# **Relationship between Mean and Variance of Travel Time in Networks**

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Understanding travel time variability has become extremely important, especially since it has been shown that drivers value travel time reliability somewhere between 50 and 80 percent as much as they value travel time [1, 2, 3]. Recent studies have developed empirical relationships between average travel time and travel time reliability which show an increasing relationship between mean travel time and travel time variance (a measure of the inverse of travel time variability) [4, 5, 6, 7]. However, more recent literature has found evidence that this relationship might be more complex. Figure 1 shows data from a single bottleneck in which the relationship between mean travel time and travel time variance follows a clockwise pattern – travel time variance is greater during the dissipation of congestion than during the onset of congestion for a given value of mean travel time. This relationship was also observed in the case of a single link [9].



Figure 1: Hysteresis observed on a 11 mile road section in Copenhagen [8]

Though the study by Fosgerau [8] provides a theoretical proof for this anti-clockwise hysteresis pattern at a single bottleneck, it is still not clear what relationship exists between the mean and variance of travel time on a particular route or even across an entire network. Some recent empirical data collected on travel times on the street network in Leeds, UK suggests the existence of anti-clockwise loops at a network level; see Figure 2 [9]. However, it is not known if this pattern is typical of urban street networks or if other types of relationships should be expected. The existence and prevalence of different patterns on urban networks is interesting and extremely relevant to model correctly to understand network reliability.



Figure 2: Hysteresis Observed in relationship between variance and mean travel time from Leeds in 2003 [Generated Using Data from 9]

Macroscopic models of urban traffic might yield some insight into the relationship between travel time and travel time reliability. One such model is the Macroscopic Fundamental Diagram (MFD) [10, 11], which describes the relationship between the average network flow and average network density. This relationship has been shown to exist in theory [11], simulation results [12], and empirical data [13] for networks in which drivers distribute themselves evenly across all links in a network. Recently, hysteresis has been observed in the MFD of urban networks [14] and freeway systems [15]. These hysteresis patterns can be attributed to natural instabilities that exist within traffic networks [12, 16]. This theoretical work also shows under what conditions the different hysteresis patterns shown in Figure 3 might arise on macroscopic traffic relationships. It was also found that a clockwise hysteresis is the most prevalent pattern for completely symmetric networks with uniform demand patterns [16].



Figure 3. Types of patterns observed in a Macroscopic Fundamental Diagram [16]

This theoretical work has also shown that networks follow a stable and higher flow-density path as vehicles initially enter the network. However at the onset of congestion the flow-density paths become unstable and less predictable [16]. Small perturbations in vehicle distributions across the network when it is highly congested or is recovering from congestion can lead to different flow-density paths on the MFD. Looking at these relationships over multiple days, day-to-day variations in supply or demand (i.e., fluctuations in the capacities of different links or intersections) can lead to stochastic variations in density in the system at a given time of day. While these variations in density should not change the path much during the stable loading period, they can lead to very different paths during congestion and recovery over different days.

The day-to-day variations in the MFD and its relationship to the hysteresis observed in variance of travel time against mean travel time is examined here using a micro-simulation of an urban traffic network. The network chosen was a part of the downtown Orlando network that was built, calibrated and validated in the VISSIM software [17]. The simulation was run 28 times to capture day-to-day in average travel time. Generalized speed, travel time, flows and density were estimated based on the vehicle trajectory data over a 120 second interval using Edie's generalized definitions [18].

The average network flow-density relationship during the loading and recovery phases are displayed in Figure 4 for three randomly chosen days of simulation runs that are indicative of the entire 28 day simulation dataset. As expected by Gayah and Daganzo [16], this relationship shows a clear clockwise pattern. Notice how the flow-density relationships overlap as vehicles initially enter the network. However, as the densities get higher, at the onset of congestion, the flow-density paths start to vary significantly over the three days. This chaotic network behavior near capacity can be attributed to quick fluctuations in flow at capacity values that might end up violating the steady state assumptions that are critical to the MFD [19]. Another reason for this chaotic behavior near maximum flow is the instability that arises as the network transitions from loading to recovery states [16]. During the recovery period, the paths followed on the three days are significantly different from each other due to this inherent instability.



Figure 4. The evolution of flow and density for three days

Loading paths on a flow-density diagram that are similar over many days indicate that travel speeds on the network will be similar during the loading period. However multiple values of flow observed at the onset of congestion and during recovery imply that over many days different network speeds will be observed at a given time. Therefore, a lower variance in travel speed (and, thus, travel time) is observed during loading as compared to during recovery. The evolution of the variance in average travel time per mile and mean travel time per mile is displayed in Figure 5. Note that, as

expected, variance in travel times is low during loading. When the average travel time becomes high (i.e., the network becomes congested), the variance in travel times starts to increase due to the various paths taken on the MFD. The variance in travel time remains high during recovery again due to the multiple recovery paths followed on the MFD.



Figure 5. Relationship between variance in travel time per mile and density

Though an anti-clockwise hysteresis pattern between mean travel time and travel time variance was observed here, there were slight deviations which can be attributed to quick fluctuations in flow at capacity levels that might have violated steady state conditions. Further research needs to be undertaken to study this phenomena more carefully. Additionally, while this study focused on day-to-day variations in travel time, there is a need to study variations in travel time across a network on a given day.

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