Capturing dependency among link boundaries in a stochastic network loading model

Carolina Osorio

Department of Civil & Environmental Engineering,

MIT Massachusetts Institute of Technology, Cambridge, MA 02139, USA

Email: osorioc@mit.edu (corresponding author)

Gunnar Flötteröd

Department of Transport Science,

KTH Royal Institute of Technology, 11428 Stockholm, Sweden

Email: gunnar.floetteroed@abe.kth.se

For presentation only. The full article has been submitted for possible journal publication.

1 Context

This article presents an analytical stochastic (i.e. probabilistic) traffic flow model, which is derived from the widely accepted kinematic wave model (KWM; Lighthill and Witham (1955); Richards (1956)). Both the KWM's original link model and its more recently developed node models (e.g. Daganzo (1995); Lebacque (1996); Lebacque and Khoshyaran (2005); Tampere et al. (2011); Flötteröd and Rohde (2011); Corthout et al. (2012)) are deterministic. They describe space/time average conditions but do not account for higher-order distributional information.

There has been a recent interest in the development of analytical stochastic link models. Most studies have considered stochastic cell-transmission models (CTMs; Boel and Mihaylova; 2006; Sumalee et al.; 2011; Jabari and Liu; 2012). While the CTM constitutes a converging numerical solution scheme for the KWM, it is left unclear to what extent a stochastic CTM converges towards a stochastic KWM.

Osorio et al. (2011) recently proposed a stochastic formulation of the link-transmission model of Yperman et al. (2007), which is an operational instance of Newell's simplified theory of kinematic waves (Newell; 1993). The present article adds important dependency structure to this previously developed model.

2 Methodology

Yperman et al. (2007) phrase Newell's simplified KWM (Newell; 1993) within the sending/receiving function framework of Daganzo (1994) and of Lebacque (1996). This framework postulates that, at any interface within the network, the instantaneously transmitted flow is the minimum of an upstream sending function and a downstream receiving function, reflecting the KWM's principle of local flow maximization (Ansorge; 1990).

This model can be equivalently rephrased based on a system of two finite capacity queues, where the so-called *upstream queue* (UQ) keeps track of the upstream boundary conditions of the link, and the so-called *downstream queue* (DQ) keeps track of the downstream boundary conditions of the link.

Both queues can hold at most ℓ vehicles. The UQ is defined such that the total amount of flow being allowed to enter the link because of possible congestion spillback is equal to the available space in the UQ. The DQ is defined such that the total amount of flow being allowed to leave the link because of a possibly limited number of vehicles in the link is equal to the available vehicles in the DQ.

The stochastic link model of Osorio et al. (2011) results from a stochastic modeling of UQ and DQ, relying on finite capacity queueing theory, where the dynamic evolution of the distribution of the number of vehicles in either queue is tracked through time. The dynamics of these queues are guided by time-dependent arrival and service rates as well as the probabilities of the queues being perfectly empty (i.e. being unable to send more flow) or perfectly full (i.e. being unable to receive more flow). The model is a simplification in the sense that the distributions of UQ and DQ are modeled independently, although both represent information about the congestion status of the link. This article overcomes this confinement.

The approach is to add only two additional dimensions to the (UQ,DQ) state space, which are called the *lagged inflow queue* (LI) and the *lagged outflow queue* (LO). The LI queue captures, at an aggregate level, the distribution of all link entries that have not yet reached the DQ (i.e. vehicles currently traveling forwards inside of the link). Symmetrically, the LO queue captures, at an aggregate level, the distribution of all link exits that have not yet been removed from the UQ

Time interval:	[0,999]	[1000,1999]	[2000,2999]
Profile 131	0.1	0.3	0.1
Profile 151	0.1	0.5	0.1
Profile 353	0.3	0.5	0.3

Table 1: Arrival rate profiles in veh/s

(i.e. “spaces” currently traveling backwards inside of the link).

The full article details the mathematical development of the new model: The state space is reduced to three dimensions by applying a mass conservation constraint. The rate at which LI discharges vehicles into DQ and the rate at which LO discharges “spaces” into UQ are modeled state dependently; this is key to the precision of our approach. The only simplifying assumption made is to neglect the stochastic temporal dependence between link inflows and outflows at different time steps. The experiments given in the following section demonstrate the very minor effect of this approximation.

3 Results

A single-lane link is considered. Nine experiments are conducted, combining three different arrival rate profiles and three different link lengths (and, hence, different space capacities ℓ). Each experiment starts with an initially empty link at time zero and runs for 3000 one-second time steps.

The link has a fixed downstream bottleneck with a service rate of $0.3\, veh/s$. The arrival profiles are displayed in Table 1. Profile 131 (resp. 151) corresponds to a step-change from undercritical to marginally critical (resp. overcritical) conditions and back. Profile 353 corresponds to a step-change from marginally critical to overcritical conditions and back.

The considered space capacities are $\ell = 10, 20, 30$, resulting in link lengths $L = 50, 100, 150\, m$. Table 2 labels the experiments for the resulting nine parameter combinations as concatenations of the respective arrival profile and space capacity.

Particular attention is paid to the stochastic dependency between up- and downstream conditions within the link, corresponding to dependency between UQ and DQ. For this, the results of the proposed analytical model are compared to empirical distributions obtained from 10^6 replications of an event-based microsimulation. Since the microsimulation perfectly captures all dependencies, it serves as a benchmark for the analytical model.

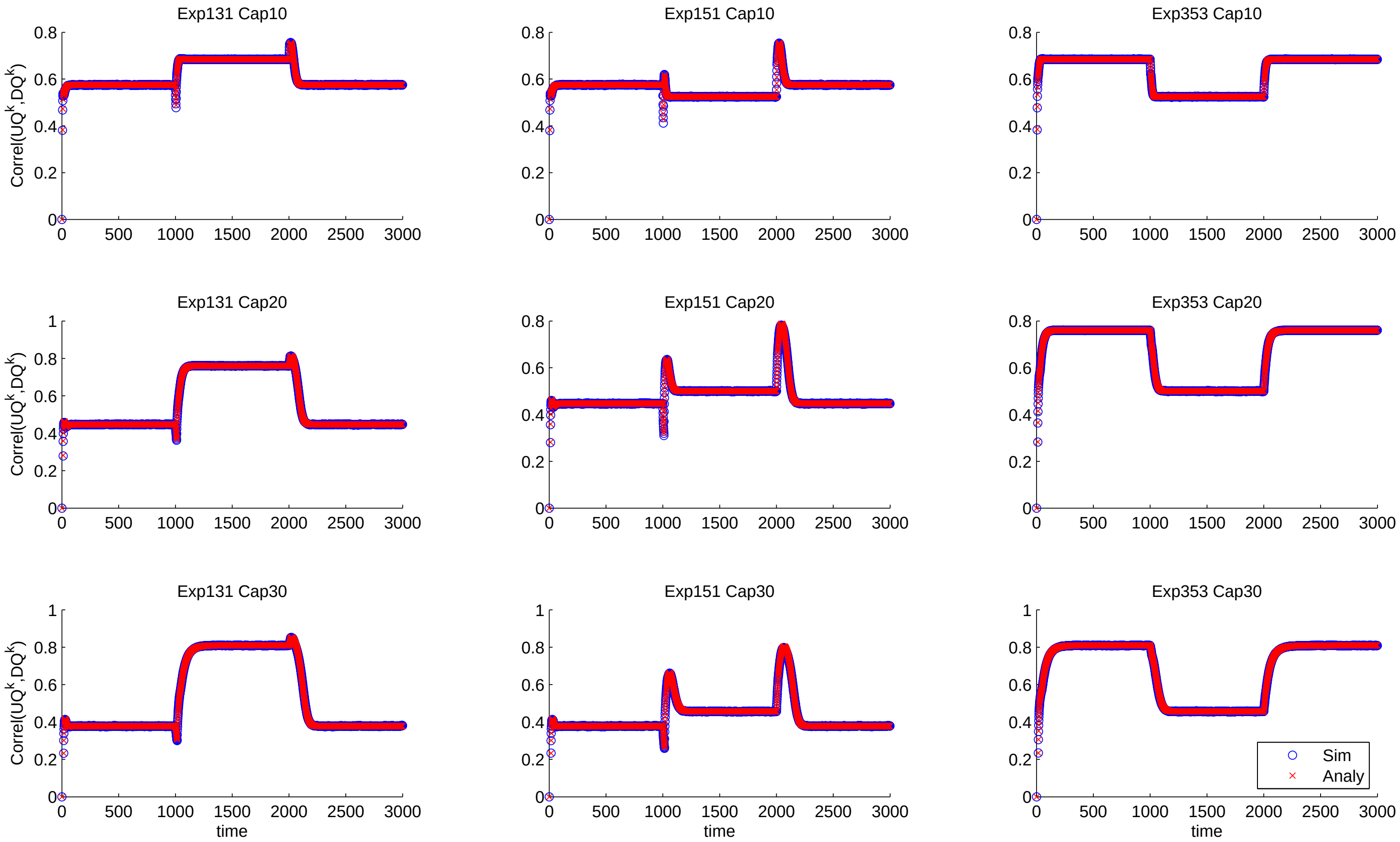

Figure 1: Correlation between UQ and DQ over time

arrival profile ℓ	131	151	353
10	“Exp 131 Cap 10”	“Exp 151 Cap 10”	“Exp 353 Cap 10”
20	“Exp 131 Cap 20”	“Exp 151 Cap 20”	“Exp 353 Cap 20”
30	“Exp 131 Cap 30”	“Exp 151 Cap 30”	“Exp 353 Cap 30”

Table 2: Experiments

Figure 1 shows for all nine experiments the evolution of the correlation between UQ and DQ over time. The red crosses represent results from the analytical model, and the blue circles represent results from the event-based simulation. The deviations between simulation and analytical model are visually negligible, indicating an excellent overall fit.

Figure 2 shows the joint distribution of LI, DQ and LO for different arrival profiles and at particularly interesting points in time (shortly after the jump-changes in the arrival profile). Only results for $\ell = 10$ are shown; the figures for $\ell = 20, 30$ do not reveal additional information. The horizontal axis represents the indices of the different states, and the vertical axis represents their probabilities. All feasible states of (LI, DQ, LO) are represented. One observes an almost perfect match between simulated and analytical results, across all experiments.

These experiments demonstrate an extremely high precision of the analytical model when approximating an event-based microsimulation of the exact stochastic KWM model for a homogeneous link. It hence is possible to analytically capture full link state distributions in consistency with a stochastic KWM.

References

Ansorge, R. (1990). What does the entropy condition mean in traffic flow theory, *Transportation Research Part B* **24**(2): 133–143.

Boel, R. and Mihaylova, L. (2006). A compositional stochastic model for real time freeway traffic simulation, *Transportation Research Part B: Methodological* **40**: 319–334.

Corthout, R., Flötteröd, G., Viti, F. and Tampere, C. (2012). Non-unique flows in macroscopic first-order intersection models, *Transportation Research Part B* **46**(3): 343–359.

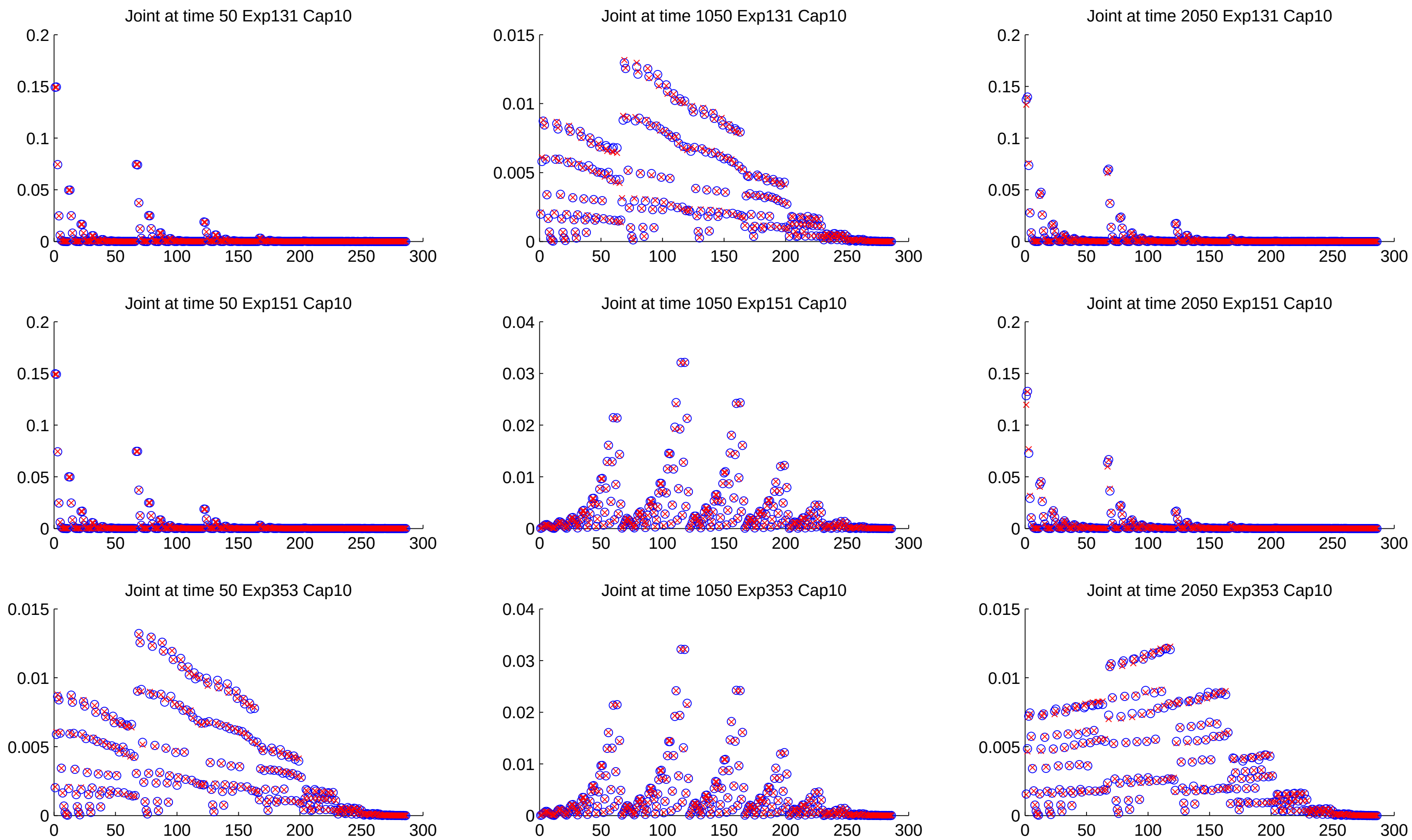

Figure 2: Joint distribution of UQ and DQ

Daganzo, C. (1994). The cell transmission model: a dynamic representation of highway traffic consistent with the hydrodynamic theory, *Transportation Research Part B* **28**(4): 269–287.

Daganzo, C. (1995). A finite difference approximation of the kinematic wave model of traffic flow, *Transportation Research Part B* **29**(4): 261–276.

Flötteröd, G. and Rohde, J. (2011). Operational macroscopic modeling of complex urban intersections, *Transportation Research Part B* **45**(6): 903–922.

Jabari, S. and Liu, H. (2012). A stochastic model of traffic flow: theoretical foundations, *Transportation Research Part B* **46**(1): 156–174.

Lebacque, J. (1996). The Godunov scheme and what it means for first order traffic flow models, *in* J.-B. Lesort (ed.), *Proceedings of the 13th International Symposium on Transportation and Traffic Theory*, Pergamon, Lyon, France.

Lebacque, J. and Khoshyaran, M. (2005). First–order macroscopic traffic flow models: intersection modeling, network modeling, *in* H. Mahmassani (ed.), *Proceedings of the 16th International Symposium on Transportation and Traffic Theory*, Elsevier, Maryland, USA, pp. 365–386.

Lighthill, M. and Witham, J. (1955). On kinematic waves II. a theory of traffic flow on long crowded roads, *Proceedings of the Royal Society A* **229**: 317–345.

Newell, G. (1993). A simplified theory of kinematic waves in highway traffic, part I: general theory, *Transportation Research Part B* **27**(4): 281–287.

Osorio, C., Flötteröd, G. and Bierlaire, M. (2011). Dynamic network loading: a stochastic differentiable model that derives link state distributions, *Transportation Research Part B* **45**(9): 1410–1423.

Richards, P. (1956). Shock waves on highways, *Operations Research* **4**: 42–51.

Sumalee, A., Zhong, R. X., Pan, T. L. and Szeto, W. Y. (2011). Stochastic cell transmission model (SCTM): a stochastic dynamic traffic model for traffic state surveillance and assignment, *Transportation Research Part B* **45**(3): 507–533.

Tampere, C., Corthout, R., Cattrysse, D. and Immers, L. (2011). A generic class of first order node models for dynamic macroscopic simulations of traffic flows, *Transportation Research Part B* **45**(1): 289–309.

Yperman, I., Tampere, C. and Immers, B. (2007). A kinematic wave dynamic network loading model including intersection delays, *Proceedings of the 86. Annual Meeting of the Transportation Research Board*, Washington, DC, USA.