Solution Non-uniqueness of Intersection Models for First-Order Macroscopic Dynamic Network Loading

Ruben Corthout *

Department of Mechanical Engineering, CIB/Traffic & Infrastructure,

KULeuven, Leuven, Belgium

Gunnar Flötteröd

Division of Traffic and Logistics,

KTH – Royal Institute of Technology, Stockholm, Sweden

Francesco Viti

Department of Mechanical Engineering, CIB/Traffic & Infrastructure,

KULeuven, Leuven, Belgium

Chris M.J. Tampère

Department of Mechanical Engineering, CIB/Traffic & Infrastructure,

KULeuven, Leuven, Belgium

* Email: ruben.corthout@cib.kuleuven.be

1 Introduction

In first-order macroscopic dynamic network loading (DNL) models, the link model provides the demand S_i of incoming links i (the maximum flow (veh/h) that can be sent) and the supply R_j of outgoing links j (the maximum flow that can be received) as constraints to the flow solution of the intersection (or node) model. In urban environments, the intersection model should additionally impose internal supply constraints due to limited supply of conflict points on the intersection itself.

This paper presents a general methodology to include internal supply constraints into the DNL intersection model, analogous to how the external R_j are typically treated. Most importantly, it is shown that – under realistic behavioral assumptions - the solution of the intersection model may be non-unique.

2 State-of-the-art

[1] shows that virtually all existing models fail to comply with some fundamental requirements for macroscopic DNL intersection models. [1] and [2] present intersection

models that do comply with these requirements. The details of these requirements and models are provided in the full paper. Here, we only explain what is essential to understand the remainder.

The intersection models of [1] and [2] find the flows q_i – which fully determine all q_{ij} given the turning fractions f_{ij} from each i to each j – given the external constraints S_i and R_j. Finite, strictly positive priority parameters α_{ij} (together with f_{ij}) determine the strength of each competing i for the supply in j. This core modeling principle is shared by most other existing DNL intersection models. This makes for the general nature of the findings in this paper.

Although the internal supply constraints are largely responsible for the traffic problems in many regional and urban networks, they are rarely considered in state-of-the-art models. Intersection models that do include internal supply constraints are presented by [2]-[7]. However, only [2] complies with all of the modeling requirements of [1]. [2] observes the possibility of solution non-uniqueness in the presence of internal supply constraints. In Section 4 of this paper, this problem is more generally analyzed and a uniqueness condition is found. First, Section 3 briefly discusses the general introduction of these internal constraints into the intersection model.

3 Internal supply constraints

The merging conflicts into outgoing links are typically considered as external conflicts (captured by the supplies R_j). At roundabouts, however, internal merging conflicts are to be considered at the entering points of the roundabout arcs. Other internal conflicts include crossing conflicts and traffic controls (the latter is not considered here).

We generally introduce the following formulation of the internal supply constraint function for an internal conflict point k:

$$\widehat{N}_k(\mathbf{q}) \leq 0 \tag{1}$$

where $\mathbf{q}$ is the vector of all q_i. Further detailing (1) is a complex problem in itself. [8] provides more elaboration. Here, it is important to note that:

- The internal supply constraints depend on the resulting flows q_i, not the demands S_i.
- All q_i that compete in k may be restricted; in contrast to formulations of restricted minor flows as functions of unrestricted prioritized flows.

The latter implies that (1) is analogous to external supply constraints. Hence, analogously, the distribution of the internal supply in k over all competing i can be expressed via priority parameters α_{ik}. This is illustrated in the following section.

4 Solution non-uniqueness

Figure 1 depicts a 2x4 example, where the solution is bounded by the demand constraints and two internal, crossing conflicts[1]. The priority rules for such conflicts typically state that the left-turning movements have to yield to the straight movements. This is modeled by setting $\alpha_{23} = \alpha_{14} = 1$ and α_{13} and α_{24} arbitrarily small.

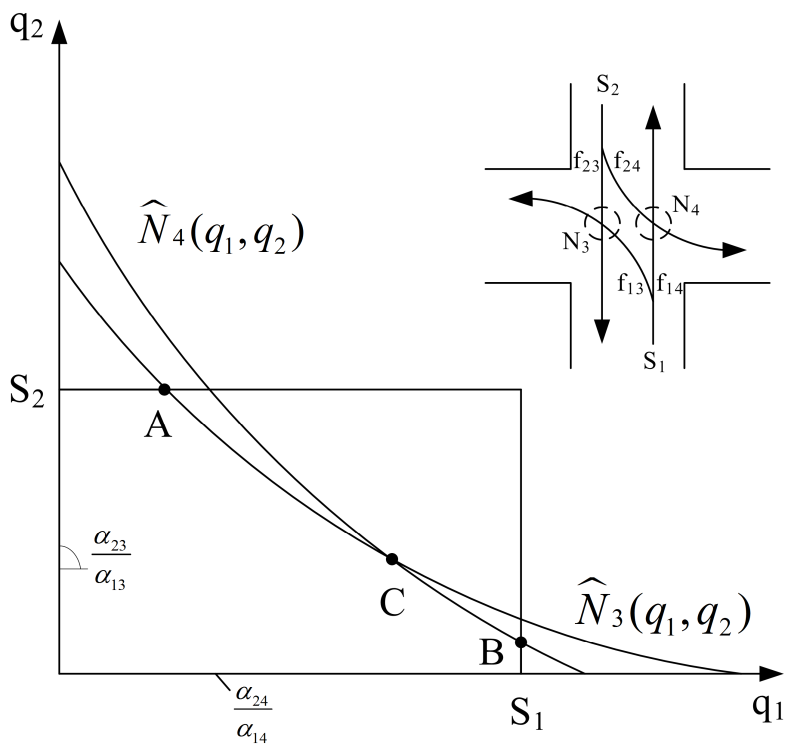

Figure 1: Solution non-uniqueness of the intersection model

Now, this problem has three possible solutions (A, B, and C). In A, the solution (q_1, q_2) is dictated by $\widehat{N}_3$, with q_2 having priority ($q_2 = S_2$), leaving the remaining supply for q_1.

[1] The internal supply constraint functions are not necessarily linear. For generality, we depict non-linear constraints that form an upper bound to the solution (q_1, q_2).

Likewise, B results from $\widehat{N}_4$. Even in C the model definitions are met, with q_1 and q_2 being constrained by $\widehat{N}_3$ and $\widehat{N}_4$ respectively, each leaving the remaining supply for the other flow. In summary, realistic behavioral assumptions (corresponding to the priority rules) can lead to multiple solutions.

We identify the source of the solution non-uniqueness as the fact that flows q_i are faced with multiple, ambiguous priority ratios in the distribution of different supplies in outgoing links j or internal conflicts k. Given arbitrary boundary conditions (demands, supplies and turning fractions) the following is a sufficient condition for solution uniqueness:

$$\begin{aligned} &\exists \alpha_i, \beta_j, \beta_k: \quad \alpha_{ij} = \alpha_i \beta_j \quad \forall i, j \\ &\qquad\qquad\qquad\ \alpha_{ik} = \alpha_i \beta_k \quad \forall i, k \\ &with: \quad \alpha_i > 0 \quad \forall i \\ &\qquad\quad\ \ \beta_j > 0 \quad \forall j \\ &\qquad\quad\ \ \beta_k > 0 \quad \forall k \end{aligned} \tag{2}$$

Condition (2) implies that the same priority ratio is used in the distribution of any supply. Specification (2) is also a necessary condition for the vast majority of real intersection topologies where the flows of at least two incoming links are mutually dependent in at least two common supply constraints (as in the example in Figure 1). Proof is provided in the full paper.

5 Conclusion

In this paper, internal supply constraints are introduced that generally extend (the main principles of) most existing DNL intersection models. It is found that only under condition (2) solution uniqueness is guaranteed. Existing models that do not meet this condition are [5] and [9]-[11].

However, condition (2) appears behaviorally unrealistic when introducing internal supply constraints. Indeed, it is (often) in contradiction with how one would naturally define the priority parameters to govern the distribution of internal supplies (see the example in Figure 1). Blindly imposing single-valued priority ratios without any consideration of the ambiguity that seems inherent to reality is therefore unadvisable. Hence, at least in deterministic DNL modeling, a transformation of the non-unique solutions into one prevailing flow pattern is needed. We distinguish two types of approaches:

- pre-processing of the priority parameters so that the model produces a unique solution
- computing non-unique solutions that result from ambiguous priority parameters and then post-processing these into one solution

Preferably, the decision of how to treat the solution non-uniqueness, which appears in the model under realistic behavioral assumptions, should be supported by empirical research.

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