Demand Based Timetabling of Passenger Railway Service

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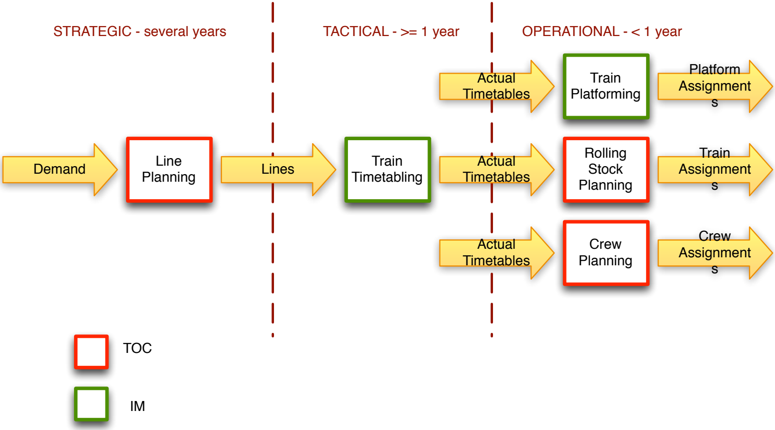
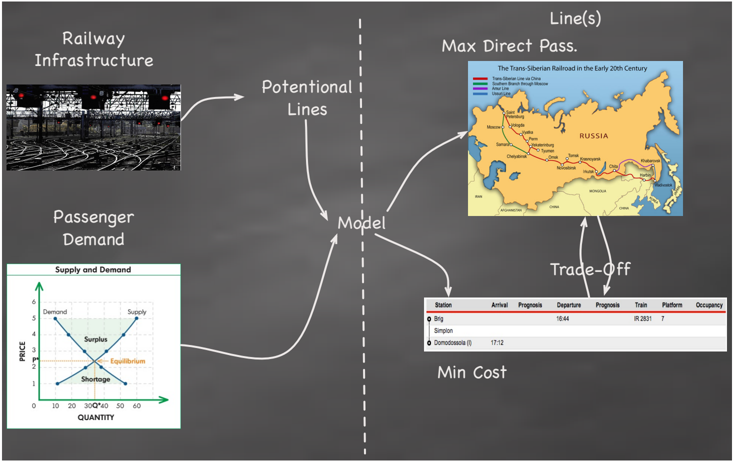
Railway Planning



Line Planning Problem



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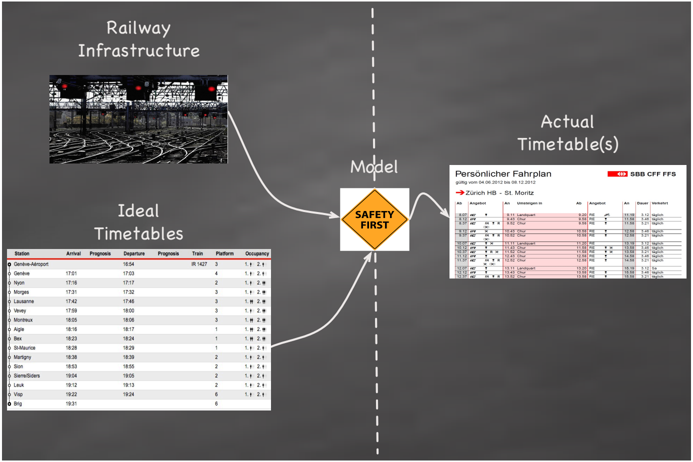
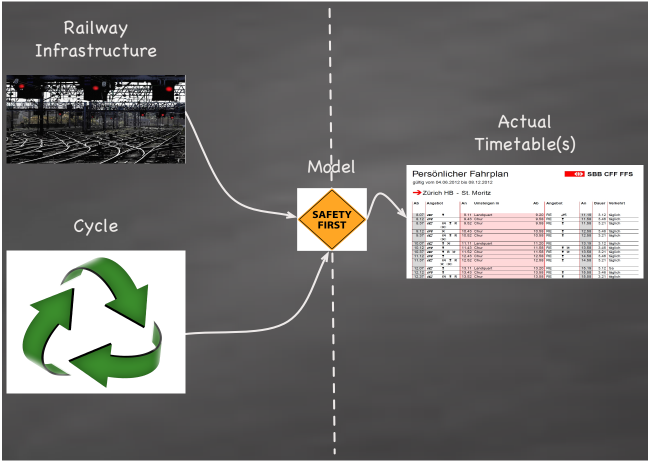
Train Timetabling Problem – Non-Cyclic



Station	Arrival	Prognosis	Departure	Prognosis	Train	Platform	Occupancy
Genève-Aéroport			16:54		IR 1427	3	1. 2.
Genève	17:01		17:03			4	1. 2.
Nyon	17:16		17:17			2	1. 2.
Morges	17:31		17:32			3	1. 2.
Lausanne	17:42		17:46			3	1. 2.
Vevey	17:59		18:00			3	1. 2.
Montreux	18:05		18:06			3	1. 2.
Aigle	18:16		18:17			1	1. 2.
Bex	18:23		18:24			1	1. 2.
St-Maurice	18:28		18:29			1	1. 2.
Martigny	18:38		18:39			2	1. 2.
Sion	18:53		18:55			2	1. 2.
Sierre/Siders	19:04		19:05			2	1. 2.
Leuk	19:12		19:13			2	1. 2.
Visp	19:22		19:24			6	1. 2.
Brig	19:31					6	

Train Timetabling Problem – Cyclic



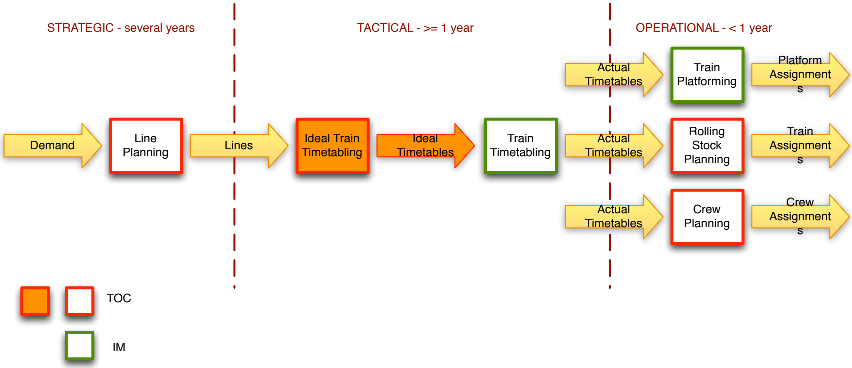
Arising Issues

Figure : Outside peak hour

Figure : Inside peak hour

Figure : Train station in China

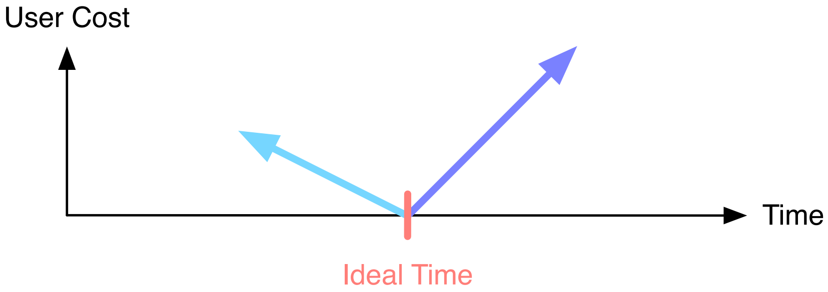
Railway Planning Improved



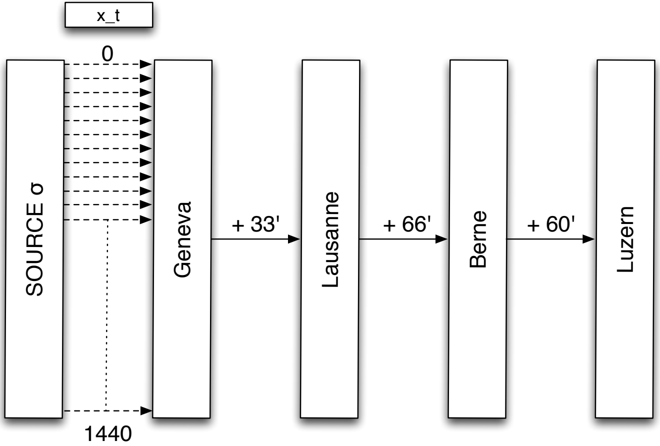
Agenda

1. Motivation
2. Ideal Train Timetabling Problem
3. Conclusions
4. Future Work

Assumptions I



Assumptions II



Inputs

$t \in T$	–	set of time steps
$l \in L$	–	set of lines
f	–	fraction by which it is better to be early
d_t^l	–	demand captured along the line l, when scheduling a train at time t
$d_t^{ll'}$	–	connection demand captured along the line l and l', when scheduling a train at time t on the line l
n^l	–	number of trains available for line l
$h_l^{l'}$	–	relative headway to reach a connection point of lines l and l' from the first station on line l and l'
c^l	–	size of the cycle on line l
s	–	preferred start of the planning horizon
$M \in \mathbb{M}$	–	set of sufficiently large numbers

Primary Decision(s)

$$x_t^l = \begin{cases} 1 & \text{if a train on line } l \text{ is scheduled at time } t, \\ 0 & \text{otherwise.} \end{cases}$$

Secondary Decisions I

- $y_t^{lb} \in \mathbb{R}^+$ – cost of the passengers wanting to travel at time t on the line l, when taking a closest train at t or before
- $y_t^{la} \in \mathbb{R}^+$ – cost of the passengers wanting to travel at time t on the line l, when taking a closest train after t
- $y_t^{l} \in \mathbb{R}^+$ – cost of the passengers wanting to travel at time t on the line l

Secondary Decisions II

$$z_t^l = \begin{cases} 1 & \text{if passengers wanting} \\ & \text{to travel at time } t \\ & \text{on the line } l \text{ take the} \\ & \text{closest train} \\ & \text{after the time } t, \\ \\ 0 & \text{otherwise.} \end{cases}$$

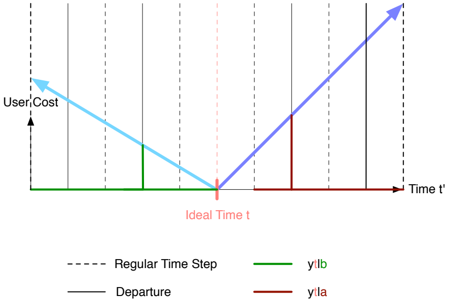
Objective

$$\min \sum_{l \in L} \sum_{t \in T} y_t^l \cdot d_t^l$$

Constraints I

$$y_t^{lb} \geq (t - t')\,/f \cdot \left(x_{t'}^{l} - \sum_{t''=t'+1}^{t} x_{t''}^{l} \right) \qquad \forall l \in L, \forall t, \forall t' \in T : t \geq t',$$

$$y_t^{la} \geq (t' - t) \cdot \left(x_{t'}^{l} - \sum_{t''=t+1}^{t'-1} x_{t''}^{l} \right) \qquad \forall l \in L, \forall t, \forall t' \in T : t < t',$$



Constraints II

$$y_t^{lb} \geq M_1 \cdot \left(1 - \sum_{t'=s}^{t} x_{t'}^{l}\right) \qquad \forall l \in L, \forall t \in T,$$

$$y_t^{la} \geq M_1 \cdot \left(1 - \sum_{t'=t}^{T} x_{t'}^{l}\right) \qquad \forall l \in L, \forall t \in T,$$

Constraints III

$$
\begin{aligned}
y_t^l &\geq y_t^{lb} - z_t^l \cdot M_2 && \forall l \in L, \forall t \in T, \\
y_t^l &\geq y_t^{la} - \left(1 - z_t^l\right) \cdot M_2 && \forall l \in L, \forall t \in T, \\
M_2 &> M_1
\end{aligned}
$$

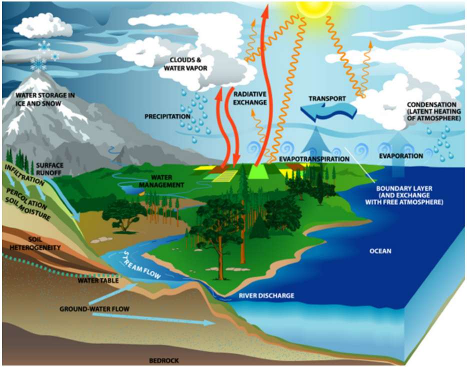
Constraints IV

$$\sum_{t \in T} x_t^l \leq n^l \qquad \forall l \in L,$$

Introducing Cyclicity

$$x^l_{t+c^l} = x^l_t \qquad \forall l \in L, \forall t \in T : t + c^l \leq T : t \geq s,$$

$$\sum_{t'=t+1}^{min\left(t+c^l, T\right)} x^l_{t'} \leq \left(1 - x^l_t\right) \cdot M_3 \qquad \forall l \in L, \forall t \in T : t \geq s,$$



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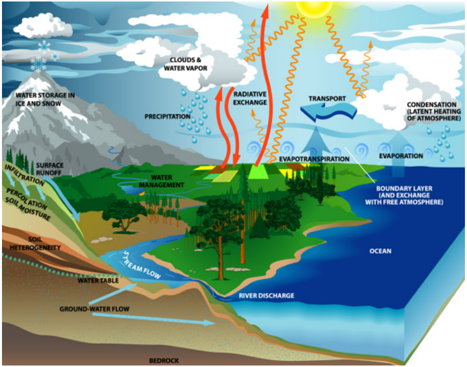

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Extra Decisions I

- $y_t^{ll'b} \in \mathbb{R}^+$ – cost of the passengers wanting to travel at time t on the line l, when taking a closest train at t or before and connecting to line l'
- $y_t^{ll'a} \in \mathbb{R}^+$ – cost of the passengers wanting to travel at time t on the line l, when taking a closest train after t and connecting to line l'
- $y_t^{ll'} \in \mathbb{R}^+$ – cost of the passengers wanting to travel at time t on the line l and connecting to line l'

Extra Decisions II

$$z_t^{ll'} = \begin{cases} 1 & \text{if passengers wanting to travel at time } t \text{ on the line } l \text{ take the closest train after the time } t \text{ and connecting to line } l', \\ 0 & \text{otherwise.} \end{cases}$$

Objective

$$\min \sum_{l \in L} \sum_{t \in T} y_t^l \cdot d_t^l + \sum_{l \in L} \sum_{l' \in L} \sum_{t \in T} y_t^{ll'} \cdot d_t^{ll'}$$

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Extra Constraints I

$$y_t^{ll'b} \geq (t - t') / f \cdot \left(x_{t'}^{l} - \sum_{t'''=t'+1}^{t} x_{t'''}^{l} \right) + \left(t'' - \left(t' + h_l^{l'} \right) \right) \cdot \left(x_{t''}^{l'} - \sum_{t'''=t'+h_l^{l'}+1}^{t''-1} x_{t'''}^{l'} \right) - M_4 \cdot \left(1 - x_{t'}^{l} + \sum_{t'''=t'+1}^{t} x_{t'''}^{l} \right)$$

$$\forall l, \forall l' \in L : l \neq l',$$

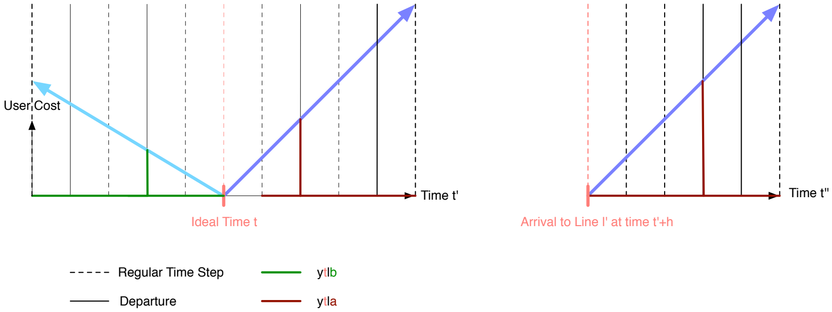
$$\forall t, \forall t', \forall t'' \in T : t \geq t' \text{ and } t' + h_l^{l'} < t'',$$

$$y_t^{ll'a} \geq (t' - t) \cdot \left(x_{t'}^{l} - \sum_{t'''=t+1}^{t'-1} x_{t'''}^{l} \right) + \left(t'' - \left(t' + h_l^{l'} \right) \right) \cdot \left(x_{t''}^{l'} - \sum_{t'''=t'+h_l^{l'}+1}^{t''-1} x_{t'''}^{l'} \right) - M_4 \cdot \left(1 - x_{t'}^{l} + \sum_{t'''=t+1}^{t'-1} x_{t'''}^{l} \right)$$

$$\forall l, \forall l' \in L : l \neq l',$$

$$\forall t, \forall t', \forall t'' \in T : t < t' \text{ and } t' + h_l^{l'} < t'',$$

Extra Constraints II



Extra Constraints III

$$y_t^{ll'} \geq y_t^{ll'b} - z_t^{ll'} \cdot M_2 \qquad \forall l, \forall l' \in L : l \neq l', \forall t \in T,$$
$$y_t^{ll'} \geq y_t^{ll'a} - \left(1 - z_t^{ll'}\right) \cdot M_2 \qquad \forall l, \forall l' \in L : l \neq l', \forall t \in T,$$

Constraints to add

- Beginning and the end of horizon, when no connections are possible

Conclusions

- New planning phase, based on the demand
- User cost rather than demand to capture (no need for discrete choice model)
- Can handle bot non- and cyclic timetables
- Connections are demand imposed

Future Work

- Methodology design (cyclic is tighter than the non-)
- Actually solving the problem
- Analysis of the general results
- Analysis of the connections

Thank you for your attention.