameter assumes not so much a behavioural connotation as much as a way of controlling the computation time of the algorithm by allowing to drop routes that become highly costly (and little used). On the other hand, when choosing a lower threshold, the parameter assumes more behavioural weight, since a low value will cause the exclusion of some routes with moderate cost (threshold will decide that these are unlikely to be used).

Solution methods

A corresponding class of path-based solution algorithms are proposed to solve for solutions fulfilling the RSUET(Φ,Ω) solutions. The algorithm is an extension of the RSUE solution algorithm proposed in Rasmussen et al. (2015), modified to ensure that the cost-threshold is fulfilled among used paths at equilibrium. An additional step is added that checks for the fulfilment of the additional cost threshold and removes violating routes, if relevant. An iteration of the proposed solution algorithm consists of 4 steps, namely the *Column generation phase*, the *Restricted master problem phase,* the *Network loading phase* and the *Threshold condition phase*.

<table>
<tr><th>Algorithm</th><th colspan="3"></th></tr>
<tr><td>Step 0</td><td colspan="3">Initialisation. Iteration n=1. Perform deterministic all-or-nothing assignment for all m=1, 2,..., M OD-pairs and obtain the flow vector for all utilised paths $\mathbf{X}_n$.
Perform network loading, compute link travel costs $t_a\left(\mathbf{f}_n\right)$ on all network links $a \in A$, and compute generalised path travel costs $c_{mr}\left(\mathbf{X}_n\right)$. Set n=2.</td></tr>
<tr><td rowspan="2">Step 1</td><td colspan="3">Column generation phase. Let $k_{m,n-1}$ denote the current number of unique paths in the choice set of used paths for OD-pair m=1, 2,..., M in iteration n-1.</td></tr>
<tr><td>For RSUET(min, Ω):
For each origin, perform a shortest path search to all destinations based on actual link travel costs $t_a\left(\mathbf{f}_{n-1}\right)$. If for any OD-pair m=1, 2,..., M a new unique path r is generated, add it to the choice set with flow $x_{mr,n-1}$=0.</td><td>For RSUET(Φ, Ω):
For each OD-pair m=1, 2,..., M, based on actual link travel costs $t_a\left(\mathbf{f}_{n-1}\right)$, check for a new route to add to the choice set $\tilde{R}_{m,n}$ by applying some path generation method which supports the fulfilment of the Φ criterion. If for any OD- pair m=1, 2,..., M a new unique path r is generated, add it to the choice set with flow $x_{mr,n-1}$=0; if several routes are possible, add only the shortest one.</td><td>For RSUET(max, Ω):
Perform $k_{m,n-1}$- shortest path search for each OD-pair m=1, 2,..., M based on actual link travel costs $t_a\left(\mathbf{f}_{n-1}\right)$.
If for any OD-pair m=1, 2,..., M a new unique path r is generated among the k generated paths, add it to the choice set with flow $x_{mr,n-1}$=0; if several new unique paths are possible, add only the shortest one.</td></tr>
<tr><td>Step 2</td><td colspan="3">Restricted master problem phase. Given the choice sets $\tilde{R}_{m,n}$ for all m=1, 2,..., M, apply the selected inner assignment component and averaging scheme to find the new flow solution $\mathbf{X}_n$.</td></tr>
<tr><td>Step 3</td><td colspan="3">Network loading phase. Perform the network loading to obtain $\mathbf{f}_n$ from $\mathbf{X}_n$. Compute the link travel costs $t_a\left(\mathbf{f}_n\right)$, the generalised path travel costs $\mathbf{C}\left(\mathbf{X}_n\right)$ and (if relevant/included) the path-size factors.</td></tr>
<tr><td>Step 4</td><td colspan="3">Threshold condition phase. Given the choice sets $\tilde{R}_{m,n}$ for all m=1, 2,..., M, check whether the threshold condition $c_{mr}\left(\mathbf{X}_n\right) \le \Omega(\{c_{ms}(\mathbf{x}) : s \in \tilde{R}_m\}; \varsigma_m)$ is violated for any $r \in \tilde{R}_{m,n}$ for m= 1, 2,..., M. Remove relevant routes (maximum 1 route per OD-pair), redistribute the flow on routes removed among the remaining routes in the respective choice sets. If no routes have been removed for any of the M OD-pairs, continue. Else, perform the network loading, compute the link travel costs $t_a\left(\mathbf{f}_n\right)$, the generalised path travel costs $\mathbf{C}\left(\mathbf{X}_n\right)$ and (if relevant/included) the path-size factors.</td></tr>
<tr><td>Step 5</td><td colspan="3">Convergence evaluation phase. If the gap measure consisting of the sum of $Rel.Gap_n^{Stoch.}$ and $Rel.Gap_n^{Unused,Stoch.}$ is below a pre-specified threshold ξ, Stop[1]. Else, set n=n+1 and return to Step 1.</td></tr>
</table>

[1] See Rasmussen et al. (2015) for the computation of the two gap measures

Please note that the path flow vector is denoted by **X** rather than **x,** in order to emphasise that in practical implementations it is not possible/practical to operate with the vector **x**, as this requires enumerating the universal choice set for all OD-pairs to obtain the dimension of the **x** vector. Rather, in practical implementations, the dimension of the flow vector is not pre-specified, but it is allowed to increase as the algorithm progresses. The same occurs for the path cost vector **c**(**x**), which we have denoted **C**(**X**) to highlight that this might grow as the algorithm progresses. The elements x_{mr} and c_{mr} thus refer to the elements of the vectors **X** and **C**, respectively.

The application of the RSUET considers the following *Threshold condition phase.* In this specification the flow on routes to be removed are redistributed among remaining routes according to the flow proportion on these.

Step 4.0	Set *m*=1
Step 4.1	For each route *r* in the choice set $\tilde{R}_{m,n}$, check whether the threshold condition $c_{mr}\left(\mathbf{X}_n\right) \leq \Omega(\{c_{ms}(\mathbf{x}) : s \in \tilde{R}_m\}; \varsigma_m)$ is violated. If any route *r* violates this condition, flag the route that violates the threshold condition the most.
Step 4.2	If no route is flagged by *Step 4.1* and if *m*<*M*, set *m*=*m*+1 and return to *Step 4.1*. If no routes are flagged by *Step 4.1* and if *m*=*M*, continue to *Step 4.3*. If a route *r* is flagged by *Step 4.1*, remove the route from the choice set and redistribute flow $x_{mr,n}$ among the remaining currently-used routes *s* according to the following: $x_{ms,n} = x_{ms,n} + x_{mr,n} \cdot \dfrac{x_{ms,n}}{d_m - x_{mr,n}}$. If *m*<*M*, set *m*=*m*+1 and return to *Step 4.1*. If *m*=*M*, continue.
Step 4.3	If no routes have been removed for any of the *M* OD-pairs, continue. Else, perform the network loading, compute the link travel costs $t_a\left(\mathbf{f}_n\right)$, the generalised path travel costs $\mathbf{C}\left(\mathbf{X}_n\right)$ and (if relevant/included) the path-size factors.

Application

We performed numerical tests on the large-scale Zealand network. Approximately 2.5 million people live in the area covering 9200 km^2, and the digitised road network representation consists of 12,015 links and 429 zones. The demand matrix applied covered a 24 hour period and contained a total of 3.2 million trips across 19 different user classes and two vehicle types (car and lorry) to be assigned to the road network (1.6 million OD relations)[2]. The study area consists of urban as well as rural areas, and the congestion level is spatially distributed as well as distributed across road type classifications.

Several variants of the implemented algorithm were tested on different configurations of the network demand. All applications focused on the RSUET(min, τ·min) using the method of successive weighted averages with parameter *d* (Liu et al., 2009) and the closed-form probability expression in the *Restricted Master Problem Phase* (as found to perform well for the RSUE(min) in Rasmussen et al. (2015)). The multinomial logit (MNL) as well as the path-size logit (PSL) choice models were evaluated.

[2] The Zealand network is a subset of the network to be used in the Danish National Model, currently under development at DTU Transport.

The solution algorithm is attractive as it does not require simulation, which has also the benefit of limiting the number of model parameters. In the following, we treat the three model parameters τ, d and θ in a subsequent manner. Then, we analyse the effect of correcting for path overlapping and give an example of the effect of adding the threshold condition on the composition of the equilibrium choice sets. Last, we present an analysis of the robustness towards the congestion level in the network. Note that the convergence can be consistently evaluated using the two-part convergence measure proposed for the RSUE(min) in Rasmussen et al. (2015).

Threshold condition

The threshold value τ is not determined from an optimisation routine, but rather from insights learned from analysing the choice of non-optimal paths in real-life observed route choices. Moreover, the threshold value was defined based on a comparison between costs on observed paths and costs on the corresponding minimum cost path. Figure3 illustrates the cumulative share of observations as a function of the ratio between the cost on the observed path (path obtained from GPS data) and the cost on the minimum cost path between the corresponding locations. The observed paths were constituted by the 16,618 routes obtained from the GPS data. The cheapest path was found by, for each GPS trip, performing a shortest-path search in the congested network, between the origin and destination of the corresponding GPS trip. It is e.g. seen that 71% of the observed paths were less than 5% longer than the corresponding optimal path.

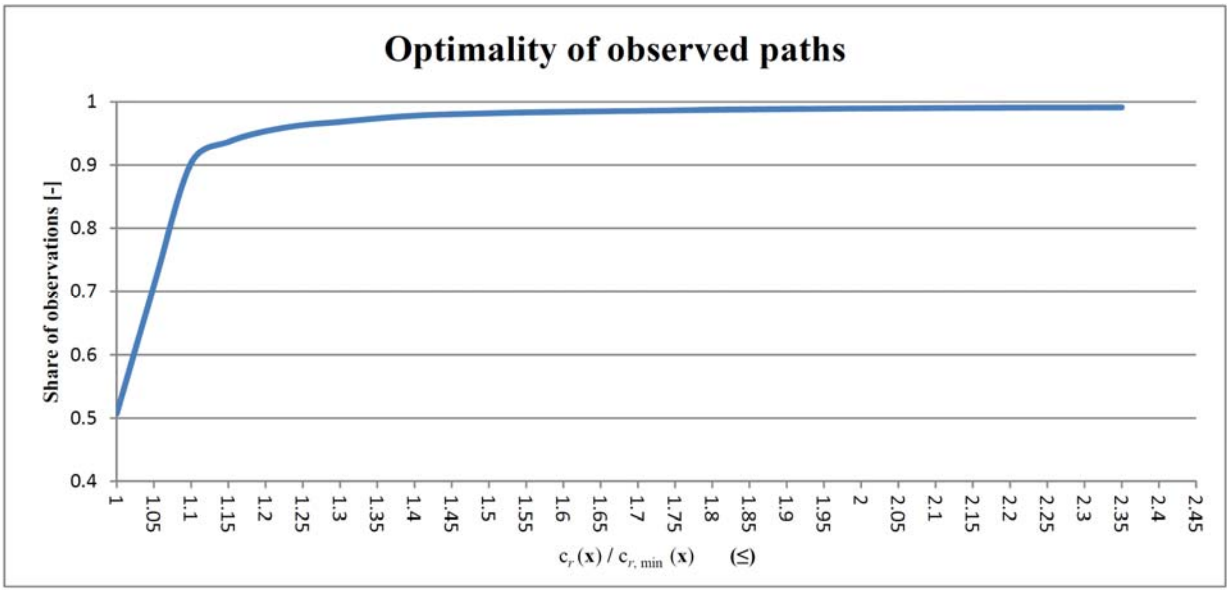

Figure 3 – Cumulative share of observations as a function of the ratio between the cost on the observed path r and the cost on the corresponding minimum cost path $c_{r,\,min}$ ***(x)***

The distribution of the 'non-optimality' of the observed routes is assumed to be representative of how (relatively) expensive paths have to be in order for the travellers not to consider and use them. We specified the threshold based on this: using a 95% interval induces a choice of τ=1.2 (i.e. the relative cost on 95% of all observed paths is within this threshold), which has then been used in the remainder of the paper.

Step-size strategy

If the model parameters τ and θ are kept constant (τ=1.2, θ=0.2), the convergence measures can be directly compared across *d*-values for the RSUET. Figure 4 and Figure 5 illustrate the convergence pattern of the MNL RSUET(min, 1.2·min) for different step-size strategies.

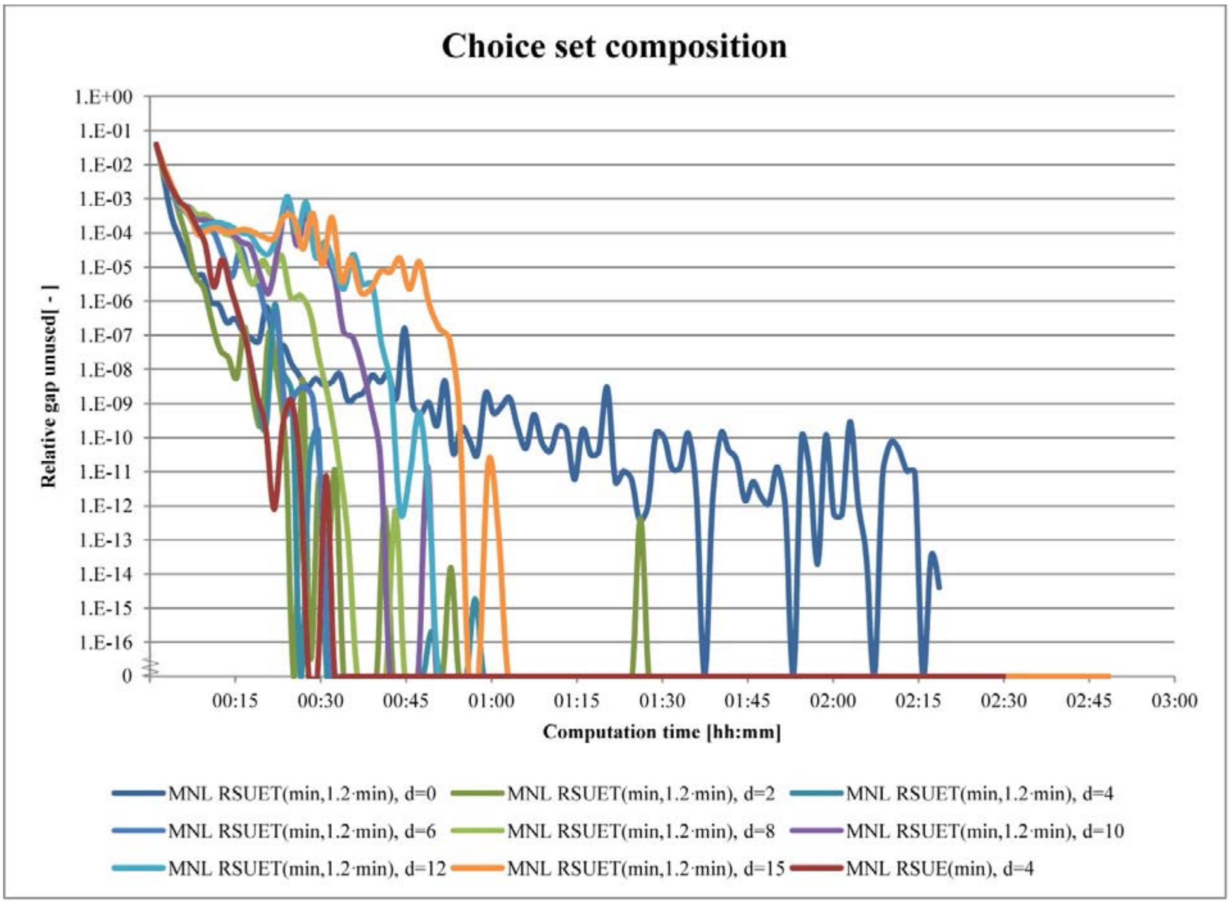

Figure 4 – Relative gap measure for convergence of choice set composition as function of computation time, Zealand application. MNL RSUET(min, 1.2·min) for various values of step-size parameter d as well as the MNL RSUE(min) with d=4. All with θ=0.2. Notice the log-scale on the vertical axis

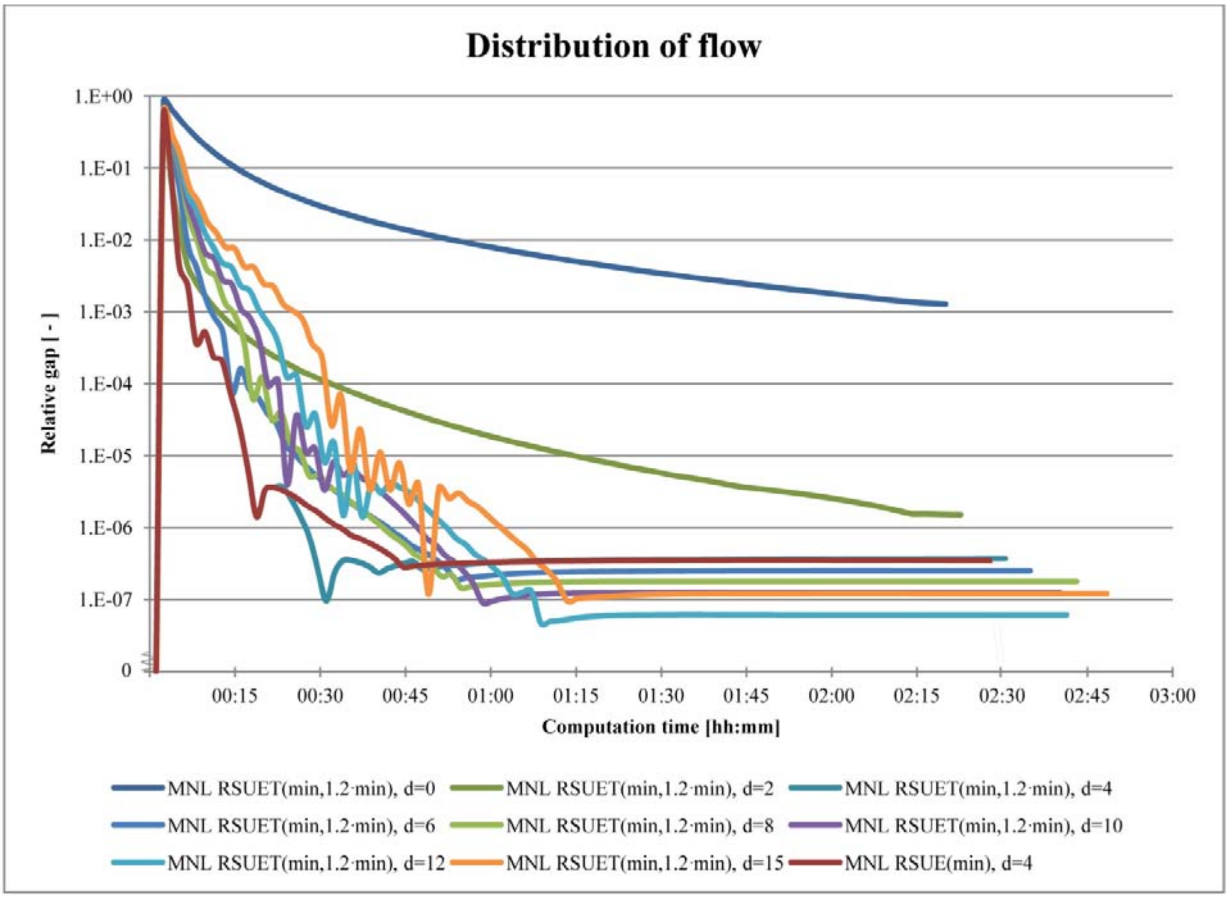

Figure 5 – Relative gap measure for convergence of flow distribution among routes in the choice set as a function of computation time, Zealand application. MNL RSUET(min, 1.2·min) for various values of step-size parameter d as well as the MNL RSUET(min) with d=4. All with θ=0.2. Notice the log-scale on the vertical axis

The choice set composition converged fast for all step-sizes, however with *d*=0 (MSA) being somewhat slower. Also the distribution of the flow among the paths in the choice set converged to a stable low level of approximately $1.0\text{-}3.5\cdot10^{-7}$, except for low values of *d* (*d*=0 and *d*=2) which were far from reaching this level at termination. Using *d*=4 caused the fastest convergence, as the final choice sets were generated within less than 30 minutes and the flow distribution converged within 35-40 minutes of calculation time. Consequently, the analyses presented in the remainder of the paper have been done using *d*=4.

The relative gap associated with the distribution of flow among paths did not seem to converge to zero, but rather stabilised at approximately $1\text{-}3.5\cdot10^{-7}$. This number is very low, and we do not see this stabilisation to a non-zero value as an indication of the algorithm not converging, but rather an issue arising due to the limitations of the computer used; the relative gap is computed using exponential functions of the costs, which causes very small deviations to be amplified into large numbers. We performed a disaggregate analysis of the changes in flow and costs on routes between iterations when *d*=4. This showed that the average/maximum change in absolute cost and flow on the paths across all OD movements is a very low $2.9\cdot10^{-12}/2.3\cdot10^{-10}$ for cost and $6.2\cdot10^{-12}/1.0\cdot10^{-9}$ for flow. These numbers are at the limit of the C# software, and we expect the non-zero gap measure to be a consequence hereof.

Scale parameter

Several different values of the scale parameter were tested, with each application using the same value across all OD movements, i.e. $\theta_m=\theta$ for *m=1,2,…,M*. The relative gap measures were used to verify that all tests converged within reasonable computation time, and extremely fast and well-behaved convergence were seen. The convergence measures can however not be compared across applications, as the scale parameter influences the relative gap measure. We therefore performed a series of alternative analyses to evaluate the performance of the solution algorithm for different values of the scale parameter. This also facilitated the comparison to the link-based multinomial probit (MNP) SUE and mixed MNP SUE solution methods.

1,169 observed link counts were available, and these were distributed throughout the case-study area. Figure 6 reports coefficient of determination (R^2) between the modelled and observed link counts. In general, very high correspondence was observed (all $R^2 \geq 0.942$), demonstrating that the RSUE/RSUET applications are successful in distributing the flow in a way that matches the observed counts. The best performance was obtained when using θ=0.2. While the mixed MNP SUE performed better than the MNP SUE, it is prevailing that both MNP SUE applications performed slightly worse than all RSUE/RSUET applications in reproducing link counts.

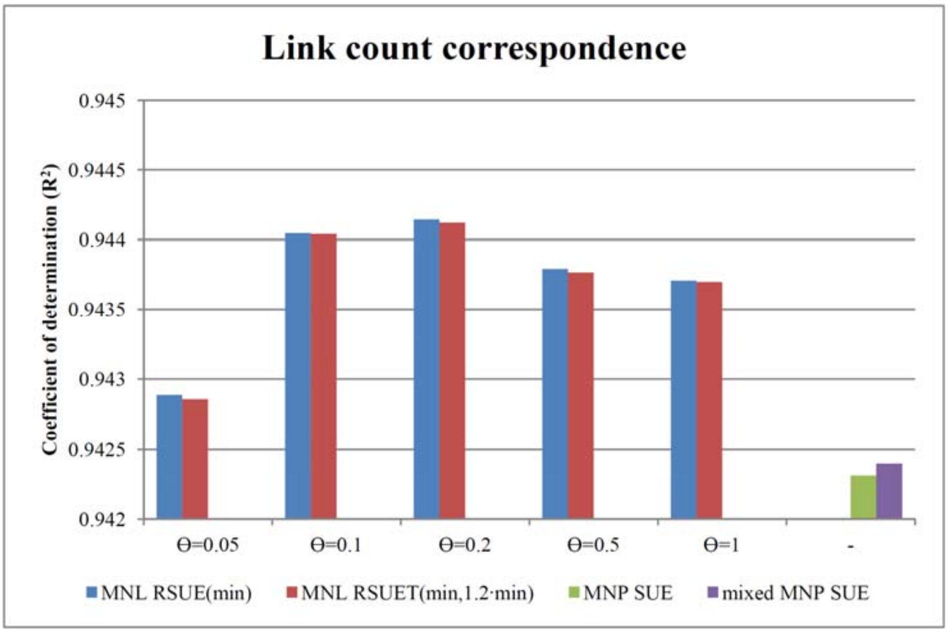

Figure 6 – Correspondence between modelled and observed link flows for various RSUE and RSUET configurations as well as the MNP SUE and mixed MNP SUE. Iteration 100, Zealand application

Moving to a disaggregate level, the solution algorithms should also be able to reproduce rational real-life route choices. We evaluated their ability to do so by comparing with the 16,618 observed route choices collected via GPS, under the hypothesis that the observed routes should be represented in the corresponding choice sets generated. The coverage measure captures this, and Table 1 reports characteristics of the solution generated, including the coverage obtained at iterations 25 and 100 when using a 80% overlap threshold. In general, high coverage levels were produced for all values of the parameter θ. It can be seen that adding the threshold on the relative costs does not seem to reduce the coverage for any of the chosen values of θ. This indicates that the paths removed by the threshold condition are in general non-relevant. Furthermore, the coverage seems to increase with an increase of the scale parameter, a phenomenon probably related to the larger fluctuations in flow in the initial iterations caused by the larger scale parameter; more weight is put on differences in costs (closer to DUE), leading to more 'extreme' auxiliary flows and thereby also larger fluctuations. These fluctuations cause more routes to be generated (seen through larger average choice set sizes) but also more routes to violate the threshold at equilibrium (and thus be removed, see Table 1).

Table 1 – Coverage, choice set size, efficiency index and number of routes removed (when relevant) for various scale parameters in MNL RSUET(min, 1.2·min) and the MNL RSUE(min). The relevant measures are also reported for the MNP SUE and the mixed MNP SUE. Zealand application

		Coverage, λ=0.8		Choice set size			Excluded paths
		Ite 25	Ite 100	Min.	Avg.	Max.	
θ=0.05	RSUE	0.8431	0.8431	1	2.364	10	-
	RSUET	0.8431	0.8431	1	2.367	10	1165
θ=0.1	RSUE	0.8452	0.8452	1	2.484	10	-
	RSUET	0.8452	0.8452	1	2.484	10	1180
θ=0.2	RSUE	0.8487	0.8487	1	2.696	13	-
	RSUET	0.8487	0.8487	1	2.695	12	1989
θ=0.5	RSUE	0.8535	0.8535	1	2.968	14	-
	RSUET	0.8535	0.8535	1	2.967	13	3784
θ=1.0	RSUE	0.8548	0.8548	1	3.059	13	-
	RSUET	0.8548	0.8548	1	3.057	13	4640
MNP SUE		0.8959	0.8959	1	14.894	100	-
mixed MNP SUE		0.8959	0.8959	1	25.365	100	-

The MNP SUE and mixed MNP SUE produced coverage levels which were considerably better than those of the RSUE and RSUET applications. This was however at the cost of generating large choice sets, which continued to grow without any clear tendency towards stabilisation. The RSUE and RSUET on the other hand produce choice sets having a very computationally reasonable size, and which are equilibrated. The equilibrated choice sets were generated within a few iterations, which is also indicated by non-changing coverage from iteration 25 to iteration 100 (Table 1). The flow distribution also converged within a few iterations, highlighting that there is no need to perform many iterations to obtain an equilibrated RSUE/RSUET solution.

Path overlap correction

Our tests also consisted of a comparison between the MNL and the PSL choice models. Similar results were seen with the two choice models, however with a more reasonable distribution of flow across used paths for the PSL (as anticipated) and comparable computation times. However, the PSL application have a higher per-iteration calculation time in the initial iterations due to the need for recalculation of the path-size correction terms when new paths are introduced into the choice sets (Figure 7).

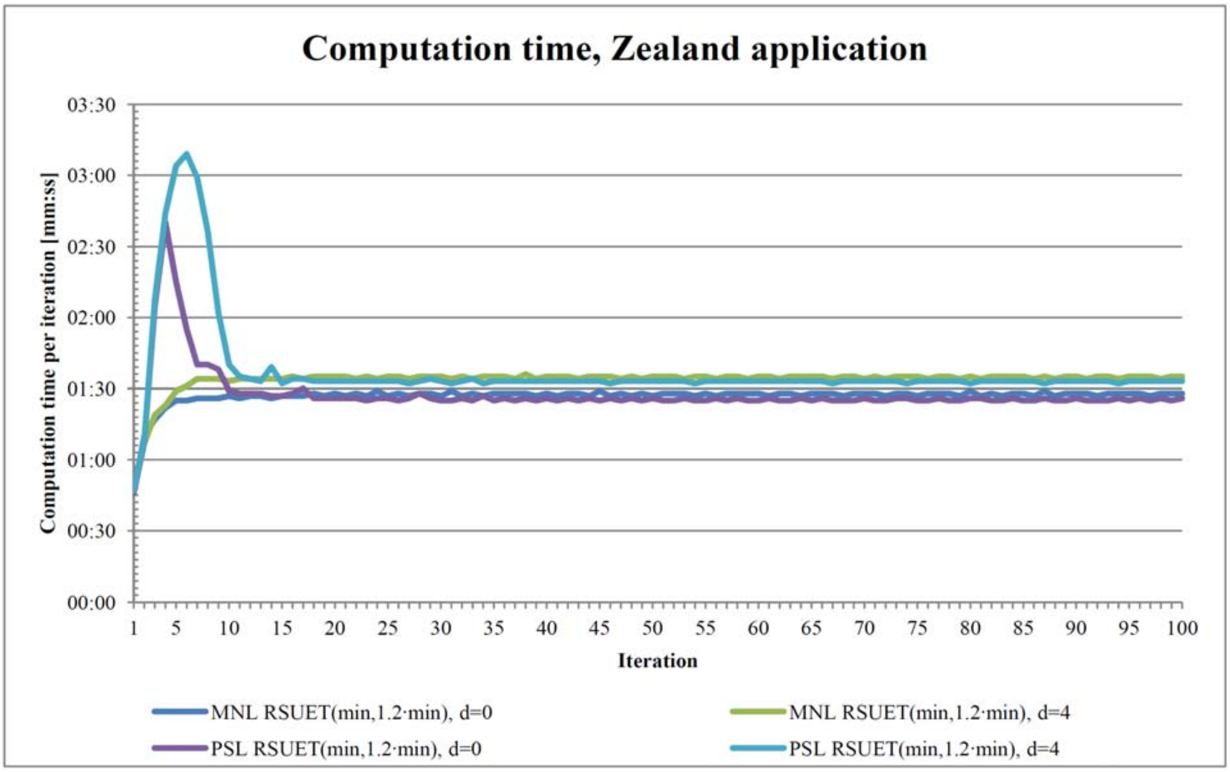

Figure 7 – Computation time per iterations for the MNL as well as PSL RSUET(min, 1.2·min) with d=0 and d=4. Zealand application

Example of route exclusion, threshold condition

1,989 unique routes were removed by the threshold condition when using τ=1.2, d=4 and the MNL choice model with ϑ=0.2. Note, however, that the same unique path may have been generated and subsequently excluded several times during the iterations of the solution algorithm. This section presents an example of an OD movement (commercial business trip undertaken in van), for which a previously generated route was removed by the threshold condition at equilibrium.

Figure 8 illustrates the four unique routes generated (each of these has been the most attractive at some iteration), and Table 2 reports the corresponding equilibrium cost components, generalised cost and route flow share on each of these. All 4 routes were however not included in the equilibrated choice set, as flow was only distributed among paths 1, 2 and 4. Comparing the generalised costs, we see that Path 3 is considerably more expensive than the others. Accordingly, since this path is 32% more expensive than the cheapest path, the threshold condition removed it from the final choice set. We have verified that the flow distribution among the three remaining paths constitutes a MNL flow solution, and that the relative costs of these are below the threshold.

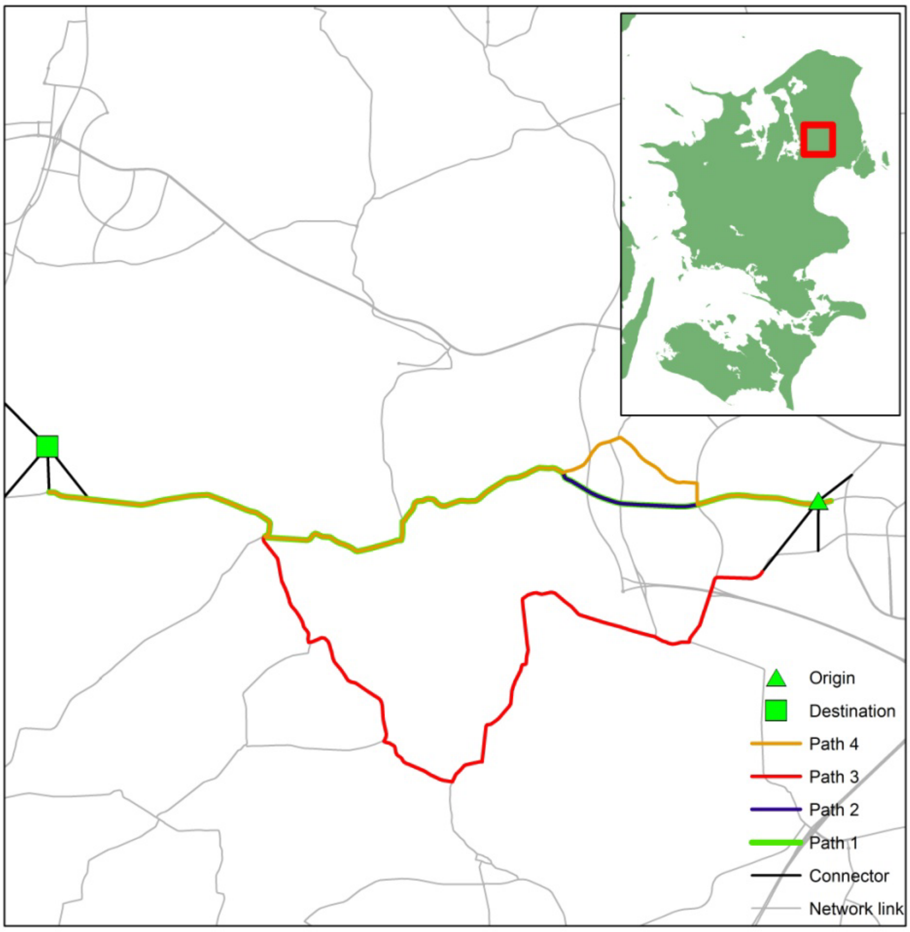

Figure 8 – Example of excluded route. 4 paths generated, but 3 utilised at convergence. MNL RSUET(min, 1.2·min), Zealand application

Table 2 – Specification of cost components, generalised costs, relative costs as well as flows at equilibrium. MNL RSUET(min, 1.2·min), Zealand application. l_{1r}, $t_{FreeTT,\ 1r}$ and $t_{CongTT,\ 1r}$ refer to the length, free-flow travel time and congested travel time of route r, respectively. $c_{1r}(\mathbf{x})$ and $c_{1,min}(\mathbf{x})$ refer to the cost on route r and the minimum cost across the used routes, respectively

Path	Category ID	l_{1r} [km]	$t_{FreeTT,\ 1r}(\mathbf{x})$ [min]	$t_{CongTT,\ 1r}(\mathbf{x})$ [min]	$c_{1r}(\mathbf{x})$	$c_{1r}(\mathbf{x})/c_{1,min}(\mathbf{x})$	Flow [%]
1	6	13.80	12.85	16.39	81.40	1.01	32.23
2	6	13.61	13.42	15.40	81.82	1.02	29.64
3	6	18.02	17.09	20.24	106.07	1.32	-
4	6	14.43	13.64	16.44	80.56	1.00	38.13

Stability to congestion level

We have applied the tested variant of the proposed solution algorithm with d=4 and θ=0.2 to a variety of scaled versions of the original demand matrices (the scale-factors tested are 1.25, 1.5, 1.75 and 2.0). This was done to test the robustness towards the general congestion level in the network.

There was a clear tendency for slower convergence as the demand increased, both in terms of number of iterations needed as well as calculation time (Figure 9-10). However, a nice convergence pattern was seen for all the tested levels of demand. The travel times in the network fluctuated more in the initial iterations and caused larger choice sets to be generated. The higher fluctuations and travel time differences in the network also caused the distribution of flow among paths to require more iterations to converge for increasing demand levels, but even the highest congestion case converged nicely once the final choice sets were generated. The larger choice sets caused an increase in the calculation time *per iteration to* also increase, and the average calculation time per iteration was approximately 90/105/130/145/180 seconds for scale-parameter 1.0/1.25/1.5/1.75/2.0, respectively.

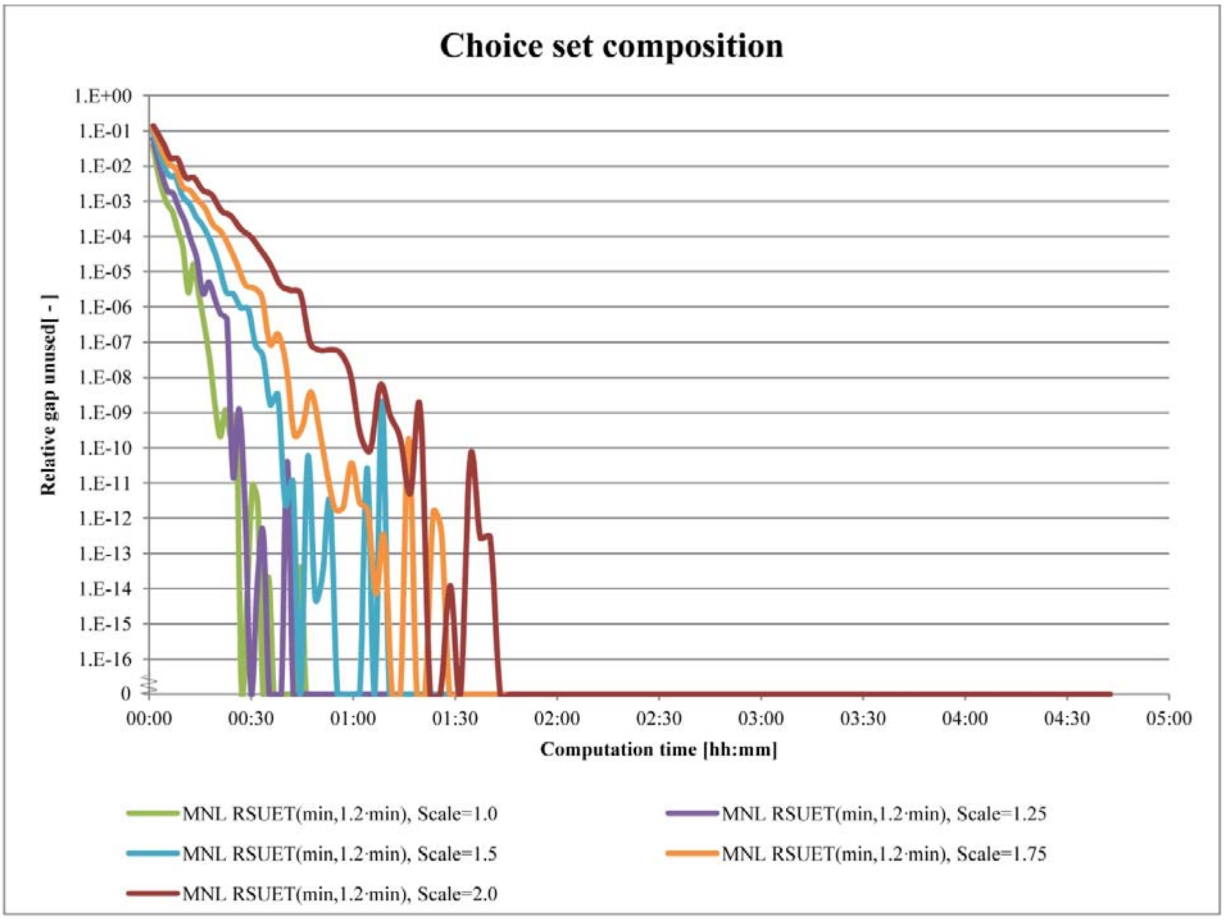

Figure 9 – Development of relative gap measuring convergence of the choice sets for various values of the factor scaling the demand, MNL RSUET(min, 1.2·min) with d=4, Zealand application

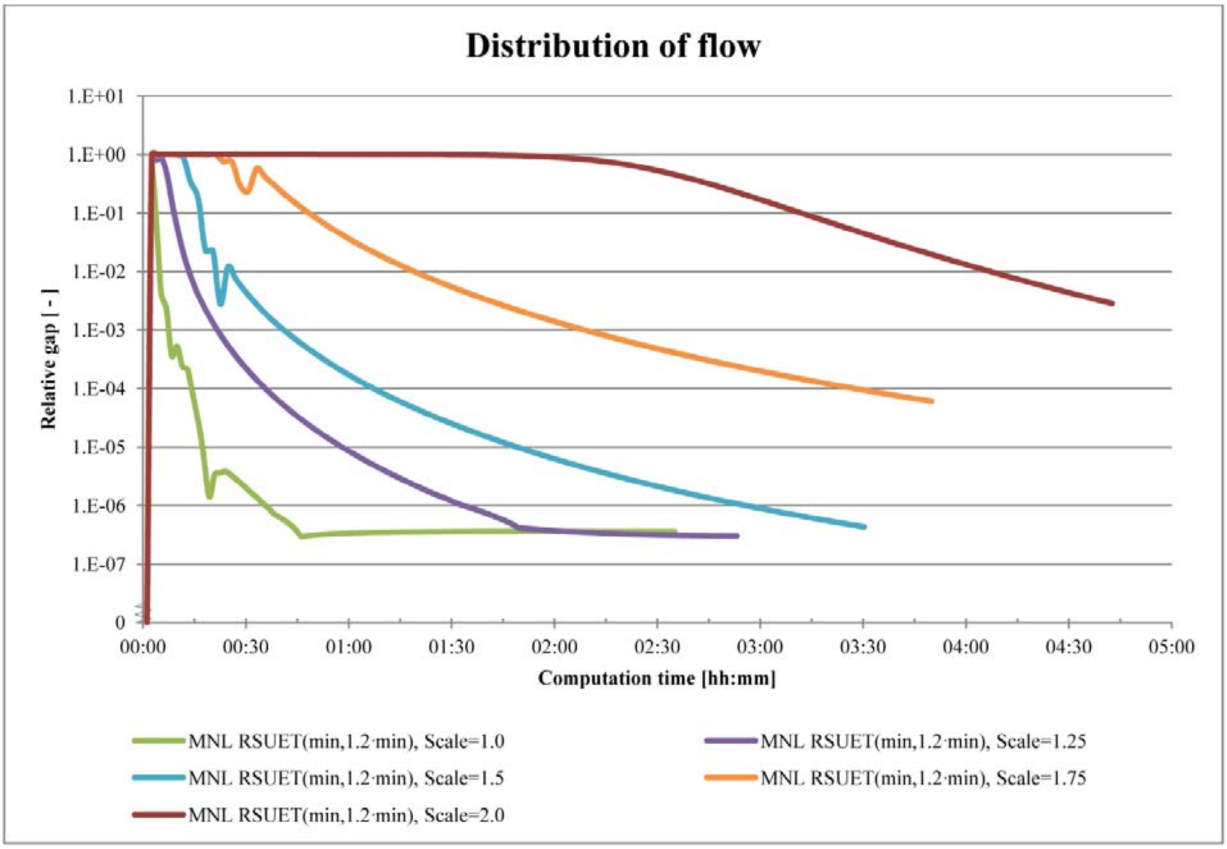

Figure 10 – Development of relative gap measuring convergence of the distribution of flow between paths for various 'scaled' demands, MNL RSUET(min, 1.2·min) with d=4, Zealand application

Conclusions

The paper tackles the challenge of obtaining equilibrated RUM flow solutions among choice sets which do not leave attractive paths unused and contain *only* attractive paths. The RSUE only partially obtains this; no attractive paths are left unused, but some unattractive paths may be used at equilibrium. We overcome this problem by proposing the RSUET (RSUE with Threshold), as an extension to the RSUE. The extension adds a behaviourally realistic threshold condition that must be fulfilled by the costs on used routes. This ensures that only attractive paths fulfilling the cost threshold are kept in the choice set and thus are assigned traffic. We have demonstrated that the modification supports an improvement of the behavioural realism in disaggregate large-scale applications, especially for high-congestion cases. We proposed a corresponding generic solution algorithm and verified several variants of this in different parameter settings in a highly complex network. The algorithm converged extremely fast to an equilibrated solution satisfying the underlying conditions (across different scale parameters, step-size strategies, and congestion levels).

References

Bar-Gera, H., 2013. Transportation Network Test Problems. Accessed Juli 1, 2013 at URL: http://www.bgu.ac.il/~bargera/tntp/.

Daganzo, C.F., Sheffi, Y., 1977. On stochastic models of traffic assignment. *Transportation Science*, 11, 351-372.

Liu, H. X., He, X., He, B., 2009. Method of Successive Weighted Averages (MSWA) and Self Regulated Averaging Schemes for Solving Stochastic User Equilibrium Problem. *Networks and Spatial Economics*, 9 (4), 485-503.

Rasmussen, T.K., Watling, D.P., Prato, C.G., Nielsen, O.A., in press. Stochastic User Equilibrium with equilibrated choice sets: Part II – Solving the Restricted SUE for the logit family. *Transportation Research Part B: Methodological*, doi: 10.1016/j.trb.2015.03.009.

Wardrop, J.G., 1952. Some theoretical aspects of road traffic research. *Proceedings of Institution of Civil Engineers*, Part II 1, 325–378.

Watling, D.P., Rasmussen, T.K., Prato, C.G., Nielsen, O.A., in press. Stochastic User Equilibrium with equilibrated choice sets: Part I – Model formulations under alternative distributions and restrictions. *Transportation Research Part B: Methodological*, doi: 10.1016/j.trb.2015.03.008.