Fundamental diagram for pedestrians: a two-level approach

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Abstract

The performance of walking facilities, in both the planning and management phase, is assessed by the utilization of indicators such as speed (v), density (k), flow (q) and the relationship between them, the fundamental diagram (FD). The FD is also a required input or a calibration criterion for most models of pedestrian dynamics. Despite its importance, the contradictions regarding the fundamental pedestrian flow quantities, existing in the literature, suggest that pedestrian flow characterization still remains not well explained. When it comes to measuring pedestrian flow indicators, the approaches are usually developed on the grounds of drawing the parallel between pedestrian and vehicular traffic. Although the field of vehicular flow modelling is quite well established, this comparison is considered useful only to some extent due to large differences which exist between the two types of traffic flow. Consequently, most of the studies have been focused only on unidirectional flows where the empirical data have been used so as to fit certain deterministic curves. The empirical and theoretical studies that focus on bidirectional and multidirectional flow are very scarce. In this manner, the existing approaches consider pedestrian population as being completely homogenous, additionally constraining their flow patterns to the unidirectional case. This paper proposes a two-level approach to coping with the existing drawbacks, and it is further supported by concrete methodological concepts and results.

At the disaggregated level the paper addresses the heterogeneity of pedestrian population which is to be reflected through the pedestrian flow indicators. To support this objective the great potential of the available pedestrian tracking data collected in a Lausanne railway station has been exploited (Alahi et al., 2011). The space and time representation are certainly the important aspects which affect the analysis of flow indicators. The existing approaches to discretization in both domain, however, are usually based on an arbitrary choice thus ignoring the fact that the results are not independent of the aggregation level. Therefore, we suggest the disaggregated and data-driven discretization framework that is specifically designed to provide comprehensive pedestrian-oriented definitions of density and speed indicators in the first place. In order to obtain a good resolution space is discretized through the use of Voronoi tessellations (Okabe et al., 2009). The Voronoi-based structures have been investigated in the pedestrian flow theory (Nakamura et al., 2011,Steffen and Seyfried, 2010), but in this paper they are employed for the sake of enabling flow characterization through the pedestrian-oriented approach. Voronoi space decomposition assigns the personal region (V_i) to each pedestrian (p_i), in such a way that all space locations are associated with the closest pedestrian in respect of the Euclidean distance. The personal density k_i is subsequently defined as the inverse of the personal area:

$$k_i = \frac{1}{|V_i|}. \tag{1}$$

Certain issues are associated with the fact that the ordinary Voronoi diagram is defined in an obstacle-free plane and with the potential numerical instability of the results due to degeneration of Voronoi

cells in very dense areas. As for these particular cases we: (i) combine the ideas of Voronoi diagram for the set of points (pedestrians) and Voronoi diagram for the set of areas (obstacles), in order to create personal regions which do not overlap with obstacles; (ii) assess a threshold distance between neighbouring pedestrians when more polygons, which are critical in terms of the area, merge together through the means of sensitivity analyses. On the other hand, the choice of the time discretization (Δt), needed for the speed computation:

$$v_i = \frac{\|p_i(t+\Delta t) - p_i(t-\Delta t)\|}{2\Delta t} \tag{2}$$

is justified by using the statistically based approach. The speed distributions obtained through different time discretization are analysed on the basis of the statistical test of moments. The outcome is the range of appropriate time discretization intervals that do not cause 'critical loss' of information (Table 1).

Table 1: Raw moments of the speed distributions*

Moment	$v_{\Delta t=0.1s}$	$v_{\Delta t=0.2}$	$v_{\Delta t=0.3s}$	$v_{\Delta t=0.4s}$	$v_{\Delta t=0.5s}$	$v_{\Delta t=0.6s}$	$v_{\Delta t=0.7s}$	$v_{\Delta t=0.8s}$	$v_{\Delta t=0.9s}$	$v_{\Delta t=1s}$
1	1.1161	1.1158	1.1156	1.1155	1.1153	1.1152	1.1150	1.1149	1.1148	1.1147
2	0.4175	0.3296	0.2956	0.2747	0.2591	0.2465	0.2358	0.2263	0.2179	0.2104
3	5.7853	2.5957	1.7703	1.4310	1.2544	1.1476	1.0740	1.0188	0.9744	0.9363
4	134.4926	31.2621	15.5319	10.9042	9.0167	8.0657	7.4917	7.0994	6.8045	6.5660

* The Kruskal-Wallis test (0.87% confidence level) on these set of moments revealed that they represent the same population

The described framework provides a discretization with the homogenous conditions within a personal region hence at this level the hydrodynamic relation $q = k \cdot v$ holds.Using the definitions (1) and (2) the empirical speed-density relationship is established; one that exhibits a high scattering phenomenon. In order to capture the effects of a large number of factors that involves randomness (personal characteristics, geometric settings, environmental conditions, etc.) a probabilistic methodology has been employed. In particular, the Kumaraswamy specification (Kumaraswamy, 1980) of speed-density relationship is proposed:

$$V \sim f(\alpha(k), \beta(k), l(k), u(k)) \tag{3}$$

where f corresponds to Kumaraswamy probability density function, α and β are shape parameters, l refers to lower boundary and u to upper boundary parameters that change with the density level. The maximum likelihood estimation methodology has been performed by using a pedestrian tracking input on a set of different parameters' specifications. Based on the loglikelihood value, the specification resulted in the following:

$$\alpha(k) = a_\alpha k^3 + b_\alpha k^2 + c_\alpha k + d_\alpha \tag{4}$$

$$\beta(k) = a_\beta \exp(b_\beta k) \tag{5}$$

$$u(k) = a_u \exp(b_u k) \tag{6}$$

$$l(k) = 0 \tag{7}$$

$$a_\alpha = 0.248, b_\alpha = -0.697, c_\alpha = 0.160, d_\alpha = 2.245, a_\beta = 68.894, b_\beta = -0.875, a_u = 8.061, b_u = -0.283. \tag{8}$$

The model robustness is validated by application on additional data set. Its performance is also compared with the well-accepted models published in the literature against empirical data.

The described framework as such is however insufficient to explain the multidirectional nature of pedestrian flows. This issue is addressed at the aggregated level through a stream-based concept. Given that the speed and flow indicators represent the vectors in finite-dimensional space $\mathfrak{R}^2$, the standard cell-based aggregation seems to be inappropriate. For instance, in the case of two pedestrians in a cell passing each other and having the same magnitude of the corresponding speed vectors, it would lead to a situation with an average speed value of zero. Therefore, we consider pedestrian traffic as being composed of different streams that interact within the same space. Pedestrians are assumed to contribute to the streams to some extent. The definition of a stream is completely exogenous; it might be designed depending on the type of problem, learned from sample sets of data or even chosen from an existing catalogue. Here we propose a stream definition that is direction-based given a directional representation system. The standard choice for the representation system would be (orthonormal) basis, implying the number of representative vectors (φ_j) that is equal to the dimension of space. However, in some cases the basis might be unfavourable since it does not allow any flexibility in design, leading to the notation of frame (the redundant counterpart of a basis):

$$(\varphi_j)_{j=1}^{S}, S \geq 2. \tag{9}$$

The decomposition of individual indicators according to the given representation system is as follows:

$$v_i \mapsto (\langle v_i, \varphi_j \rangle)_{j=1}^{S} \tag{10}$$

where $\langle \cdot, \cdot \rangle$ represents the inner-product operator giving the individual contribution to the direction (stream) φ_j. The exact choice of the stream configuration is however left to the modeller. Within a stream-based framework, we start from the bidirectional case and the first principals building upon a model of stationary bidirectional flow proposed by Flötteröd and Lämmel (2014). The performances of the model within the context of a new data set (Daamen and Hoogendoorn, 2003) will be presented together with the necessary enhancements concerning the streams interaction, leading towards its multidirectional generalization. One of the objectives is the integration of the stream-based concept with the developed probabilistic framework so as to be able to comply with the observed heterogeneity. The real case study will be presented and analysed in details.

In the contrast to the existing research, here we provide a two-level approach allowing FD formulation from the pedestrian-oriented prospective. As such, it also has the implications on dynamic continuum and discrete models for pedestrians that combine a conservation principle with a fundamental diagram. Moreover, the evaluation and optimization of the level of service of pedestrian facilities can largely benefit from the findings presented in this paper.

Keywords

pedestrian flow indicators - fundamental diagram - heterogeneous population- multidirectional streams - individual trajectories

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