Estimating exponential scheduling preferences

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Extended abstract

Choice of departure time is a travel choice dimension that transportation planners often need to forecast in appraisal. A traveller may shift departure time in response to changes in expected travel time or travel time variability (TTV) or in response to time-differentiated congestion pricing. The direction and size of such shifts depend on the traveller's scheduling preferences, i.e. his preferences for travelling and being at the origin and destination at different times of day (Vickrey, 1973). Moreover, the traveller's response to and economic value of TTV can be derived from the underlying scheduling preferences (Noland and Small, 1995, Bates et al., 2001, Fosgerau and Karlström, 2010).

The scheduling preferences can be formally represented as time-dependent rates of utility derived at different locations. Assuming that the travellers are rational and choose departure time by maximising expected total utility over the day, their departure times are conditional on rates of utility derived at these locations. For forecasting and economic evaluation of planning alternatives, it is desirable to have simple forms of utility rates with few parameters. Several forms of the utility rates have been used in theoretical and empirical research, in particular a step function (the popular α-β-γ model from Small, 1982) and an affine function (Fosgerau and Engelson, 2011). Recently, Engelson and Fosgerau (2011) showed that scheduling preferences represented as a constant utility rate at the origin and an exponential utility rate at the destination imply valuations of TTV proportional to the cumulant generating function of the travel time, which is uniquely convenient because the expected generalised travel cost for a route is then obtained as a sum of the expected generalised travel costs of the used links, if independence of the link travel times is assumed. The purpose of this paper is to empirically estimate the exponential form of utility rates and to compare it both to a more general form and to conventional forms as regards conformity to collected data on travellers' departure time choice.

The assumption underlying the scheduling approach is that the traveller rationally maximises her total utility obtained during a period of time. The total utility depends on time of departure from the origin and time of arrival to the destination. The total utility is usually assumed to be separable, i.e. equal to the sum of utility obtained at the origin until the departure time, and utility acquired at the destination since the arrival time. To facilitate the analysis the utilities are further assumed to be absolutely continuous functions of time. This implies that the departure time choice problem can be formulated in terms of minimizing the sum of two integrals with integrands representing the marginal utility of time at origin (H) and at destination (W) relative to the time spent travelling. Since the utility is derived at variable rates, H and W are functions of time. This formulation was first proposed by Vickrey (1973) and further developed by Tseng and Verhoef (2008). Figure 1 provides an illustration. In the Figure, t_d indicates departure time, and T travel time. If travel time is known with certainty, the optimal departure time satisfies $H(t_d)=W(t_d+T)$.

The popular α-β-γ scheduling formulation proposed by Vickrey (1969) and Small (1982) is a special case of the model suggested by Vickrey (1973), where H is constant and W is a piecewise constant function with two fixed utility rates for time spent at the destination before and after the Preferred Arrival Time (PAT). For the case when the travel time is random, Noland and Small (1995) suggested using expected utility theory to derive the reduced form of expected travel time cost that includes the cost of TTV. For the α-β-γ formulation of scheduling preferences and exponential or uniform distribution of travel time, Noland and Small (1995) showed that the cost of

TTV is proportional to the scale of travel time distribution. Fosgerau and Karlström (2010) generalised this result to a general but fixed shape of distribution of travel time. Fosgerau and Engelson (2011) showed that affine H and W functions imply that the disutility of TTV is proportional to variance, with a coefficient independent on the distribution shape. Engelson (2011) derived the explicit formulation for the valuation of TTV when both functions H and W are grade two polynomials or exponential functions. These are probably the most general functions for H and W that still have a closed form expression for the reduced-form measure of TTV.

In transport network applications, TTV and other components of the generalised travel costs are usually measured on links, while the travellers face the travel cost on the trip level. In order to convert generalised travel cost from link to trip level, network assignment algorithms assume additive travel costs and evaluate the trip cost as a sum of the travel costs along the links constituting the trip path. Hence, to be able to add the cost arising from TTV for each link to the trip cost, the measure of TTV should be additive.

Engelson and Fosgerau (2011) showed that the only form of scheduling preferences providing additive travel costs when link travel times are arbitrarily distributed independent random variables is a constant H and an exponential or affine W. The constant-exponential formulation implies that the reduced-form disutility of TTV is proportional to the cumulant generating function $\ln(Ee^{\eta T})$ where T is the random travel time and E is the expectation. As mentioned, this measure is additive over independent links. The constant-affine utility rates yield the reduced-form disutility of TTV proportional to the variance of travel time, which is also additive.

Most empirical studies estimating scheduling models have used the α-β-γ specification, and no previous study has estimated the constant-exponential scheduling model implying the reduced form disutility of travel time that possesses the attractive additivity property. The purpose of this paper is to explore how well these scheduling preferences explain behaviour, compared to other possible scheduling models, and whether empirical estimation of the more complex exponential scheduling preferences is feasible.

We use data from a stated preference survey conducted among car drivers commuting to work in the morning in central Stockholm. The survey contains observations of choices between car and public transport travel alternatives, which differ in terms of departure time, monetary cost, and the distribution of travel time. We develop a discrete choice model to describe behaviour, based on a theoretical model that has an exponential-exponential specification of H and W and so is more general than the models with the additivity property. We adapt the model to take into account that the marginal utilities of travelling by car and public transport may be different, and consider how it should be normalised across individuals. The resulting discrete choice model is thus a combined model for mode and departure time choice. We use it to estimate the preference parameters of H and W and to compare the goodness-of-fit of the constant-exponential and constant-affine specifications to two benchmarks: The generalised exponential-exponential specification and the conventional α-β-γ specification.

As could be expected, exponential preferences are difficult to estimate: The estimated parameters of H and W have large standard errors, and some types of models exhibit severe convergence problems. However, the signs of the obtained parameter estimates are consistent with theory. We have no problems estimating the simpler constant-affine specification, where all parameter estimates have signs consistent with theory and low standard errors. Whether the models with the additivity property perform better or worse than the two benchmarks depends on i) whether we include a mode-specific constant and ii) whether we analyse the full sample or the subsamples of people having a fixed arrival time or fixed work time and their complements. As could be expected, the α-β-γ specification outperforms the other specifications when it comes to modelling commuters with fixed arrival times or fixed work times, while it performs worse than the other specifications when modelling people without a fixed arrival time or fixed work time.

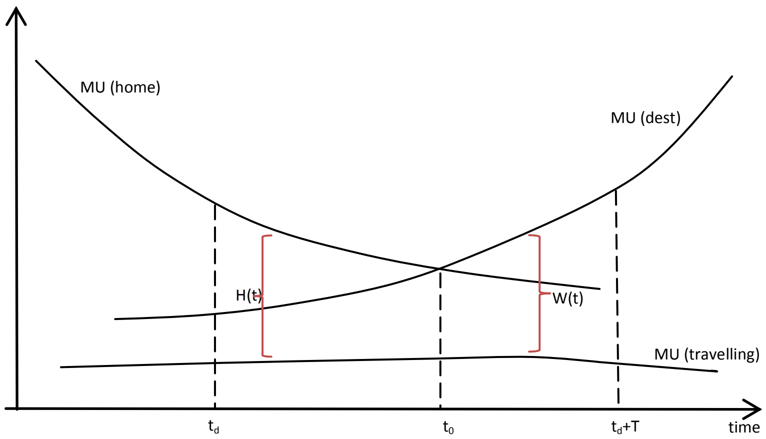

Figure 1: Illustration of scheduling preferences

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