ANALYSING ROUTE CHOICE DECISIONS ON METRO NETWORKS

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ABSTRACT

Understanding travellers' behaviour is key element in transportation planning. This article presents a route choice model for metro networks that considers different time components as well as variables related to the transferring experience, train crowding, and network topology. The route choice model is applied to the London Underground and Santiago Metro networks, to make a comparison of the decision making process of the users on both cities. As all the variables are statistically significant, it is possible to affirm that public transport users take into account a wide variety of elements when choosing routes. While in London the travellers prefer to spend time walking, in Santiago is preferable to spend time waiting. Santiago Metro users are more willing to travel in crowded trains than London Underground users. Both user groups have a similar dispreference to transfers after controlling for the time spent on transfer, but different attitudes to ascending and descending transfers. Topological factors presented on a distorted Metro map are more important than actual topology to passengers' route choice decisions.

KEYWORDS: decision making; metro networks, network topology, route choice.

1. INTRODUCTION

Understanding how public transport users make their travel decisions and being able to predict their behaviour is essential in transportation planning. The purpose of this study is to advance our understanding of the behaviour of public transport users when choosing a route in a Metro network and to quantify the impacts of the underlying explanatory variables that influence their decisions.

The route choice variables normally included in traditional route choice models limit to some basic service levels attributes of the alternative routes, such as travel time and fare (Ortúzar and Willumsen, 2011). However, other variables, related to both the level of service and the travellers' perceptions, influence the user's route choice process but are generally ignore in traditional modelling.

This study presents a route choice analysis on Metro networks, incorporating variables related to the different times involved (travel, waiting and walking times), trains and stations usage, transfer environment and network topology. It also conducts an empirical analysis to compare route choice decision making on the Santiago Metro system and the London Underground system, using the same modelling approach and specification. Even though behaviour comparisons are common on literature (mainly between results such as values of time and demand elasticities), these are generally made based on models with different specification and context.

The rest of the paper is organized as follows. In section 2 we address the modelling approach and present the modelling variables considered. In section 3 we present and discuss the route choice results for Santiago Metro and London Underground. Finally, in section 4 we present our main conclusions.

2. ROUTE CHOICE MODELLING

Understanding how people make their travel decisions regarding route choices, and being able to predict their behaviour, is essential in transportation planning. Our objective is to identify and quantify the different aspects of travelling that are taken into account by public transport users. For this, we specify and estimate (through a Maximum Likelihood approach) Discrete Choice Models (Multinomial Logit, MNL) to study travellers' preferences.

2.1 Metro Networks for Analysis

We conduct an empirical analysis to compare route choice decision making on the Santiago Metro system (base on the work of Raveau *et al.*, 2011) and the London Underground system (based on the work of Guo, 2011). Both systems have high ridership (3 million daily trips in London Underground and 2.3 million daily trips in Santiago Metro) but differ in size (London Underground has 402 Km of length, while Santiago Metro only 103 Km) and complexity. Based on their characteristics, the results from the comparison could be generalized to public transport systems in general. Also, as both systems conduct similar route choice surveys for planning purposes, we can specify and estimate the same utility function on both networks to compare the behaviour characteristics of the respective travellers.

The Santiago's trip database was obtained from an origin-destination survey conducted at Metro stations in October 2008. When the survey was conducted the network consisted of 5 Metro lines, 85 stations and 7 transfer stations. The database obtained from the survey consists of 28,961 individual trips along the day between the 1,365 different OD pairs (19% of the total OD pairs of the network) which have two or more alternatives routes to travel (i.e. those OD pairs where there is a route choice decision). A specific analysis of the peak-hours travels can be found in Raveau *et al*. (2011).

The London's trip database is based on data collected by Transport for London (TfL) from 1998 to 2005. During the period of analysis, the London Underground network had 11 lines, 255 stations and 72 transfer stations. The database consists of 16,300 individual trips along the day between 2,127 OD pairs (3% of the total OD pairs of the network) with two or more alternatives routes. The details of the database can be found in Guo and Wilson (2011).

2.2 Topology and Map Effects

A key element to be analyzed is the influence of the network's topology on the route choice, as travellers might tend to prefer alternatives that seem more direct between the origin and destination stations (aside from other variables such as travel time). To test this hypothesis we define an angular cost (Raveau *et al.*, 2011) to measure how direct/indirect a given route is.

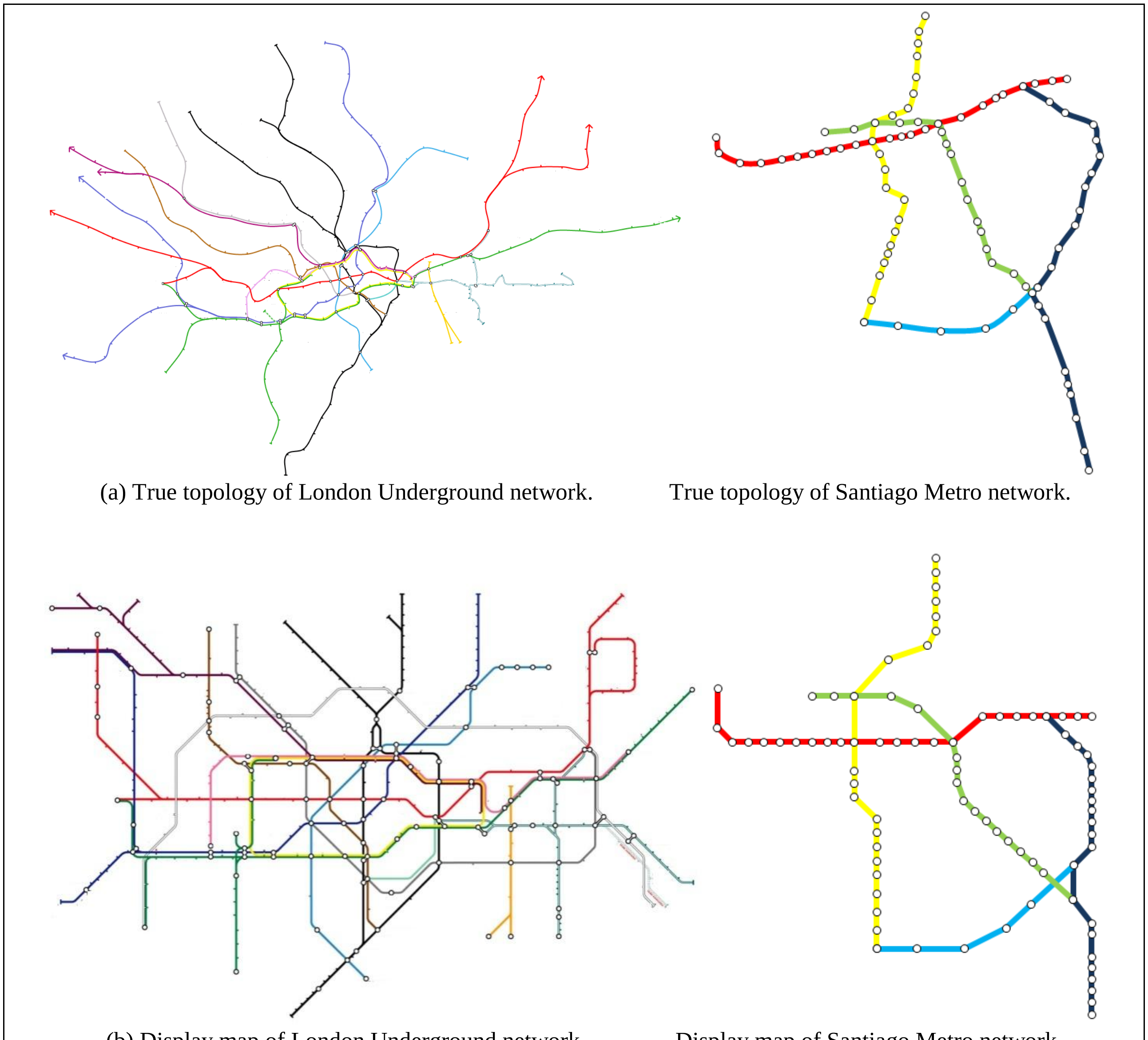
On most public transport systems in the world the network is presented to the users through display maps, where all the relevant information for travelling (different lines, stations, transfer points, etc) is included. To enhance the understanding and make the maps easier to read, the public transport network tends to be distorted in the maps (Ovenden, 2007). As public transport maps usually do not include service information (such as travel time or crowding levels), their distortion might affect the decisions made by the travellers.

Figure 1 shows the true geographic (panel *a*) and display map (panel *b*) depictions of the London underground and Santiago Metro networks. On both cases the map displayed in the stations and consulted (or at least seen) by users on a day-to-day basis contains distortions, both in the relative location of the stations and the distances between them. This way, users' angular and geometric perceptions may also diverge from the geographical reality, which may induce them to choose sub-optimal routes.

Although the Santiago Metro network has grown considerably over the last decade, and will continue growing, it is still far away of being as complex and dense as the London Underground network. Also, the distortion is less severe (the correlation between the true and map distances in Santiago Metro is 94%, while in London Underground is only 22%). This way, the topology and map impacts on travel decisions should be more significant in London.

2.3 Set of Alternative Routes

When dealing with probabilistic route choice models (such as a MNL model) it is fundamental to explicitly define an appropriate set $A(q)$ of available alternatives. This is not trivial, as there is no sure way of knowing which routes were considered by the travellers but were not chosen. For this, many approaches (both deterministic and stochastic) have been developed for constructing sets of alternatives routes, trying to replicate the mental selection process undertaken by travellers (for a complete review on these approaches see Prato, 2009).



(a) True topology of London Underground network. True topology of Santiago Metro network.

(b) Display map of London Underground network. Display map of Santiago Metro network.

Figure 1. London Underground and Santiago Metro networks topologies

For the Santiago Metro study case the set of alternative routes for each OD pair was generated based on the all the routes chosen by the travellers of that OD pair. This was possible due to the large ratio between trips and OD pairs (21.2 trips per OD pair, in average), so in every one of the 1,365 considered OD pair there were at least two routes used. This way, no methodological approach for generating alternative routes was necessary, and the set obtained comes directly from the observed traveller's behaviour.

On the London Underground study case the ratio between trips and OD pairs is lower (7.7 trips per OD pair, in average) and in most OD pairs there is only one chosen route. Therefore, a methodological approach for obtaining a set of available alternatives is needed. A labelling approach (Ben-Akiva *et al.*, 1984) was used to generate the set of available alternatives, by obtaining the shortest path for different definitions of "link costs" (e.g. different combinations of time, length and/or transfers). This way, a minimum number of two and up to six different routes were obtained for each of the 2,127 OD pairs.

The distribution of OD pairs and trips according to the number of available routes is presented in Table 1. As the London Underground network is denser, there are more alternatives routes for travelling than in the Santiago Metro network. It can be seen as well that in both cities the OD pairs distribution and the trips distribution is very similar: no particular OD structure (in terms of available alternatives) tends to concentrate more trips than another.

Table 1. Route availability statistics

	Santiago Metro				**London Underground**			
Available Routes	**OD pairs**		**Trips**		**OD pairs**		**Trips**	
2	1,322	(96.9%)	26,950	(93.1%)	1,161	(54.6%)	9,164	(56.2%)
3	37	(2.7%)	1,892	(6.5%)	694	(32.6%)	5,319	(32.6%)
4	6	(0.4%)	119	(0.4%)	202	(9.5%)	1,395	(8.6%)
5	0	(0.0%)	0	(0.0%)	58	(2.7%)	335	(2.1%)
6	0	(0.0%)	0	(0.0%)	12	(0.6%)	87	(0.5%)
Total	1,365	(100.0%)	28,961	(100.0%)	2,127	(100.0%)	16,300	(100.0%)

2.4 Model Description

In the literature there are multiple route choice models (mainly MNL models or extensions of it) based on users' socioeconomic characteristics and route attributes (Ramming, 2001; Prashker and Bekhor, 2004; Liu *et al.*, 2010). However, the attributes included in these models are all tangible and quite limited to travel time components, fare and transfers. This study presents a more complete route choice analysis on Metro networks, incorporating variables related to the different times involved (travel, waiting and walking times), trains and stations usage, transfer environment and network topology. The additional variables, included to improve the ability to explain and predict route choice decisions, are discussed below. It's important to take into account that no fare variable was included because all alternative routes between a given OD pair have the same monetary cost.

The most traditional and important variable used to explain route choice behaviour is the travel time. Users tend to look for the fastest way of getting from their origin to their destination, and the travel time is the main criterion to discard unattractive (i.e. slow) alternatives. We consider three different time components: the **in-vehicle time**, the **waiting time** at the origin station and all subsequent transfer stations, and the **walking time** when transferring (as the origins and destinations are fixed, we obviate access and egress times). These different time components are considered separately to address their different perception and importance in the travellers' decision making process.

Regarding the transferring experience, we firstly consider the total **number of transfers** of each alternative route; as the actual transferring time is captured by the walking and waiting time variables, this variable solely captures the displeasure of having to transfer. To further understand the transferring valuation, we differentiate between possible types of transfers. In terms of stations layout, the transfers can either be made between **ascending levels** (i.e. going up), **even levels** (usually walking across the platform) and **descending levels** (i.e. going down). In terms of stations infrastructure, the transfers can be **assisted** (made completely using escalator and/or lift), **semi-assisted** (made partially using escalator and/or lift and partially on foot), and **non-assisted** (made completely on foot).

To address the level of comfort and crowding experienced by the public transport users during their trip, the **mean occupancy** along the route was included in the models. This variable is defined as the distance-weighted ratio between passengers load and train capacity in peak hour (even though not all trips are made in peak hour, this specification presented the best goodness-of-fit). By definition the rate can vary between 0 (trains travelling empty along all arcs of the route) and 1 (trains travelling fully loaded along all arcs of the route).

Two additional variables related to train usage and extreme crowding levels were included in the model. We distinguish those transfer stations where there is the **possibility of getting a seat**, depending on the occupancy of the trains leaving those stations. On London Underground this happens when the occupancy is 20% or less, while on Santiago Metro this happens when the occupancy is 15% or less (these percentages represent the percentage of the capacity that corresponds to seats). On the other hand, we distinguish those transfers where there is the **possibility of not boarding** the first train, and having to wait for the next train. We observe that on London Underground this happens when the occupancy is 70% or more, while on Santiago Metro this happens when the occupancy is 85% or more. Based on these thresholds, it may seem that Santiago Metro's users are more willing to board crowded trains, instead of simply waiting for the next one.

To deal with the topology's effect on the route choices of the travellers, we include different topological factors. Following Guo (2011), we include the **distance** and **number of stations** between the origin and the destination along the different routes. Due to variable dwelling times and spacing between stations, these variables are not highly correlated with the travel time components and can be included in the models without problems. We also include an **angular cost** to measure how direct a certain route is. The angular cost is defined accordingly with Raveau *et al*. (2011) as shown by (2.5), where s represents a leg of the route, d_s is the distance of the leg s, and θ_s is the angle formed between the destination station, the first station of leg s and the last station of leg s.

$$\textit{Angular Cost} = \sum_{s} d_s \cdot \sin\left(\frac{\theta_s}{2}\right) \tag{1}$$

Finally, also related to the topology's effect, two additional variables were considered. We say that a route **turns back** if it has a transfer station (or the destination, if it is the last leg) that is closer to the origin than the transfer station immediately before it. We say that a route **turns away** if it has a transfer station that is further from the destination than the transfer station (or the origin, if it is the first leg) immediately before it. These variables are an application of Dial's definition of reasonable routes (Dial, 1971).

3. RESULTS AND ANALYSIS

Based on the data gathered on route choice in the London Underground and Santiago networks, MNL models were estimated to understand the travellers' decision making process. While different models that address path correlation due to overlapping have been widely applied to route choice in road networks (Prato, 2009), we do not consider correlation among alternatives. Paths in public transport networks (especially in underground systems) usually involve only a few links and, in most cases, no more than two transfer stations. This way, in public transport networks the correlation across nodes (i.e. transfer stations) is likely to be more influential than the correlation across links. Because of this, we specify the transfer environment in great detail to control, to some extent, the possible correlation over transfer stations.

When specifying the model, all topological variables were defined based on the display maps of the networks, rather than based on their true topology, as they resulted on better results (Raveau *et al*., 2011). The estimated parameters, their *t*-values and goodness-of-fit indicators for the model are presented in Table 2.

Table 2. Parameters estimates

Attribute	**London Underground**		**Santiago Metro**	
	Parameter	***t*-value**	**Parameter**	***t*-value**
Travel Time	- 0.188	- 16.02	- 0.095	- 19.57
Waiting Time	- 0.311	- 7.39	- 0.139	- 5.07
Walking Time	- 0.216	- 6.14	- 0.155	- 8.23
Number of Transfers	- 1.240	- 4.37	- 0.632	- 4.06
Ascending Transfers	- 0.138	- 2.57	- 0.323	- 2.73
Even Transfers	0.513	3.53	n. a. (2)	n. a.
Descending Transfers	0.000 (1)	n. a.	0.000 (1)	n. a.
Assisted Transfers	0.000 (1)	n. a.	0.000 (1)	n. a.
Semi-Assisted Transfers	- 0.328	- 6.83	n. a. (2)	n. a.
Non-Assisted Transfers	- 0.541	- 6.79	- 0.262	- 6.23
Mean Occupancy	- 2.911	- 3.48	- 1.018	- 5.60
Getting a Seat	0.098	2.08	0.092	3.41
Not Boarding	- 0.430	- 6.06	- 0.380	- 2.97
Angular Cost	- 0.065	- 5.87	- 0.024	- 5.48
Map Distance	- 0.358	- 5.76	- 0.274	- 5.69
Number of Stations	- 0.316	- 5.52	- 0.147	- 3.10
Turning Back	- 0.725	- 8.12	- 0.141	- 9.76
Turning Away	- 0.968	- 8.00	- 0.226	- 7.11
Sample Size	16,300		28,961	
Log-Likelihood	- 6,412		- 12,927	
Corrected ρ^2	0.566		0.382	

(1): These parameters where fixed as base categories for the layout and infrastructure variables.
(2): These kinds of transfers do not exist in Santiago Metro.

All the parameters presented in Table 2 have the expected signs and are statistically significant at 95% confidence level. For the station layout, the descending transfer parameter was fixed in zero as the base category; for the station infrastructure, the assisted transfer was defined as base category. It can be seen that an even transfer is preferred over changing levels; if a change in levels must be made, users prefer to descend rather than ascend, as ascending seems to be mentally associated with a greater effort. In terms of infrastructure, as the grade of assistance increases, the transfer experience improves.

The parameters obtained for the London Underground and Santiago Metro are not directly comparable between each other, as the MNL models have different scales from their Gumbel errors variances (Train, 2009), but marginal rates of substitution can be derived from these parameters and compared for both networks. These marginal rates are presented in Table 3. For this direct comparison, the angular cost and the map distance are excluded as they don't have a measurement scale.

Table 3. Marginal rates of substitution

Attribute	London Underground	Santiago Metro
1 min of waiting time	1.65 min in-vehicle	1.46 min in-vehicle
1 min of walking time	1.15 min in-vehicle	1.62 min in-vehicle
1 base transfer (2)	6.60 min in-vehicle	6.63 min in-vehicle
1 ascending transfer	0.73 min in-vehicle	3.39 min in-vehicle
1 even transfer	2.73 min in-vehicle (1)	n.a. (3)
1 non-assisted transfer	2.88 min in-vehicle	2.75 min in-vehicle
1 semi-assisted transfer	1.74 min in-vehicle	n.a. (3)
1 % of occupancy	0.15 min in-vehicle	0.11 min in-vehicle
Possibility of seating	0.52 min in-vehicle (1)	0.97 min in-vehicle (1)
Possibility of not boarding	2.29 min in-vehicle	3.99 min in-vehicle
1 station along the way	1.68 min in-vehicle	1.54 min in-vehicle
Turning back	3.86 min in-vehicle	1.48 min in-vehicle
Turning away	5.15 min in-vehicle	2.37 min in-vehicle

(1): Absolute value, these variables represent a gain in utility.
(2): Based on the model specification, this base transfer corresponds to an assisted descending transfer where there is no possibility of either getting a seat or not being able to board.
(3): These kinds of transfers do not exist in Santiago Metro.

On both cities the waiting and walking times are more valuable than the in-vehicle travel time, but while in London the value of the waiting time is greater than the value of the walking time, in Santiago the relationship is the opposite. When choosing transferring alternatives, Santiago Metro users will be more willing to wait and London Underground users will be more willing to walk, in part because Santiago Metro has fewer transfer options and their environment is generally simpler, while London Underground presents many transfer alternatives and the connection between platforms could be longer and more complex due to the incremental addition of lines over the time.

Even though in both cities the valuation of a base transfer (this is, an assisted descending transfer where there is no possibility of either getting a seat or not being able to board) might seem very similar, it's necessary to take into account the marginal valuations of the layout and infrastructure variables. As in London Underground the most used parts of the network are underground, a descending change of levels will tend to have associated a corresponding ascending change of levels; this way the valuation of ascending transfers is lower in London Underground than in Santiago Metro. For example, while London Underground is completely underground in the city centre of London, in the city centre of Santiago almost 30% of the Santiago Metro network is overground.

To further analyze the users' valuation of transfer experiences, Table 4 presents the marginal rates of substitution for all possible transfer types. There is a high variability in London transfer's valuations (between 3.35 and 12.49 minutes of in-vehicle travel time) and Santiago transfer's valuations (between 5.67 and 16.76 minutes of in-vehicle travel time). This way, the necessity of distinguishing by station layout, infrastructure and occupancy is clear; otherwise the heterogeneity in preferences would not be captured. Weighting each valuation for the number of transfer of each type made on a regular day, the average transfer penalty in London Underground is 7.7 minutes and in Santiago Metro is 11.3 minutes. It seems that, in general, London Underground users are more willing to transfer than Santiago Metro users. As the London Underground network is bigger and denser, there are more transfer possibilities and Londoners are more used to transferring: 69% of the trips in London Underground involve at least one transfer, while in Santiago Metro is the 47% of the trips.

Table 4. Transferring valuations

London Underground				
Characteristics		**Getting a Seat**	**Intermediate**	**Not Boarding**
Ascending	**Assisted**	6.81 min	7.33 min	9.62 min
Ascending	**Semi-assisted**	8.56 min	9.07 min	11.36 min
Ascending	**Non-assisted**	9.69 min	10.21 min	12.49 min
Even		3.35 min	3.87 min	6.15 min
Descending	**Assisted**	6.08 min	6.60 min	8.88 min
Descending	**Semi-assisted**	7.82 min	8.34 min	10.63 min
Descending	**Non-assisted**	8.95 min	9.47 min	11.76 min
Santiago Metro				
Characteristics		**Getting a Seat**	**Intermediate**	**Not Boarding**
Ascending	**Assisted**	9.05 min	10.02 min e	14.01 min
Ascending	**Non-assisted**	11.80 min	12.77 min	16.76 min
Descending	**Assisted**	5.67 min	6.63 min	10.62 min
Descending	**Non-assisted**	8.41 min	9.38 min	13.37 min

Also from Table 3 it can be seen that occupancy is more valuated in London, as the London Underground users are less willing to travel in crowded trains than Santiago Metro users. This also explains why Londoners are more sensitive towards not boarding a train due to crowding, consistent with the lower threshold obtained when defining the possibility of not boarding. As an example of this, TfL defines a "crowded train" as 4 passengers per m^2, while Santiago Metro defines a "crowded train" as 6 passengers per m^2. On the other hand, extreme occupancies (getting a seat or not being able to board) are more valuated in Santiago, probably because in London the occupancies are lower.

Avoiding one station along the way (this is, taking a route with one station less between the origin and the destination) is valuated similarly on both cities. To further understand this marginal rate, the mean travel time between two consecutive stations (considering dwelling time) in London Underground and Santiago Metro are 1.8 minutes and 1.6 minutes, respectively. The similarity between these values and the marginal rates obtained indicates that the public transport users have a good notion of the operation times of the networks.

Londoners are much more sensitive to taking routes that seem unreasonable in terms of their geometry, either because they turn back to the origin or turn away from the destination. This could be due to the high distortion of the London Underground map. Another possible explanation is that the London Underground network generates more unreasonable alternatives routes due to its complexity and density, which in reality are barely considered by the users.

4. CONCLUSIONS

Route choice modelling variables are traditionally limited to some tangible factors such as time and fare that, although relevant, fail to accommodate different aspects of traveller's behaviour. In this study we specify and estimate route choice models for metro networks that consider four different kinds of variables: travel time components, transfer experience characteristics, occupancy and comfort indicators, and network topology variables. All these variables result significant understanding for travellers' behaviour. This reassures the idea that public transport users take into account a wide variety of attributes when choosing routes.

An incomplete model specification can result on biased results, especially attributes valuations, which could have a great repercussion when valuating transportation projects (such as network expansions or operational changes). Also related to the specification, the topological variables have a greater explanatory power when specified based on the networks' display maps rather than their true topology. Although distortions in these display maps can induce route choice decisions that reduce utility for some users, they could also be exploited by planners to induce optimal route choices to make better use of public transport system capacity (Jankowski *et al.*, 2001). This is particularly relevant in London Underground, where the map distortion is extremely big (probably bigger than any other public transportation system in the world). Schematic maps can also be complemented with information regarding level of service, to enable a more informed decision by the travellers (Hochmair, 2009).

As the route choice model was estimated for the London Underground and Santiago Metro networks, a comparison between traveller's behaviour in both cities was possible. London Underground users seem to be more willing to transfer than Santiago Metro users, as they are more used to required transfers when travelling. When transferring in London, users will tend to prefer to spend more walking than waiting, while in Santiago users will tend to prefer to waiting time over walking time. In terms of occupancy and comfort, Londoners are less willing to travel in crowded trains and will tend to stop boarding when the trains reach a lower threshold than in Santiago; on the other hand, they care less about getting a seat than Santiago Metro users.

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REFERENCES

Ben-Akiva, M.E., Bergman, M.J., Daly, A.J. and Ramaswamy, R. (1984). Modeling inter-urban route choice behaviour. In Volmuller, J. and Hamerslag, R. (eds.), Proceedings of the 9th International Symposium on Transportation and Traffic Theory. VNU Science Press, Utrecht, The Netherlands, 299-330.

Dial, R. B. (1971). A probabilistic multipath traffic assignment model which obviates path enumeration. Transportation Research 5, 83-113.

Guo, Z. (2011). Mind the map! The impact of transit maps on path choice in public transit. Transportation Research 45A, 625-639.

Guo, Z. and Wilson, N.H.M. (2011). Assessing the cost of transfer inconvenience in public transport systems: a case study of the London Underground. Transportation Research 45A, 91-104.

Hochmair, H. (2009). The influence of map design on route choice from public transportation maps in urban areas. The Cartographic Journal 46, 242-256.

Jankowski, P., Andrienko, N. and Andrienko, G. (2001). Map-centred exploratory approach to multiple criteria spatial decision making. International Journal of Geographical Information Science 15, 101-127.

Liu, Y., Bunker, J. and Ferreira, L. (2010). Transit users' route-choice modelling in transit assignment: a review. Transport Reviews 30, 753-769.

Ortúzar, J. de D. and L. Willumsen. (2011). Modelling Transport. John Wiley and Sons, Chichester.

Ovenden, M. (2007). Transit Maps of the World. Penguin Group, New York.

Prato, C. G. (2009) Route choice modeling: past, present and future research directions. Journal of Choice Modelling 2, 65-100.

Prashker J. N. and Bekhor, S. (2004). Route choice models used in the stochastic user equilibrium problem: a review. Transport Reviews 24, 437-463.

Ramming, M. S. (2001). Network Knowledge and Route Choice. Unpublished Ph.D. Thesis, Massachusetts Institute of Technology, Cambridge, MA.

Raveau, S., Muñoz, J. C. and de Grange, L. (2011). A topological route choice model for Metro. Transportation Research 45A, 138-147.

Train, K. (2009). Discrete Choice Methods with Simulation. Cambridge University Press, Cambridge.