Stability of Equilibrium in Transportation Systems: A Dynamic Switching System Point of View

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Extended Abstract

In the last decades, the problem of studying the stability of equilibrium patterns in transportation systems has attracted the interest of many researchers. In the relevant studies, inter-period assignment problems have been modelled as dynamic processes that can be interpreted as non-linear discrete systems, in which the time discretisation coincides with the length of the periods considered in the assignment problem (see [1] and the references therein).

In such studies, the traffic demand is assumed to be constant, and the stability of the equilibrium is assumed to depend on the parameters characterising the demand and the supply models, such as the cost functions, the uncertainties of the user behaviours, the probability that users take different decisions in different periods, and so on.

Following [1], the above processes can be represented as

$$\begin{cases} \boldsymbol{c}_{k+1} = g[c(\boldsymbol{f}_k), \boldsymbol{c}_k] \\ \boldsymbol{f}_{k+1} = h[f(\boldsymbol{c}_k), \boldsymbol{f}_k] \end{cases} \tag{1}$$

where $\boldsymbol{c}_k$ and $\boldsymbol{f}_k$ are the forecasted costs and the flows on the network, $c(\boldsymbol{f}_k)$ and $f(\boldsymbol{c}_k)$ are the (non-linear) functions giving the costs of arcs and giving the flows in the network. Finally, $g(\cdot)$ and $h(\cdot)$ are the relations providing the costs and the flows in the period $k+1$ as functions of those in the period k.

In this framework, a hypothesis considered in such models consists of assuming that all the periods have the same characteristics, that is, the same mobility demand, the same classes of users, and so on.

Actually, such an assumption is realistic when long periods are considered (days, weeks, or months), in which the natural variations of both the demand and the user characteristics are averaged.

In other cases, periods with the same characteristics are not consecutive in the time domain, whereas consecutive periods have different characteristics. It is the case of rush hour periods. For instance, all the

morning peak hours show the same characteristics, but are separated by periods with different parameters, as depicted in Figure 1.

Figure 1: Sequence of periods: each colour represents a period with fixed characteristics.

This class of systems can be considered, in this framework, to be a subclass of the more general time-variant systems in which the above parameters assume the same constant values, periodically.

With the same formalism of the one introduced in Eq. 1, such processes can be modelled as

$$\begin{cases} \boldsymbol{c}_{k+1} = g[c(\boldsymbol{f}_k, \boldsymbol{\zeta}), \boldsymbol{c}_k] \\ \boldsymbol{f}_{k+1} = h[f(\boldsymbol{c}_k, \boldsymbol{\zeta}), \boldsymbol{f}_k] \end{cases} \tag{2}$$

being $\boldsymbol{\zeta}$ the set of non-constant parameters characterising the different periods.

The aim of this paper is to understand the behaviour of the assignment equilibrium in the more general framework of switching systems. In doing so, it is worth noting that, in general, although all the single dynamic processes are characterised by stable equilibrium, the whole system is not necessarily globally stable (see [2] for a basic introduction on switching systems and the relevant properties).

Then, in details, in the present paper some simulation analysis will be performed with the aim of showing different possible system configurations, discussing their properties and, when possible, identify conditions that guarantee the complete stability of the system.

References

[1] Cantarella, G.E., P. Velonà. *Stability of equilibrium patterns in a transportation network*. In Proc. AET European Transport Conference, 2003.

[2] Liberzon, D., A.S. Morse. *Basic problems in stability and design of switched systems*. In Control Systems, IEEE , vol.19, no.5, pp.59-70, Oct 1999.